Computer algebra independent integration tests

4-Trig-functions/4.2-Cosine/4.2.12-e-x-^m-a+b-cos-c+d-x^n-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [99]. This is test number [85].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (99)	% 0.00 (0)
Mathematica	% 100.00 (99)	% 0.00 (0)
Maple	% 87.88 (87)	% 12.12 (12)
Maxima	% 81.82 (81)	% 18.18 (18)
Fricas	% 91.92 (91)	% 8.08 (8)
Sympy	% 33.33 (33)	% 66.67 (66)
Giac	% 51.52 (51)	% 48.48 (48)
Mupad	% 30.30 (30)	% 69.70 (69)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

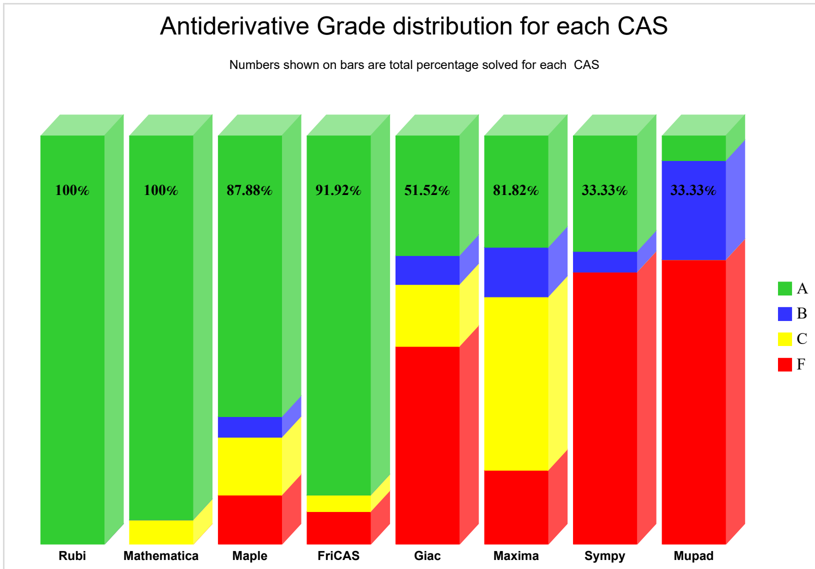
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

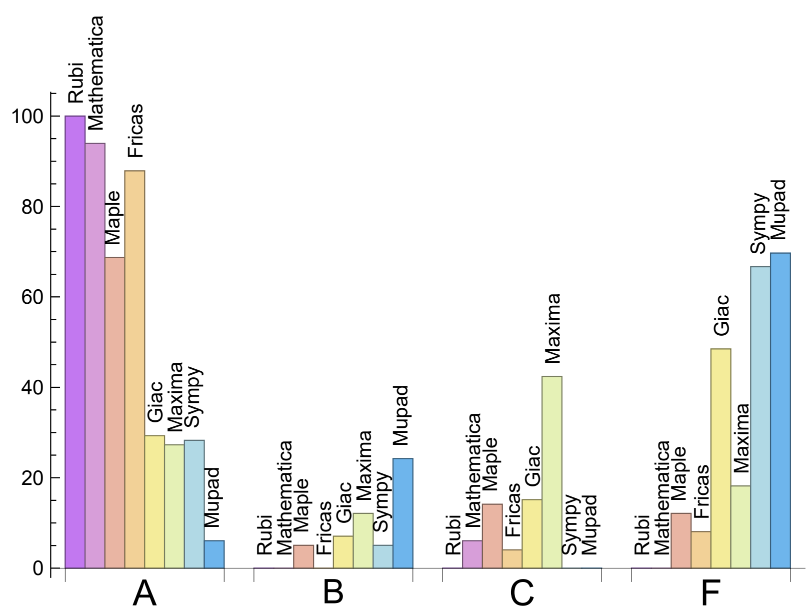
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	93.94	0.00	6.06	0.00
Maple	68.69	5.05	14.14	12.12
Maxima	27.27	12.12	42.42	18.18
Fricas	87.88	0.00	4.04	8.08
Sympy	28.28	5.05	0.00	66.67
Giac	29.29	7.07	15.15	48.48
Mupad	6.06	24.24	0.00	69.70

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	12	100.00 %	0.00 %	0.00 %
Maxima	18	100.00 %	0.00 %	0.00 %
Fricas	8	100.00 %	0.00 %	0.00 %
Sympy	66	90.91 %	9.09 %	0.00 %
Giac	48	100.00 %	0.00 %	0.00 %
Mupad	69	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

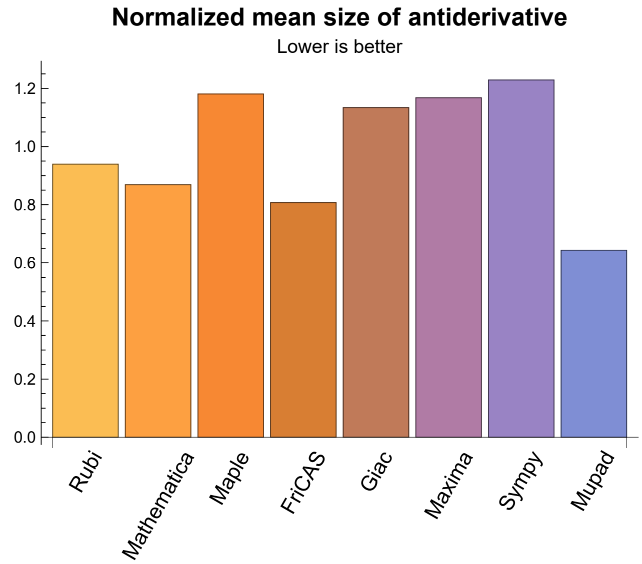
1.3 Performance

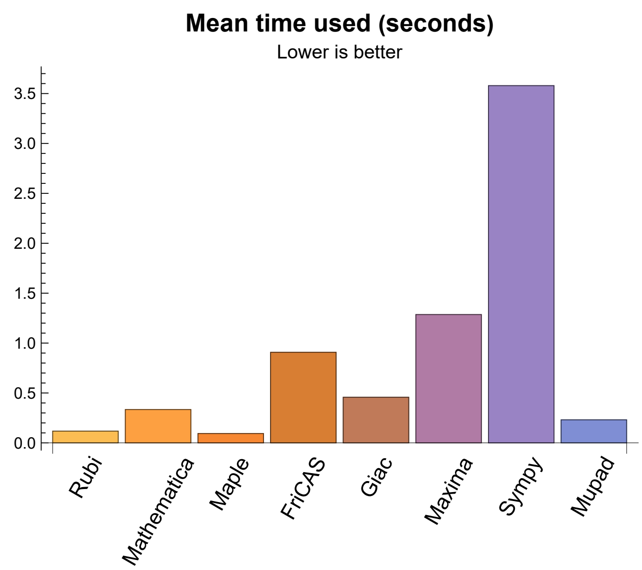
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	102.90	0.94	85.00	1.00
Mathematica	0.33	89.70	0.87	81.00	0.95
Maple	0.09	144.82	1.18	65.00	0.88
Maxima	1.29	114.11	1.17	83.00	1.00
Fricas	0.91	76.38	0.81	62.00	0.79
Sympy	3.58	86.79	1.23	46.00	1.06
Giac	0.46	112.63	1.13	55.00	0.99
Mupad	0.23	28.03	0.64	25.00	0.76

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{63, 64, 66, 68, 88, 89}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {67, 98}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

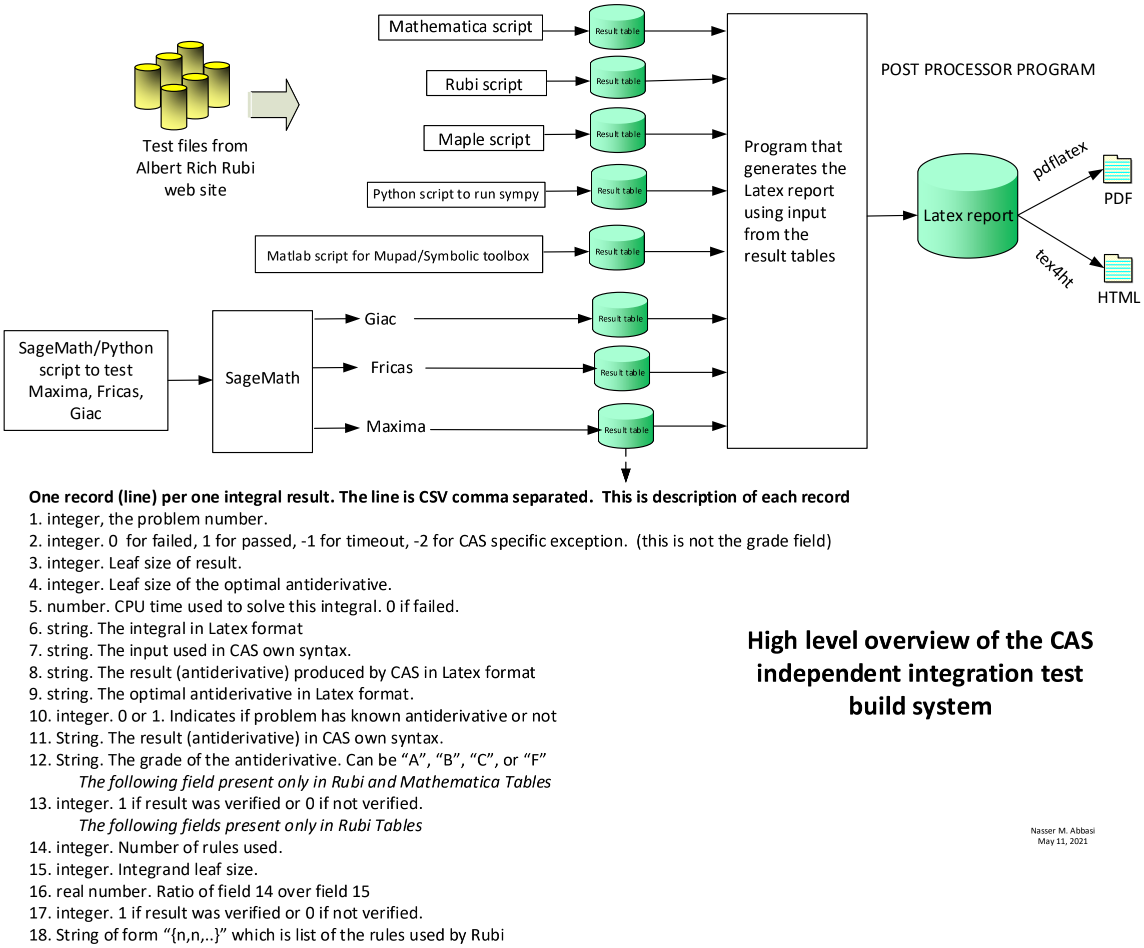
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 96, 97 }

B grade: { }

C grade: { 90, 93, 94, 95, 98, 99 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 16, 17, 18, 20, 22, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 68, 69, 70, 71, 72, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 97 }

B grade: { 90, 93, 94, 95, 96 }

C grade: { 12, 14, 19, 21, 23, 24, 25, 26, 27, 28, 73, 76, 98, 99 }

F grade: { 29, 30, 31, 32, 33, 34, 65, 67, 74, 75, 77, 78 }

2.1.4 Maxima

A grade: { 1, 3, 8, 10, 15, 17, 22, 29, 33, 34, 37, 43, 45, 46, 47, 48, 61, 62, 63, 64, 66, 68, 88, 89, 91, 92, 97 }

B grade: { 23, 24, 25, 26, 27, 28, 30, 31, 32, 90, 95, 96 }

C grade: { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 35, 36, 38, 39, 40, 41, 42, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 69, 70, 71, 72, 85, 86, 87 }

F grade: { 65, 67, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 93, 94, 98, 99 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 68, 69, 70, 71, 72, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 97 }

B grade: { }

C grade: { 93, 94, 98, 99 }

F grade: { 65, 67, 73, 74, 75, 76, 77, 78 }

2.1.6 Sympy

A grade: { 1, 3, 4, 8, 10, 11, 15, 17, 18, 22, 36, 37, 38, 39, 43, 46, 47, 48, 62, 63, 64, 66, 88, 89, 90, 91, 92, 97 }

B grade: { 2, 9, 16, 45, 61 }

C grade: { }

F grade: { 5, 6, 7, 12, 13, 14, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 40, 41, 42, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 93, 94, 95, 96, 98, 99 }

2.1.7 Giac

A grade: { 1, 3, 5, 8, 10, 12, 15, 17, 19, 22, 37, 38, 45, 46, 47, 48, 61, 62, 63, 64, 66, 68, 88, 89, 90, 91, 92, 96, 97 }

B grade: { 7, 14, 21, 35, 36, 39, 95 }

C grade: { 2, 4, 9, 11, 16, 18, 49, 50, 51, 55, 56, 57, 85, 86, 87 }

F grade: { 6, 13, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 43, 44, 52, 53, 54, 58, 59, 60, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 93, 94, 98, 99 }

2.1.8 Mupad

A grade: { 63, 64, 66, 68, 88, 89 }

B grade: { 1, 3, 4, 8, 10, 15, 17, 22, 37, 38, 39, 42, 43, 45, 46, 47, 48, 61, 62, 85, 86, 87, 92, 97 }

C grade: { }

F grade: { 2, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 44, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 90, 91, 93, 94, 95, 96, 98, 99 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	29	31	27	27	36	27	27
normalized size	1	1.00	0.85	0.91	0.79	0.79	1.06	0.79	0.79
time (sec)	N/A	0.032	0.049	0.023	1.111	0.569	0.769	0.376	0.101

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	58	67	72	209	135	-1
normalized size	1	1.00	0.90	0.64	0.74	0.79	2.30	1.48	-0.01
time (sec)	N/A	0.069	0.149	0.020	2.053	0.639	1.944	0.540	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	19	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	1.27	0.87	0.87
time (sec)	N/A	0.014	0.003	0.020	0.428	0.855	0.170	0.380	0.060

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	57	44	48	61	61	95	51
normalized size	1	1.00	0.81	0.63	0.69	0.87	0.87	1.36	0.73
time (sec)	N/A	0.021	0.091	0.019	0.806	0.822	0.442	0.394	0.357

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	22	43	29	0	21	-1
normalized size	1	1.00	0.96	0.88	1.72	1.16	0.00	0.84	-0.04
time (sec)	N/A	0.029	0.050	0.024	1.142	0.882	0.000	0.418	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	57	73	70	0	0	-1
normalized size	1	1.00	1.01	0.71	0.91	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.181	0.026	1.577	0.872	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	39	48	57	0	87	-1
normalized size	1	1.00	1.00	0.93	1.14	1.36	0.00	2.07	-0.02
time (sec)	N/A	0.089	0.071	0.022	1.075	0.930	0.000	0.487	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	40	42	42	45	78	55	41
normalized size	1	1.00	0.78	0.82	0.82	0.88	1.53	1.08	0.80
time (sec)	N/A	0.052	0.114	0.040	0.898	0.940	1.548	0.429	0.148

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	87	63	89	84	201	118	-1
normalized size	1	1.00	0.96	0.69	0.98	0.92	2.21	1.30	-0.01
time (sec)	N/A	0.097	0.171	0.035	1.226	0.750	2.598	0.824	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	34	23	28	60	26	22
normalized size	1	1.00	0.87	1.10	0.74	0.90	1.94	0.84	0.71
time (sec)	N/A	0.029	0.043	0.028	0.591	1.326	0.407	0.400	0.272

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	67	45	70	59	56	82	-1
normalized size	1	1.00	0.96	0.64	1.00	0.84	0.80	1.17	-0.01
time (sec)	N/A	0.044	0.054	0.046	1.455	0.950	0.800	0.315	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	68	51	39	0	35	-1
normalized size	1	1.00	0.92	1.84	1.38	1.05	0.00	0.95	-0.03
time (sec)	N/A	0.052	0.062	0.146	1.306	0.898	0.000	0.410	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	62	83	66	0	0	-1
normalized size	1	1.00	1.00	0.82	1.09	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.161	0.037	1.544	0.889	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	50	89	61	65	0	107	-1
normalized size	1	1.00	0.88	1.56	1.07	1.14	0.00	1.88	-0.02
time (sec)	N/A	0.118	0.119	0.165	1.738	0.671	0.000	0.398	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	55	66	58	58	92	58	66
normalized size	1	1.00	0.70	0.84	0.73	0.73	1.16	0.73	0.84
time (sec)	N/A	0.074	0.138	0.040	1.047	0.865	2.775	0.366	0.410

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	160	130	143	148	439	259	-1
normalized size	1	1.00	0.85	0.69	0.76	0.79	2.34	1.38	-0.01
time (sec)	N/A	0.182	0.425	0.036	0.963	0.661	4.380	0.522	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	27	25	44	26	28
normalized size	1	1.00	1.00	0.79	0.82	0.76	1.33	0.79	0.85
time (sec)	N/A	0.028	0.016	0.028	0.473	0.717	0.768	0.369	0.289

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	116	101	112	121	129	185	-1
normalized size	1	1.00	0.76	0.66	0.73	0.79	0.84	1.21	-0.01
time (sec)	N/A	0.089	0.231	0.035	1.292	1.372	1.139	0.600	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	125	89	63	0	47	-1
normalized size	1	1.00	0.91	2.27	1.62	1.15	0.00	0.85	-0.02
time (sec)	N/A	0.083	0.094	0.171	1.509	1.524	0.000	0.608	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	166	128	151	136	0	0	-1
normalized size	1	1.00	0.99	0.76	0.90	0.81	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.640	0.043	3.088	0.861	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	90	162	98	108	0	185	-1
normalized size	1	1.00	0.99	1.78	1.08	1.19	0.00	2.03	-0.01
time (sec)	N/A	0.201	0.175	0.196	1.061	0.682	0.000	0.463	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	50	55	51	94	52	55
normalized size	1	1.00	0.81	0.75	0.82	0.76	1.40	0.78	0.82
time (sec)	N/A	0.054	0.086	0.030	1.098	0.937	7.818	0.373	0.745

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	229	156	58	0	0	-1
normalized size	1	1.00	1.02	2.06	1.41	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.195	0.109	2.188	0.880	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	111	290	158	56	0	0	-1
normalized size	1	1.00	1.00	2.61	1.42	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.152	0.077	1.335	0.603	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	89	290	138	44	0	0	-1
normalized size	1	1.00	1.10	3.58	1.70	0.54	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.084	0.059	1.159	0.988	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	89	338	135	44	0	0	-1
normalized size	1	1.00	1.10	4.17	1.67	0.54	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.066	0.079	2.843	0.953	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	114	338	133	56	0	0	-1
normalized size	1	1.00	1.16	3.45	1.36	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.173	0.071	1.336	0.796	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	117	358	133	61	0	0	-1
normalized size	1	1.00	1.12	3.44	1.28	0.59	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.154	0.061	1.802	0.915	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	142	0	174	78	0	0	-1
normalized size	1	1.00	1.08	0.00	1.32	0.59	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.404	0.197	1.753	1.054	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	142	0	182	77	0	0	-1
normalized size	1	1.00	1.08	0.00	1.38	0.58	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.328	0.164	1.971	0.872	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	122	0	156	50	0	0	-1
normalized size	1	1.00	1.22	0.00	1.56	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.218	0.154	2.598	0.727	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	120	0	159	50	0	0	-1
normalized size	1	1.00	1.25	0.00	1.66	0.52	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.221	0.183	2.447	0.892	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	137	0	143	59	0	0	-1
normalized size	1	1.00	1.17	0.00	1.22	0.50	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.337	0.136	1.869	0.975	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	137	0	145	63	0	0	-1
normalized size	1	1.00	1.18	0.00	1.25	0.54	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.343	0.147	0.910	0.659	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	39	57	45	0	132	-1
normalized size	1	1.00	1.00	1.26	1.84	1.45	0.00	4.26	-0.03
time (sec)	N/A	0.072	0.032	0.051	1.902	1.153	0.000	0.345	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	43	28	15	41	-1
normalized size	1	1.00	1.00	1.05	2.15	1.40	0.75	2.05	-0.05
time (sec)	N/A	0.027	0.042	0.041	2.010	1.433	0.966	0.413	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	15	15	15	13
normalized size	1	1.00	1.00	1.08	1.00	1.15	1.15	1.15	1.00
time (sec)	N/A	0.014	0.003	0.021	2.000	0.925	0.975	0.356	0.264

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	42	51	33	31	49	30
normalized size	1	1.00	0.97	1.40	1.70	1.10	1.03	1.63	1.00
time (sec)	N/A	0.025	0.051	0.039	1.695	0.746	1.803	0.393	0.279

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	92	50	43	46	107	47
normalized size	1	1.00	1.00	2.00	1.09	0.93	1.00	2.33	1.02
time (sec)	N/A	0.047	0.005	0.043	1.227	0.713	3.041	0.428	0.372

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	80	57	127	73	0	0	-1
normalized size	1	1.00	1.01	0.72	1.61	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.133	0.025	2.371	0.802	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	22	43	29	0	0	-1
normalized size	1	1.00	0.96	0.88	1.72	1.16	0.00	0.00	-0.04
time (sec)	N/A	0.028	0.042	0.025	1.336	0.568	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	62	48	98	65	0	0	55
normalized size	1	1.00	0.84	0.65	1.32	0.88	0.00	0.00	0.74
time (sec)	N/A	0.032	0.107	0.023	1.242	0.694	0.000	0.000	0.406

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	17	22	0	13
normalized size	1	1.00	1.00	0.93	0.87	1.13	1.47	0.00	0.87
time (sec)	N/A	0.015	0.003	0.022	0.665	0.993	2.816	0.000	0.268

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	64	74	84	0	0	-1
normalized size	1	1.00	0.91	0.66	0.76	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.157	0.023	1.288	0.680	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	14	12	13	39	12	12
normalized size	1	1.00	0.95	0.74	0.63	0.68	2.05	0.63	0.63
time (sec)	N/A	0.021	0.029	0.044	0.300	0.534	0.263	0.379	0.367

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.009	0.002	0.023	0.904	0.778	0.249	0.382	0.033

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	20	16	16
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.91	0.73	0.73
time (sec)	N/A	0.011	0.015	0.026	0.849	0.623	0.239	0.489	0.278

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	31	34	23	24	51	23	23
normalized size	1	1.00	0.86	0.94	0.64	0.67	1.42	0.64	0.64
time (sec)	N/A	0.021	0.024	0.025	0.656	1.261	0.254	0.406	0.345

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	165	196	135	145	0	241	-1
normalized size	1	1.00	0.70	0.83	0.57	0.62	0.00	1.03	-0.00
time (sec)	N/A	0.348	0.578	0.035	0.808	0.597	0.000	2.210	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	141	131	111	118	0	193	-1
normalized size	1	1.00	0.83	0.78	0.66	0.70	0.00	1.14	-0.01
time (sec)	N/A	0.196	0.354	0.029	1.238	0.768	0.000	0.496	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	94	64	73	78	0	143	-1
normalized size	1	1.00	0.95	0.65	0.74	0.79	0.00	1.44	-0.01
time (sec)	N/A	0.113	0.154	0.022	1.027	0.894	0.000	0.517	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	78	74	96	0	0	-1
normalized size	1	1.00	1.00	0.71	0.67	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.253	0.030	1.961	1.262	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	180	129	76	134	0	0	-1
normalized size	1	1.00	0.98	0.70	0.41	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.238	0.249	0.035	1.622	0.809	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	238	180	76	164	0	0	-1
normalized size	1	1.00	0.95	0.72	0.30	0.66	0.00	0.00	-0.00
time (sec)	N/A	0.337	0.336	0.030	1.288	0.650	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	174	219	161	184	0	224	-1
normalized size	1	1.00	0.56	0.71	0.52	0.59	0.00	0.72	-0.00
time (sec)	N/A	0.360	0.645	0.062	1.237	0.979	0.000	1.249	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	148	145	137	144	0	176	-1
normalized size	1	1.00	0.68	0.67	0.63	0.66	0.00	0.81	-0.00
time (sec)	N/A	0.251	0.438	0.060	1.237	0.732	0.000	0.451	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	103	67	95	90	0	124	-1
normalized size	1	1.00	1.01	0.66	0.93	0.88	0.00	1.22	-0.01
time (sec)	N/A	0.170	0.184	0.057	1.266	1.469	0.000	0.532	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	116	87	87	100	0	0	-1
normalized size	1	1.00	1.00	0.75	0.75	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.246	0.056	1.297	0.872	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	185	146	89	154	0	0	-1
normalized size	1	1.00	0.81	0.64	0.39	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.250	0.274	0.059	1.715	0.856	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	249	207	89	192	0	0	-1
normalized size	1	1.00	0.76	0.63	0.27	0.59	0.00	0.00	-0.00
time (sec)	N/A	0.351	0.384	0.064	1.829	1.237	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	66	58	47	48	513	47	62
normalized size	1	1.00	0.77	0.67	0.55	0.56	5.97	0.55	0.72
time (sec)	N/A	0.068	0.098	0.037	0.851	1.042	2.605	0.386	0.496

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.010	0.002	0.022	0.985	0.977	75.626	0.357	0.427

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	1.010	0.985	0.000	0.657	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.022	1.286	0.895	0.000	1.537	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	89	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.174	1.051	0.000	1.702	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.049	0.933	0.914	0.000	0.935	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	149	0	0	0	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.428	1.000	0.000	0.910	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	1.142	0.943	0.000	1.004	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	25	90	35	0	0	-1
normalized size	1	1.00	0.92	0.96	3.46	1.35	0.00	0.00	-0.04
time (sec)	N/A	0.034	0.066	0.027	1.741	1.311	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	45	99	48	0	0	-1
normalized size	1	1.00	0.86	1.05	2.30	1.12	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.094	0.054	2.361	0.652	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	53	52	179	74	0	0	-1
normalized size	1	1.00	0.79	0.78	2.67	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.127	0.053	1.863	0.820	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	66	77	188	87	0	0	-1
normalized size	1	1.00	0.84	0.97	2.38	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.137	0.054	2.619	0.900	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	92	75	0	0	0	0	-1
normalized size	1	1.00	1.11	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.082	0.119	0.000	1.136	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	94	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.246	0.231	0.000	0.934	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	173	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.242	0.284	0.000	0.815	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	115	111	0	0	0	0	-1
normalized size	1	1.00	1.10	1.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.205	0.160	0.000	0.993	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	129	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.520	0.188	0.000	0.942	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	221	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.559	0.253	0.000	0.681	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	45	0	62	0	0	-1
normalized size	1	1.00	0.96	0.96	0.00	1.32	0.00	0.00	-0.02
time (sec)	N/A	0.086	0.096	0.041	0.000	0.838	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	53	65	0	72	0	0	-1
normalized size	1	1.00	0.77	0.94	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.192	0.059	0.000	0.962	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	95	101	0	117	0	0	-1
normalized size	1	1.00	0.84	0.89	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.238	0.058	0.000	1.038	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	65	0	90	0	0	-1
normalized size	1	1.00	0.90	0.83	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.137	0.043	0.000	0.923	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	82	89	0	106	0	0	-1
normalized size	1	1.00	0.86	0.94	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.224	0.059	0.000	0.702	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	141	144	0	167	0	0	-1
normalized size	1	1.00	0.85	0.87	0.00	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.253	0.360	0.067	0.000	0.924	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	76	131	256	112	0	159	80
normalized size	1	1.00	0.77	1.32	2.59	1.13	0.00	1.61	0.81
time (sec)	N/A	0.068	0.285	0.027	3.190	0.815	0.000	0.457	0.146

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	63	197	63	0	119	39
normalized size	1	1.00	0.89	1.34	4.19	1.34	0.00	2.53	0.83
time (sec)	N/A	0.032	0.049	0.026	1.870	0.990	0.000	0.550	0.071

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	36	84	40	0	55	32
normalized size	1	1.00	1.00	1.24	2.90	1.38	0.00	1.90	1.10
time (sec)	N/A	0.006	0.008	0.020	0.823	0.951	0.000	0.400	0.062

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.008	2.647	0.150	0.000	0.921	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.009	5.219	0.133	0.000	0.655	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	224	825	672	103	269	480	-1
normalized size	1	1.00	0.65	2.38	1.94	0.30	0.78	1.39	-0.00
time (sec)	N/A	0.305	0.831	0.050	0.645	1.750	1.647	0.454	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	71	299	263	67	151	166	-1
normalized size	1	1.00	0.43	1.79	1.57	0.40	0.90	0.99	-0.01
time (sec)	N/A	0.136	0.283	0.043	0.365	0.796	0.517	0.474	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	61	60	42	66	42	42
normalized size	1	1.00	0.89	1.13	1.11	0.78	1.22	0.78	0.78
time (sec)	N/A	0.027	0.074	0.021	0.300	0.916	0.403	0.329	0.344

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	145	271	0	149	0	0	-1
normalized size	1	1.00	1.15	2.15	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.239	0.671	0.058	0.000	0.924	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	240	714	0	210	0	0	-1
normalized size	1	1.00	1.30	3.88	0.00	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.349	1.205	0.081	0.000	0.975	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	537	537	382	1809	1349	182	0	1104	-1
normalized size	1	1.00	0.71	3.37	2.51	0.34	0.00	2.06	-0.00
time (sec)	N/A	0.511	1.098	0.050	0.750	0.951	0.000	0.496	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	117	655	523	110	0	370	-1
normalized size	1	1.00	0.45	2.51	2.00	0.42	0.00	1.42	-0.00
time (sec)	N/A	0.231	0.425	0.045	0.696	0.773	0.000	1.259	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	65	131	118	57	94	81	68
normalized size	1	1.00	0.76	1.54	1.39	0.67	1.11	0.95	0.80
time (sec)	N/A	0.055	0.113	0.034	0.436	1.093	1.161	0.429	0.382

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	243	279	0	287	0	0	-1
normalized size	1	1.00	1.04	1.19	0.00	1.23	0.00	0.00	-0.00
time (sec)	N/A	0.468	0.428	0.067	0.000	1.089	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	138	933	0	406	0	0	-1
normalized size	1	1.00	0.42	2.81	0.00	1.22	0.00	0.00	-0.00
time (sec)	N/A	0.741	0.602	0.097	0.000	0.791	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [35] had the largest ratio of [.6250]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	12	0.250
2	A	4	4	1.00	12	0.333
3	A	2	2	1.00	10	0.200
4	A	3	3	1.00	8	0.375
5	A	3	3	1.00	12	0.250
6	A	4	4	1.00	12	0.333
7	A	5	5	1.00	12	0.417
8	A	3	3	1.00	14	0.214
9	A	6	5	1.00	14	0.357
10	A	3	3	1.00	12	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
11	A	5	4	1.00	10	0.400
12	A	5	4	1.00	14	0.286
13	A	6	6	1.00	14	0.429
14	A	7	6	1.00	14	0.429
15	A	4	4	1.00	14	0.286
16	A	10	5	1.00	14	0.357
17	A	3	2	1.00	12	0.167
18	A	8	4	1.00	10	0.400
19	A	8	4	1.00	14	0.286
20	A	9	5	1.00	14	0.357
21	A	12	6	1.00	14	0.429
22	A	3	2	1.00	12	0.167
23	A	4	3	1.00	14	0.214
24	A	4	3	1.00	14	0.214
25	A	3	2	1.00	14	0.143
26	A	3	2	1.00	14	0.143
27	A	4	3	1.00	14	0.214
28	A	4	3	1.00	14	0.214
29	A	7	5	1.00	16	0.312
30	A	7	5	1.00	16	0.312
31	A	6	4	1.00	16	0.250
32	A	6	4	1.00	16	0.250
33	A	7	5	1.00	16	0.312
34	A	7	6	1.00	16	0.375
35	A	5	5	1.00	8	0.625
36	A	3	3	1.00	12	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
37	A	2	2	1.00	12	0.167
38	A	3	3	1.00	12	0.250
39	A	4	3	1.00	12	0.250
40	A	5	5	1.00	8	0.625
41	A	3	3	1.00	12	0.250
42	A	4	4	1.00	12	0.333
43	A	2	2	1.00	12	0.167
44	A	5	5	1.00	12	0.417
45	A	3	3	1.00	14	0.214
46	A	2	2	1.00	12	0.167
47	A	3	3	1.00	6	0.500
48	A	3	3	1.00	8	0.375
49	A	13	7	1.00	16	0.438
50	A	10	7	1.00	16	0.438
51	A	7	7	1.00	16	0.438
52	A	8	7	1.00	16	0.438
53	A	11	7	1.00	16	0.438
54	A	14	7	1.00	16	0.438
55	A	15	10	1.00	18	0.556
56	A	12	9	1.00	18	0.500
57	A	9	8	1.00	18	0.444
58	A	10	9	1.00	18	0.500
59	A	12	10	1.00	18	0.556
60	A	16	9	1.00	18	0.500
61	A	7	5	1.00	8	0.625
62	A	2	2	1.00	12	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
63	A	0	0	0.00	0	0.000
64	A	0	0	0.00	0	0.000
65	A	3	3	1.00	20	0.150
66	A	0	0	0.00	0	0.000
67	A	5	5	1.00	22	0.227
68	A	0	0	0.00	0	0.000
69	A	3	3	1.00	12	0.250
70	A	5	4	1.00	14	0.286
71	A	8	4	1.00	14	0.286
72	A	8	4	1.00	14	0.286
73	A	3	2	1.00	8	0.250
74	A	5	3	1.00	10	0.300
75	A	8	3	1.00	10	0.300
76	A	3	2	1.00	12	0.167
77	A	5	3	1.00	14	0.214
78	A	8	3	1.00	14	0.214
79	A	5	5	1.00	16	0.312
80	A	7	6	1.00	18	0.333
81	A	12	6	1.00	18	0.333
82	A	6	5	1.00	16	0.312
83	A	8	6	1.00	18	0.333
84	A	14	6	1.00	18	0.333
85	A	7	6	1.00	12	0.500
86	A	5	4	1.00	10	0.400
87	A	1	1	1.00	8	0.125
88	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
89	A	0	0	0.00	0	0.000
90	A	14	3	1.00	18	0.167
91	A	8	3	1.00	16	0.188
92	A	3	3	1.00	14	0.214
93	A	8	4	1.00	18	0.222
94	A	10	6	1.00	18	0.333
95	A	20	4	1.00	18	0.222
96	A	11	4	1.00	16	0.250
97	A	4	3	1.00	14	0.214
98	A	11	4	1.00	18	0.222
99	A	13	6	1.00	18	0.333

Chapter 3

Listing of integrals

3.1 $\int x^3 \cos\left(a + bx^2\right) dx$

Optimal. Leaf size=34

$$\frac{\cos\left(a + bx^2\right)}{2b^2} + \frac{x^2 \sin\left(a + bx^2\right)}{2b}$$

```
[Out] 1/2*cos(b*x^2+a)/b^2+1/2*x^2*sin(b*x^2+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3380, 3296, 2638}

$$\frac{\cos\left(a + bx^2\right)}{2b^2} + \frac{x^2 \sin\left(a + bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[a + b*x^2],x]
```

```
[Out] Cos[a + b*x^2]/(2*b^2) + (x^2*Sin[a + b*x^2])/(2*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^3 \cos\left(a+bx^2\right) dx &= \frac{1}{2}\operatorname{Subst}\left(\int x\cos(a+bx)\,dx, x, x^2\right)\\ &= \frac{x^2\sin\left(a+bx^2\right)}{2b} - \frac{\operatorname{Subst}\left(\int \sin(a+bx)\,dx, x, x^2\right)}{2b}\\ &= \frac{\cos\left(a+bx^2\right)}{2b^2} + \frac{x^2\sin\left(a+bx^2\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.05, size = 29, normalized size = 0.85

$$\frac{bx^2\sin\left(a+bx^2\right)+\cos\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[a + b*x^2],x]
```

```
[Out] (Cos[a + b*x^2] + b*x^2*Sin[a + b*x^2])/(2*b^2)
```

fricas [A] time = 0.57, size = 27, normalized size = 0.79

$$\frac{bx^2\sin\left(bx^2+a\right)+\cos\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*sin(b*x^2 + a) + cos(b*x^2 + a))/b^2
```

giac [**A**] time = 0.38, size = 27, normalized size = 0.79

$$\frac{bx^2\sin\left(bx^2+a\right)+\cos\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(b*x^2+a),x, algorithm="giac")
[Out] 1/2*(b*x^2*sin(b*x^2 + a) + cos(b*x^2 + a))/b^2
```

maple [**A**] time = 0.02, size = 31, normalized size = 0.91

$$\frac{\cos\left(b\,x^2+a\right)}{2b^2}+\frac{x^2\sin\left(b\,x^2+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(b*x^2+a),x)
[Out] 1/2*cos(b*x^2+a)/b^2+1/2*x^2*sin(b*x^2+a)/b
```

maxima [**A**] time = 1.11, size = 27, normalized size = 0.79

$$\frac{bx^2\sin\left(bx^2+a\right)+\cos\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(b*x^2+a),x, algorithm="maxima")
[Out] 1/2*(b*x^2*sin(b*x^2 + a) + cos(b*x^2 + a))/b^2
```

mupad [**B**] time = 0.10, size = 27, normalized size = 0.79

$$\frac{\cos\left(b\,x^2+a\right)+b\,x^2\,\sin\left(b\,x^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(a + b*x^2),x)
[Out] (cos(a + b*x^2) + b*x^2*sin(a + b*x^2))/(2*b^2)
```

sympy [**A**] time = 0.77, size = 36, normalized size = 1.06

$$\begin{cases} \frac{x^2\sin\left(a+bx^2\right)}{2b}+\frac{\cos\left(a+bx^2\right)}{2b^2} & \text{for } b\neq 0 \\ \frac{x^4\cos\left(a\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(b*x**2+a),x)
```

```
[Out] Piecewise((x**2*sin(a + b*x**2)/(2*b) + cos(a + b*x**2)/(2*b**2), Ne(b, 0))
, (x**4*cos(a)/4, True))
```

3.2 $\int x^2 \cos\left(a + bx^2\right) dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{\frac{\pi}{2}} \sin(a) C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{2b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \cos(a) S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{2b^{3/2}} + \frac{x\sin\left(a+bx^2\right)}{2b}$$

```
[Out] 1/2*x*sin(b*x^2+a)/b-1/4*cos(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2
)*Pi^(1/2)/b^(3/2)-1/4*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*
Pi^(1/2)/b^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3386, 3353, 3352, 3351}

$$-\frac{\sqrt{\frac{\pi}{2}} \sin(a) \text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)}{2b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \cos(a) S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{2b^{3/2}} + \frac{x\sin\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*x^2],x]
```

```
[Out] -(Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x])/(2*b^(3/2)) - (Sqrt[Pi/
2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/(2*b^(3/2)) + (x*Sin[a + b*x^2])/
(2*b)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rubi steps

$$\begin{aligned}\int x^2\cos\left(a+bx^2\right)dx &= \frac{x\sin\left(a+bx^2\right)}{2b}-\frac{\int\sin\left(a+bx^2\right)dx}{2b}\\ &= \frac{x\sin\left(a+bx^2\right)}{2b}-\frac{\cos(a)\int\sin\left(bx^2\right)dx}{2b}-\frac{\sin(a)\int\cos\left(bx^2\right)dx}{2b}\\ &= -\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{2b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{2b^{3/2}}+\frac{x\sin\left(a+bx^2\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.15, size = 82, normalized size = 0.90

$$\frac{-\sqrt{2\pi}\sin(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{2\pi}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)+2\sqrt{b}x\sin\left(a+bx^2\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*x^2],x]
```

```
[Out] (-(Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]) - Sqrt[2*Pi]*FresnelC[
Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a] + 2*Sqrt[b]*x*Sin[a + b*x^2])/(4*b^(3/2))
```

fricas [A] time = 0.64, size = 72, normalized size = 0.79

$$-\frac{\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)S\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)+\sqrt{2}\pi\sqrt{\frac{b}{\pi}}C\left(\sqrt{2}x\sqrt{\frac{b}{\pi}}\right)\sin(a)-2bx\sin\left(bx^2+a\right)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)) + sqrt
(2)*pi*sqrt(b/pi)*fresnel_cos(sqrt(2)*x*sqrt(b/pi))*sin(a) - 2*b*x*sin(b*x^
2 + a))/b^2
```

giac [C] time = 0.54, size = 135, normalized size = 1.48

$$-\frac{i\,xe^{\left(i\,bx^2+i\,a\right)}}{4\,b}+\frac{i\,xe^{\left(-i\,bx^2-i\,a\right)}}{4\,b}-\frac{i\,\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\,x\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(i\,a)}}{8\,b\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}+\frac{i\,\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{2}\,x\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(-}}{8\,b\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*I*x*e^(I*b*x^2 + I*a)/b + 1/4*I*x*e^(-I*b*x^2 - I*a)/b - 1/8*I*sqrt(2)
*sqrt(pi)*erf(-1/2*sqrt(2)*x*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b*(-I
*b/abs(b) + 1)*sqrt(abs(b))) + 1/8*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(I
*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(I*b/abs(b) + 1)*sqrt(abs(b)))
```

maple [A] time = 0.02, size = 58, normalized size = 0.64

$$\frac{x\sin\left(b\,x^2+a\right)}{2b}-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(a)\,\mathrm{FresnelC}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{4b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(b*x^2+a),x)
```

```
[Out] 1/2*x*sin(b*x^2+a)/b-1/4/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(x*b^(1/2
)*2^(1/2)/Pi^(1/2))+sin(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2)))
```

maxima [C] time = 2.05, size = 67, normalized size = 0.74

$$\frac{8\,b^2x\sin\left(bx^2+a\right)+\sqrt{2}\,\sqrt{\pi}\left(\left(-\left(i+1\right)\,\cos(a)+\left(i-1\right)\,\sin(a)\right)\operatorname{erf}\left(\sqrt{i\,b}\,x\right)+\left(\left(i-1\right)\,\cos(a)-\left(i+1\right)\,\sin(a)\right)\operatorname{erf}}{16\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/16*(8*b^2*x*sin(b*x^2 + a) + sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)
*sin(a))*erf(sqrt(I*b)*x) + ((I - 1)*cos(a) - (I + 1)*sin(a))*erf(sqrt(-I*b
)*x))*b^(3/2))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\cos\left(b\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a + b*x^2),x)
```

```
[Out] int(x^2*cos(a + b*x^2), x)
```

sympy [B] time = 1.94, size = 209, normalized size = 2.30

$$\frac{b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\sin(a)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{b^2x^4}{4}\right)}{8\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}-\frac{\sqrt{b}\,x^3\sqrt{\frac{1}{b}}\cos(a)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{5}{4},\frac{7}{4}\end{matrix}\middle|-\frac{b^2x^4}{4}\right)}{8\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}-\frac{\sqrt{2}\sqrt{\pi}\,x^2\sqrt{\frac{1}{b}}\sin}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(b*x**2+a),x)
```

```
[Out] b**(3/2)*x**5*sqrt(1/b)*sin(a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2
, 7/4, 9/4), -b**2*x**4/4)/(8*gamma(7/4)*gamma(9/4)) - sqrt(b)*x**3*sqrt(1/
b)*cos(a)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*x*
*4/4)/(8*gamma(5/4)*gamma(7/4)) - sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*sin(a)*fr
esnels(sqrt(2)*sqrt(b)*x/sqrt(pi))/2 + sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*cos(
a)*fresnelc(sqrt(2)*sqrt(b)*x/sqrt(pi))/2
```

3.3 $\int x\cos\left(a+bx^2\right)\,dx$

Optimal. Leaf size=15

$$\frac{\sin\left(a+bx^2\right)}{2b}$$

```
[Out] 1/2*sin(b*x^2+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3380, 2637}

$$\frac{\sin\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x^2],x]
```

```
[Out] Sin[a + b*x^2]/(2*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x\cos\left(a+bx^2\right)\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int\cos(a+bx)\,dx,x,x^2\right)\\ &= \frac{\sin\left(a+bx^2\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sin\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x^2],x]
```

```
[Out] Sin[a + b*x^2]/(2*b)
```

fricas [A] time = 0.85, size = 13, normalized size = 0.87

$$\frac{\sin\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sin(b*x^2 + a)/b
```

giac [A] time = 0.38, size = 13, normalized size = 0.87

$$\frac{\sin\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sin(b*x^2 + a)/b
```

maple [A] time = 0.02, size = 14, normalized size = 0.93

$$\frac{\sin\left(b\,x^2+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x^2+a),x)
```

```
[Out] 1/2*sin(b*x^2+a)/b
```

maxima [A] time = 0.43, size = 13, normalized size = 0.87

$$\frac{\sin\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*sin(b*x^2 + a)/b
```

mupad [B] time = 0.06, size = 13, normalized size = 0.87

$$\frac{\sin\left(b\,x^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x^2),x)
```

```
[Out] sin(a + b*x^2)/(2*b)
```

sympy [A] time = 0.17, size = 19, normalized size = 1.27

$$\begin{cases} \frac{\sin\left(a+bx^2\right)}{2b} & \text{for } b \neq 0 \\ \frac{x^2\cos\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x**2+a),x)
```

```
[Out] Piecewise((sin(a + b*x**2)/(2*b), Ne(b, 0)), (x**2*cos(a)/2, True))
```

3.4 $\int \cos\left(a+bx^2\right)\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{\frac{\pi}{2}}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}}$$

```
[Out] 1/2*cos(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(1/2)-1/
2*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3354, 3352, 3351}

$$\frac{\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2],x]
```

```
[Out] (Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x])/Sqrt[b] - (Sqrt[Pi/2]*Fr
esnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/Sqrt[b]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rubi steps

$$\int \cos\left(a+bx^2\right)\,dx = \cos(a)\int \cos\left(bx^2\right)\,dx - \sin(a)\int \sin\left(bx^2\right)\,dx$$
$$= \frac{\sqrt{\frac{\pi}{2}}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\,x\right)}{\sqrt{b}} - \frac{\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\,x\right)\sin(a)}{\sqrt{b}}$$

Mathematica [A] time = 0.09, size = 57, normalized size = 0.81

$$\frac{\sqrt{\frac{\pi}{2}}\left(\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\,x\right) - \sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\,x\right)\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2],x]
```

```
[Out] (Sqrt[Pi/2]*(Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x] - FresnelS[Sqrt[b]*Sqrt[
2/Pi]*x]*Sin[a]))/Sqrt[b]
```

fricas [A] time = 0.82, size = 61, normalized size = 0.87

$$\frac{\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right) - \sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)\sin(a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x*sqrt(b/pi)) - sqrt(
2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi))*sin(a))/b
```

giac [C] time = 0.39, size = 95, normalized size = 1.36

$$-\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\,x\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(i\,a)}}{4\left(-\frac{i\,b}{|b|}+1\right)\sqrt{|b|}} - \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\,x\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}\right)e^{(-i\,a)}}{4\left(\frac{i\,b}{|b|}+1\right)\sqrt{|b|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^
(I*a)/((-I*b/abs(b) + 1)*sqrt(abs(b))) - 1/4*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt
```

```
(2)*x*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/((I*b/abs(b) + 1)*sqrt(abs(b)
))
```

maple [A] time = 0.02, size = 44, normalized size = 0.63

$$\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin(a)\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a),x)
```

```
[Out] 1/2*2^(1/2)*Pi^(1/2)/b^(1/2)*(cos(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))-s
in(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2)))
```

maxima [C] time = 0.81, size = 48, normalized size = 0.69

$$-\frac{\sqrt{2}\,\sqrt{\pi}\left(\left((i-1)\,\cos(a)+(i+1)\,\sin(a)\right)\mathrm{erf}\left(\sqrt{i\,b}\,x\right)+\left(-(i+1)\,\cos(a)-(i-1)\,\sin(a)\right)\mathrm{erf}\left(\sqrt{-i\,b}\,x\right)\right)}{8\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(pi)*(((I - 1)*cos(a) + (I + 1)*sin(a))*erf(sqrt(I*b)*x) +
 (-(I + 1)*cos(a) - (I - 1)*sin(a))*erf(sqrt(-I*b)*x))/sqrt(b)
```

mupad [B] time = 0.36, size = 51, normalized size = 0.73

$$\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)\,\cos(a)}{2\,\sqrt{b}}-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)\,\sin(a)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2),x)
```

```
[Out] (2^(1/2)*pi^(1/2)*fresnelc((2^(1/2)*b^(1/2)*x)/pi^(1/2))*cos(a))/(2*b^(1/2)
) - (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b^(1/2)*x)/pi^(1/2))*sin(a))/(2*b^(
1/2))
```

sympy [A] time = 0.44, size = 61, normalized size = 0.87

$$\frac{\sqrt{2}\,\sqrt{\pi}\,\left(-\sin{(a)}S\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)+\cos{(a)}C\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a),x)
```

```
[Out] sqrt(2)*sqrt(pi)*(-sin(a)*fresnels(sqrt(2)*sqrt(b)*x/sqrt(pi)) + cos(a)*fre
snelc(sqrt(2)*sqrt(b)*x/sqrt(pi)))*sqrt(1/b)/2
```

3.5 $\int \frac{\cos(a+bx^2)}{x}\, dx$

Optimal. Leaf size=25

$$\frac{1}{2}\cos(a)\text{Ci}\left(bx^2\right)-\frac{1}{2}\sin(a)\text{Si}\left(bx^2\right)$$

```
[Out] 1/2*Ci(b*x^2)*cos(a)-1/2*Si(b*x^2)*sin(a)
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3378, 3376, 3375}

$$\frac{1}{2}\cos(a)\text{CosIntegral}\left(bx^2\right)-\frac{1}{2}\sin(a)\text{Si}\left(bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]/x,x]
```

```
[Out] (Cos[a]*CosIntegral[b*x^2])/2 - (Sin[a]*SinIntegral[b*x^2])/2
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(a+bx^2\right)}{x}\, dx &= \cos(a)\int \frac{\cos\left(bx^2\right)}{x}\, dx-\sin(a)\int \frac{\sin\left(bx^2\right)}{x}\, dx\\ &= \frac{1}{2}\cos(a)\text{Ci}\left(bx^2\right)-\frac{1}{2}\sin(a)\text{Si}\left(bx^2\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 24, normalized size = 0.96

$$\frac{1}{2}\left(\cos(a)\text{Ci}\left(bx^2\right)-\sin(a)\text{Si}\left(bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]/x,x]
```

```
[Out] (Cos[a]*CosIntegral[b*x^2] - Sin[a]*SinIntegral[b*x^2])/2
```

fricas [A] time = 0.88, size = 29, normalized size = 1.16

$$\frac{1}{4}\left(\text{Ci}\left(bx^2\right)+\text{Ci}\left(-bx^2\right)\right)\cos(a)-\frac{1}{2}\sin(a)\,\text{Si}\left(bx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x,x, algorithm="fricas")
```

```
[Out] 1/4*(cos_integral(b*x^2) + cos_integral(-b*x^2))*cos(a) - 1/2*sin(a)*sin_in
tegral(b*x^2)
```

giac [A] time = 0.42, size = 21, normalized size = 0.84

$$\frac{1}{2}\cos(a)\,\text{Ci}\left(bx^2\right)-\frac{1}{2}\sin(a)\,\text{Si}\left(bx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/2*cos(a)*cos_integral(b*x^2) - 1/2*sin(a)*sin_integral(b*x^2)
```

maple [A] time = 0.02, size = 22, normalized size = 0.88

$$\frac{\text{Ci}\left(b\,x^2\right)\cos(a)}{2}-\frac{\text{Si}\left(b\,x^2\right)\sin(a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)/x,x)
```

```
[Out] 1/2*Ci(b*x^2)*cos(a)-1/2*Si(b*x^2)*sin(a)
```

maxima [C] time = 1.14, size = 43, normalized size = 1.72

$$\frac{1}{4}\left(\text{Ei}\left(i\,bx^2\right)+\text{Ei}\left(-i\,bx^2\right)\right)\cos(a)+\frac{1}{4}\left(i\,\text{Ei}\left(i\,bx^2\right)-i\,\text{Ei}\left(-i\,bx^2\right)\right)\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/4*(Ei(I*b*x^2) + Ei(-I*b*x^2))*cos(a) + 1/4*(I*Ei(I*b*x^2) - I*Ei(-I*b*x^
2))*sin(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\frac{\cos(a)\,\mathrm{cosint}\left(b\,x^2\right)}{2}-\frac{\sin(a)\,\mathrm{sinint}\left(b\,x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)/x,x)
```

```
[Out] (cos(a)*cosint(b*x^2))/2 - (sin(a)*sinint(b*x^2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)/x,x)
```

```
[Out] Integral(cos(a + b*x**2)/x, x)
```

3.6 $\int \frac{\cos(a+bx^2)}{x^2} dx$

Optimal. Leaf size=80

$$-\sqrt{2\pi}\sqrt{b}\sin(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{2\pi}\sqrt{b}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\frac{\cos(a+bx^2)}{x}$$

```
[Out] -cos(b*x^2+a)/x-cos(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*b^(1/2)*2^(1/2)
*Pi^(1/2)-FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*b^(1/2)*2^(1/2)*Pi^(1
/2)
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3388, 3353, 3352, 3351}

$$-\sqrt{2\pi}\sqrt{b}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)-\sqrt{2\pi}\sqrt{b}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\frac{\cos(a+bx^2)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]/x^2,x]
```

```
[Out] -(Cos[a + b*x^2]/x) - Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]
*x] - Sqrt[b]*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(a+bx^2\right)}{x^2}\,dx &= -\frac{\cos\left(a+bx^2\right)}{x}-(2b)\int \sin\left(a+bx^2\right)\,dx\\ &= -\frac{\cos\left(a+bx^2\right)}{x}-(2b\cos(a))\int \sin\left(bx^2\right)\,dx-(2b\sin(a))\int \cos\left(bx^2\right)\,dx\\ &= -\frac{\cos\left(a+bx^2\right)}{x}-\sqrt{b}\,\sqrt{2\pi}\,\cos(a)S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,x\right)-\sqrt{b}\,\sqrt{2\pi}\,C\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,x\right)\sin(a)\end{aligned}$$

Mathematica [A] time = 0.18, size = 81, normalized size = 1.01

$$-\sqrt{2\pi}\,\sqrt{b}\left(\sin(a)C\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,x\right)+\cos(a)S\left(\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,x\right)\right)+\frac{\sin(a)\sin\left(bx^2\right)}{x}-\frac{\cos(a)\cos\left(bx^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]/x^2,x]
```

```
[Out] -((Cos[a]*Cos[b*x^2])/x) - Sqrt[b]*Sqrt[2*Pi]*(Cos[a]*FresnelS[Sqrt[b]*Sqrt
[2/Pi]*x] + FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a]) + (Sin[a]*Sin[b*x^2])/x
```

fricas [A] time = 0.87, size = 70, normalized size = 0.88

$$-\frac{\sqrt{2}\,\pi x\sqrt{\frac{b}{\pi}}\,\cos(a)\,\mathrm{S}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)+\sqrt{2}\,\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)\sin(a)+\cos\left(bx^2+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)) + sqrt(2
)*pi*x*sqrt(b/pi)*fresnel_cos(sqrt(2)*x*sqrt(b/pi))*sin(a) + cos(b*x^2 + a)
)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx^2+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^2 + a)/x^2, x)
```

maple [A] time = 0.03, size = 57, normalized size = 0.71

$$-\frac{\cos\left(b\,x^{2}+a\right)}{x}-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(a)\,\mathrm{FresnelC}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)/x^2,x)
```

```
[Out] -cos(b*x^2+a)/x-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(x*b^(1/2)*2^(1/2)
/Pi^(1/2))+sin(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2)))
```

maxima [C] time = 1.58, size = 73, normalized size = 0.91

$$\frac{\sqrt{bx^{2}}\left(\left(-(i+1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},i\,bx^{2}\right)+(i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},-i\,bx^{2}\right)\right)\cos(a)+\left((i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},i\,bx^{2}\right)-(i+1)\,\sqrt{2}\,\Gamma\left(\right.\right.\right.}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(b*x^2)*((-(I + 1)*sqrt(2)*gamma(-1/2, I*b*x^2) + (I - 1)*sqrt(2)*g
amma(-1/2, -I*b*x^2))*cos(a) + ((I - 1)*sqrt(2)*gamma(-1/2, I*b*x^2) - (I +
 1)*sqrt(2)*gamma(-1/2, -I*b*x^2))*sin(a))/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\cos\left(b\,x^{2}+a\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)/x^2,x)
```

```
[Out] int(cos(a + b*x^2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(a+bx^{2}\right)}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)/x**2,x)
```

```
[Out] Integral(cos(a + b*x**2)/x**2, x)
```

3.7 $\int \frac{\cos\left(a+bx^2\right)}{x^3}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{2}b\sin(a)\text{Ci}\left(bx^2\right)-\frac{1}{2}b\cos(a)\text{Si}\left(bx^2\right)-\frac{\cos\left(a+bx^2\right)}{2x^2}$$

```
[Out] -1/2*cos(b*x^2+a)/x^2-1/2*b*cos(a)*Si(b*x^2)-1/2*b*Ci(b*x^2)*sin(a)
```

Rubi [A] time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3380, 3297, 3303, 3299, 3302}

$$-\frac{1}{2}b\sin(a)\text{CosIntegral}\left(bx^2\right)-\frac{1}{2}b\cos(a)\text{Si}\left(bx^2\right)-\frac{\cos\left(a+bx^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]/x^3,x]
```

```
[Out] -Cos[a + b*x^2]/(2*x^2) - (b*CosIntegral[b*x^2]*Sin[a])/2 - (b*Cos[a]*SinIn
tegral[b*x^2])/2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(a+bx^2\right)}{x^3}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\cos\left(a+bx^2\right)}{2x^2} - \frac{1}{2}b\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, x^2\right)\\
&= -\frac{\cos\left(a+bx^2\right)}{2x^2} - \frac{1}{2}(b\cos(a))\,\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, x^2\right) - \frac{1}{2}(b\sin(a))\,\text{Subst}\left(\int \frac{\cos(bx)}{x}\right.\\
&= -\frac{\cos\left(a+bx^2\right)}{2x^2} - \frac{1}{2}b\text{Ci}\left(bx^2\right)\sin(a) - \frac{1}{2}b\cos(a)\text{Si}\left(bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 42, normalized size = 1.00

$$-\frac{bx^2\sin(a)\text{Ci}\left(bx^2\right)+bx^2\cos(a)\text{Si}\left(bx^2\right)+\cos\left(a+bx^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]/x^3,x]
```

```
[Out] -1/2*(Cos[a + b*x^2] + b*x^2*CosIntegral[b*x^2]*Sin[a] + b*x^2*Cos[a]*SinIn
tegral[b*x^2])/x^2
```

fricas [A] time = 0.93, size = 57, normalized size = 1.36

$$-\frac{2\,bx^2\cos(a)\,\text{Si}\left(bx^2\right)+\left(bx^2\,\text{Ci}\left(bx^2\right)+bx^2\,\text{Ci}\left(-bx^2\right)\right)\sin(a)+2\,\cos\left(bx^2+a\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^2*cos(a)*sin_integral(b*x^2) + (b*x^2*cos_integral(b*x^2) + b*x
^2*cos_integral(-b*x^2))*sin(a) + 2*cos(b*x^2 + a))/x^2
```

giac [B] time = 0.49, size = 87, normalized size = 2.07

$$-\frac{\left(bx^2+a\right)b^2\,\mathrm{Ci}\left(bx^2\right)\sin(a)-ab^2\,\mathrm{Ci}\left(bx^2\right)\sin(a)+\left(bx^2+a\right)b^2\cos(a)\,\mathrm{Si}\left(bx^2\right)-ab^2\cos(a)\,\mathrm{Si}\left(bx^2\right)+b^2\cos\left(b\right.}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*((b*x^2 + a)*b^2*cos_integral(b*x^2)*sin(a) - a*b^2*cos_integral(b*x^2
)*sin(a) + (b*x^2 + a)*b^2*cos(a)*sin_integral(b*x^2) - a*b^2*cos(a)*sin_in
tegral(b*x^2) + b^2*cos(b*x^2 + a))/(b^2*x^2)
```

maple [A] time = 0.02, size = 39, normalized size = 0.93

$$-\frac{\cos\left(b\,x^2+a\right)}{2x^2}-b\left(\frac{\cos(a)\,\mathrm{Si}\left(b\,x^2\right)}{2}+\frac{\sin(a)\,\mathrm{Ci}\left(b\,x^2\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)/x^3,x)
```

```
[Out] -1/2*cos(b*x^2+a)/x^2-b*(1/2*cos(a)*Si(b*x^2)+1/2*sin(a)*Ci(b*x^2))
```

maxima [C] time = 1.07, size = 48, normalized size = 1.14

$$-\frac{1}{4}\left(\left(i\,\Gamma\left(-1,i\,bx^2\right)-i\,\Gamma\left(-1,-i\,bx^2\right)\right)\cos(a)+\left(\Gamma\left(-1,i\,bx^2\right)+\Gamma\left(-1,-i\,bx^2\right)\right)\sin(a)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*((I*gamma(-1, I*b*x^2) - I*gamma(-1, -I*b*x^2))*cos(a) + (gamma(-1, I*
b*x^2) + gamma(-1, -I*b*x^2))*sin(a))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(b\,x^2+a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)/x^3,x)
```

```
[Out] int(cos(a + b*x^2)/x^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a + bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)/x**3,x)
```

```
[Out] Integral(cos(a + b*x**2)/x**3, x)
```

3.8 $\int x^3 \cos^2\left(a+bx^2\right) dx$

Optimal. Leaf size=51

$$\frac{\cos^2\left(a+bx^2\right)}{8b^2}+\frac{x^2\sin\left(a+bx^2\right)\cos\left(a+bx^2\right)}{4b}+\frac{x^4}{8}$$

```
[Out] 1/8*x^4+1/8*cos(b*x^2+a)^2/b^2+1/4*x^2*cos(b*x^2+a)*sin(b*x^2+a)/b
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3380, 3310, 30}

$$\frac{\cos^2\left(a+bx^2\right)}{8b^2}+\frac{x^2\sin\left(a+bx^2\right)\cos\left(a+bx^2\right)}{4b}+\frac{x^4}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[a + b*x^2]^2,x]
```

```
[Out] x^4/8 + Cos[a + b*x^2]^2/(8*b^2) + (x^2*Cos[a + b*x^2]*Sin[a + b*x^2])/(4*b
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int x^3\cos^2\left(a+bx^2\right)\,dx = \frac{1}{2}\,\text{Subst}\left(\int x\cos^2(a+bx)\,dx,x,x^2\right)$$
$$= \frac{\cos^2\left(a+bx^2\right)}{8b^2}+\frac{x^2\cos\left(a+bx^2\right)\sin\left(a+bx^2\right)}{4b}+\frac{1}{4}\,\text{Subst}\left(\int x\,dx,x,x^2\right)$$
$$= \frac{x^4}{8}+\frac{\cos^2\left(a+bx^2\right)}{8b^2}+\frac{x^2\cos\left(a+bx^2\right)\sin\left(a+bx^2\right)}{4b}$$

Mathematica [**A**] time = 0.11, size = 40, normalized size = 0.78

$$\frac{2bx^2\left(\sin\left(2\left(a+bx^2\right)\right)+bx^2\right)+\cos\left(2\left(a+bx^2\right)\right)}{16b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[a + b*x^2]^2,x]
```

```
[Out] (Cos[2*(a + b*x^2)] + 2*b*x^2*(b*x^2 + Sin[2*(a + b*x^2)]))/(16*b^2)
```

fricas [**A**] time = 0.94, size = 45, normalized size = 0.88

$$\frac{b^2x^4+2\,bx^2\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)+\cos\left(bx^2+a\right)^2}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*x^4 + 2*b*x^2*cos(b*x^2 + a)*sin(b*x^2 + a) + cos(b*x^2 + a)^2)/b^
2
```

giac [**A**] time = 0.43, size = 55, normalized size = 1.08

$$\frac{2\,bx^2\sin\left(2\,bx^2+2\,a\right)+2\left(bx^2+a\right)^2-4\left(bx^2+a\right)a+\cos\left(2\,bx^2+2\,a\right)}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*b*x^2*sin(2*b*x^2 + 2*a) + 2*(b*x^2 + a)^2 - 4*(b*x^2 + a)*a + cos(
2*b*x^2 + 2*a))/b^2
```

maple [A] time = 0.04, size = 42, normalized size = 0.82

$$\frac{x^4}{8}+\frac{x^2\sin\left(2b\,x^2+2a\right)}{8b}+\frac{\cos\left(2b\,x^2+2a\right)}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(b*x^2+a)^2,x)
```

```
[Out] 1/8*x^4+1/8/b*x^2*sin(2*b*x^2+2*a)+1/16/b^2*cos(2*b*x^2+2*a)
```

maxima [A] time = 0.90, size = 42, normalized size = 0.82

$$\frac{2\,b^2x^4+2\,bx^2\sin\left(2\,bx^2+2\,a\right)+\cos\left(2\,bx^2+2\,a\right)}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*b^2*x^4 + 2*b*x^2*sin(2*b*x^2 + 2*a) + cos(2*b*x^2 + 2*a))/b^2
```

mupad [B] time = 0.15, size = 41, normalized size = 0.80

$$\frac{\cos\left(2\,b\,x^2+2\,a\right)}{16\,b^2}+\frac{x^4}{8}+\frac{x^2\,\sin\left(2\,b\,x^2+2\,a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(a + b*x^2)^2,x)
```

```
[Out] cos(2*a + 2*b*x^2)/(16*b^2) + x^4/8 + (x^2*sin(2*a + 2*b*x^2))/(8*b)
```

sympy [A] time = 1.55, size = 78, normalized size = 1.53

$$\begin{cases} \frac{x^4\sin^2\left(a+bx^2\right)}{8}+\frac{x^4\cos^2\left(a+bx^2\right)}{8}+\frac{x^2\sin\left(a+bx^2\right)\cos\left(a+bx^2\right)}{4b}+\frac{\cos^2\left(a+bx^2\right)}{8b^2} & \text{for } b\neq 0 \\ \frac{x^4\cos^2(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(b*x**2+a)**2,x)
```

```
[Out] Piecewise((x**4*sin(a + b*x**2)**2/8 + x**4*cos(a + b*x**2)**2/8 + x**2*sin
(a + b*x**2)*cos(a + b*x**2)/(4*b) + cos(a + b*x**2)**2/(8*b**2), Ne(b, 0))
, (x**4*cos(a)**2/4, True))
```

3.9 $\int x^2 \cos^2\left(a+bx^2\right) dx$

Optimal. Leaf size=91

$$-\frac{\sqrt{\pi}\,\sin(2a)C\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{\pi}\,\cos(2a)S\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)}{16b^{3/2}}+\frac{x\sin\left(2a+2bx^2\right)}{8b}+\frac{x^3}{6}$$

```
[Out] 1/6*x^3+1/8*x*sin(2*b*x^2+2*a)/b-1/16*cos(2*a)*FresnelS(2*x*b^(1/2)/Pi^(1/2
))*Pi^(1/2)/b^(3/2)-1/16*FresnelC(2*x*b^(1/2)/Pi^(1/2))*sin(2*a)*Pi^(1/2)/b
^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3404, 3386, 3353, 3352, 3351}

$$-\frac{\sqrt{\pi}\,\sin(2a)\text{FresnelC}\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{\pi}\,\cos(2a)S\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)}{16b^{3/2}}+\frac{x\sin\left(2a+2bx^2\right)}{8b}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*x^2]^2,x]
```

```
[Out] x^3/6 - (Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]])/(16*b^(3/2)) -
 (Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a])/(16*b^(3/2)) + (x*Sin
[2*a + 2*b*x^2])/(8*b)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3404

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2\cos^2\left(a+bx^2\right)\,dx &= \int\left(\frac{x^2}{2}+\frac{1}{2}x^2\cos\left(2a+2bx^2\right)\right)dx\\
&= \frac{x^3}{6}+\frac{1}{2}\int x^2\cos\left(2a+2bx^2\right)dx\\
&= \frac{x^3}{6}+\frac{x\sin\left(2a+2bx^2\right)}{8b}-\frac{\int\sin\left(2a+2bx^2\right)dx}{8b}\\
&= \frac{x^3}{6}+\frac{x\sin\left(2a+2bx^2\right)}{8b}-\frac{\cos(2a)\int\sin\left(2bx^2\right)dx}{8b}-\frac{\sin(2a)\int\cos\left(2bx^2\right)dx}{8b}\\
&= \frac{x^3}{6}-\frac{\sqrt{\pi}\cos(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{\pi}C\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)\sin(2a)}{16b^{3/2}}+\frac{x\sin\left(2a+2bx^2\right)}{8b}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 87, normalized size = 0.96

$$
\frac{-3\sqrt{\pi}\sin(2a)C\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)-3\sqrt{\pi}\cos(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)+2\sqrt{b}x\left(3\sin\left(2\left(a+bx^2\right)\right)+4bx^2\right)}{48b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*x^2]^2,x]
```

```
[Out] (-3*Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]] - 3*Sqrt[Pi]*Fresnel
C[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a] + 2*Sqrt[b]*x*(4*b*x^2 + 3*Sin[2*(a + b*
x^2)]))/(48*b^(3/2))
```

fricas [A] time = 0.75, size = 84, normalized size = 0.92

$$
\frac{8\,b^2x^3+12\,bx\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)-3\,\pi\sqrt{\frac{b}{\pi}}\cos\left(2\,a\right)\mathrm{S}\left(2\,x\sqrt{\frac{b}{\pi}}\right)-3\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right)}{48\,b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(8*b^2*x^3 + 12*b*x*cos(b*x^2 + a)*sin(b*x^2 + a) - 3*pi*sqrt(b/pi)*co
s(2*a)*fresnel_sin(2*x*sqrt(b/pi)) - 3*pi*sqrt(b/pi)*fresnel_cos(2*x*sqrt(b
/pi))*sin(2*a))/b^2
```

giac [C] time = 0.82, size = 118, normalized size = 1.30

$$\frac{1}{6}x^3 - \frac{ixe^{\left(2ibx^2+2ia\right)}}{16b} + \frac{ixe^{\left(-2ibx^2-2ia\right)}}{16b} - \frac{i\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{b}x\left(-\frac{ib}{|b|}+1\right)\right)e^{(2ia)}}{32b^{\frac{3}{2}}\left(-\frac{ib}{|b|}+1\right)} + \frac{i\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{b}x\left(\frac{ib}{|b|}+1\right)\right)e^{(-2ia)}}{32b^{\frac{3}{2}}\left(\frac{ib}{|b|}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/16*I*x*e^(2*I*b*x^2 + 2*I*a)/b + 1/16*I*x*e^(-2*I*b*x^2 - 2*I*a
)/b - 1/32*I*sqrt(pi)*erf(-sqrt(b)*x*(-I*b/abs(b) + 1))*e^(2*I*a)/(b^(3/2)*
(-I*b/abs(b) + 1)) + 1/32*I*sqrt(pi)*erf(-sqrt(b)*x*(I*b/abs(b) + 1))*e^(-2
*I*a)/(b^(3/2)*(I*b/abs(b) + 1))
```

maple [A] time = 0.04, size = 63, normalized size = 0.69

$$\frac{x^3}{6} + \frac{x\sin\left(2bx^2+2a\right)}{8b} - \frac{\sqrt{\pi}\,\left(\cos\left(2a\right)\mathrm{S}\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right) + \sin\left(2a\right)\mathrm{FresnelC}\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right)\right)}{16b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(b*x^2+a)^2,x)
```

```
[Out] 1/6*x^3+1/8*x*sin(2*b*x^2+2*a)/b-1/16/b^(3/2)*Pi^(1/2)*(cos(2*a)*FresnelS(2
*x*b^(1/2)/Pi^(1/2))+sin(2*a)*FresnelC(2*x*b^(1/2)/Pi^(1/2)))
```

maxima [C] time = 1.23, size = 89, normalized size = 0.98

$$\frac{64\,b^3x^3 + 48\,b^2x\sin\left(2\,bx^2+2\,a\right) + 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(-(3i+3)\cos\left(2\,a\right) + (3i-3)\sin\left(2\,a\right)\right)\operatorname{erf}\left(\sqrt{2i\,b}\,x\right) + \left((3i-3)\ c\right.\right.}{384\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/384*(64*b^3*x^3 + 48*b^2*x*sin(2*b*x^2 + 2*a) + 4^(1/4)*sqrt(2)*sqrt(pi)*
((-(3*I + 3)*cos(2*a) + (3*I - 3)*sin(2*a))*erf(sqrt(2*I*b)*x) + ((3*I - 3)
*cos(2*a) - (3*I + 3)*sin(2*a))*erf(sqrt(-2*I*b)*x))*b^(3/2))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \cos\left(b x^2 + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a + b*x^2)^2,x)
```

```
[Out] int(x^2*cos(a + b*x^2)^2, x)
```

sympy [B] time = 2.60, size = 201, normalized size = 2.21

$$\frac{b^{\frac{3}{2}} x^5 \sqrt{\frac{1}{b}} \sin(2a)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right) {}_2F_3\left(\begin{matrix} \frac{3}{4}, \frac{5}{4} \\ \frac{3}{2}, \frac{7}{4}, \frac{9}{4} \end{matrix} \middle| -b^2x^4\right)}{8\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)} - \frac{\sqrt{b}\, x^3 \sqrt{\frac{1}{b}} \cos(2a)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) {}_2F_3\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{5}{4}, \frac{7}{4} \end{matrix} \middle| -b^2x^4\right)}{16\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)} + \frac{x^3}{6} - \frac{\sqrt{\pi}\, x^2 \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(b*x**2+a)**2,x)
```

```
[Out] b**(3/2)*x**5*sqrt(1/b)*sin(2*a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3
/2, 7/4, 9/4), -b**2*x**4)/(8*gamma(7/4)*gamma(9/4)) - sqrt(b)*x**3*sqrt(1/
b)*cos(2*a)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*
x**4)/(16*gamma(5/4)*gamma(7/4)) + x**3/6 - sqrt(pi)*x**2*sqrt(1/b)*sin(2*a
)*fresnels(2*sqrt(b)*x/sqrt(pi))/4 + sqrt(pi)*x**2*sqrt(1/b)*cos(2*a)*fresn
elc(2*sqrt(b)*x/sqrt(pi))/4
```

3.10 $\int x\cos^2\left(a+bx^2\right)\,dx$

Optimal. Leaf size=31

$$\frac{\sin\left(a+bx^2\right)\cos\left(a+bx^2\right)}{4b}+\frac{x^2}{4}$$

```
[Out] 1/4*x^2+1/4*cos(b*x^2+a)*sin(b*x^2+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3380, 2635, 8}

$$\frac{\sin\left(a+bx^2\right)\cos\left(a+bx^2\right)}{4b}+\frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x^2]^2,x]
```

```
[Out] x^2/4 + (Cos[a + b*x^2]*Sin[a + b*x^2])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int x\cos^2\left(a+bx^2\right)dx = \frac{1}{2}\text{Subst}\left(\int\cos^2(a+bx)\,dx,x,x^2\right)$$
$$=\frac{\cos\left(a+bx^2\right)\sin\left(a+bx^2\right)}{4b}+\frac{1}{4}\text{Subst}\left(\int 1\,dx,x,x^2\right)$$
$$=\frac{x^2}{4}+\frac{\cos\left(a+bx^2\right)\sin\left(a+bx^2\right)}{4b}$$

Mathematica [A]   time = 0.04, size = 27, normalized size = 0.87

$$\frac{2\left(a+bx^2\right)+\sin\left(2\left(a+bx^2\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x^2]^2,x]
[Out] (2*(a + b*x^2) + Sin[2*(a + b*x^2)])/(8*b)
```

fricas [A]   time = 1.33, size = 28, normalized size = 0.90

$$\frac{bx^2+\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/4*(b*x^2 + cos(b*x^2 + a)*sin(b*x^2 + a))/b
```

giac [A]   time = 0.40, size = 26, normalized size = 0.84

$$\frac{2\,bx^2+2\,a+\sin\left(2\,bx^2+2\,a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x^2+a)^2,x, algorithm="giac")
[Out] 1/8*(2*b*x^2 + 2*a + sin(2*b*x^2 + 2*a))/b
```

maple [A]   time = 0.03, size = 34, normalized size = 1.10

$$\frac{\frac{\cos\left(b\,x^2+a\right)\sin\left(b\,x^2+a\right)}{2}+\frac{b\,x^2}{2}+\frac{a}{2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x^2+a)^2,x)
```

```
[Out] 1/2/b*(1/2*cos(b*x^2+a)*sin(b*x^2+a)+1/2*b*x^2+1/2*a)
```

maxima [A] time = 0.59, size = 23, normalized size = 0.74

$$\frac{2\,bx^2 + \sin\left(2\,bx^2 + 2\,a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*b*x^2 + sin(2*b*x^2 + 2*a))/b
```

mupad [B] time = 0.27, size = 22, normalized size = 0.71

$$\frac{\sin\left(2\,b\,x^2 + 2\,a\right)}{8\,b} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x^2)^2,x)
```

```
[Out] sin(2*a + 2*b*x^2)/(8*b) + x^2/4
```

sympy [A] time = 0.41, size = 60, normalized size = 1.94

$$\begin{cases} \frac{x^2 \sin^2\left(a+bx^2\right)}{4} + \frac{x^2 \cos^2\left(a+bx^2\right)}{4} + \frac{\sin\left(a+bx^2\right)\cos\left(a+bx^2\right)}{4b} & \text{for } b \neq 0 \\ \frac{x^2 \cos^2\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x**2+a)**2,x)
```

```
[Out] Piecewise((x**2*sin(a + b*x**2)**2/4 + x**2*cos(a + b*x**2)**2/4 + sin(a + 
b*x**2)*cos(a + b*x**2)/(4*b), Ne(b, 0)), (x**2*cos(a)**2/2, True))
```

3.11 $\int \cos^2\left(a+bx^2\right)\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{\pi}\cos(2a)C\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}\sin(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4\sqrt{b}}+\frac{x}{2}$$

```
[Out] 1/2*x+1/4*cos(2*a)*FresnelC(2*x*b^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(1/2)-1/4*Fres
nelS(2*x*b^(1/2)/Pi^(1/2))*sin(2*a)*Pi^(1/2)/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3358, 3354, 3352, 3351}

$$\frac{\sqrt{\pi}\cos(2a)\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}\sin(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)}{4\sqrt{b}}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]^2,x]
```

```
[Out] x/2 + (Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]])/(4*Sqrt[b]) - (S
qrt[Pi]*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a])/(4*Sqrt[b])
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
```

```
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cos^2\left(a+bx^2\right)\,dx &= \int \left(\frac{1}{2}+\frac{1}{2}\cos\left(2a+2bx^2\right)\right)dx\\ &= \frac{x}{2}+\frac{1}{2}\int \cos\left(2a+2bx^2\right)dx\\ &= \frac{x}{2}+\frac{1}{2}\cos(2a)\int \cos\left(2bx^2\right)dx-\frac{1}{2}\sin(2a)\int \sin\left(2bx^2\right)dx\\ &= \frac{x}{2}+\frac{\sqrt{\pi}\,\cos(2a)C\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)}{4\sqrt{b}}-\frac{\sqrt{\pi}\,S\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)\sin(2a)}{4\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.96

$$\frac{\sqrt{\pi}\,\cos(2a)C\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)-\sqrt{\pi}\,\sin(2a)S\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)+2\sqrt{b}\,x}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^2,x]
```

```
[Out] (2*Sqrt[b]*x + Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]] - Sqrt[Pi
]*FresnelS[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a])/(4*Sqrt[b])
```

fricas [A] time = 0.95, size = 59, normalized size = 0.84

$$\frac{\pi\sqrt{\frac{b}{\pi}}\,\cos\left(2\,a\right)\mathrm{C}\left(2\,x\sqrt{\frac{b}{\pi}}\right)-\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(2\,x\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right)+2\,bx}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(pi*sqrt(b/pi)*cos(2*a)*fresnel_cos(2*x*sqrt(b/pi)) - pi*sqrt(b/pi)*fre
snel_sin(2*x*sqrt(b/pi))*sin(2*a) + 2*b*x)/b
```

giac [C] time = 0.32, size = 82, normalized size = 1.17

$$\frac{1}{2}\,x-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{b}\,x\left(-\frac{i\,b}{|b|}+1\right)\right)e^{(2\,i\,a)}}{8\,\sqrt{b}\left(-\frac{i\,b}{|b|}+1\right)}-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-\sqrt{b}\,x\left(\frac{i\,b}{|b|}+1\right)\right)e^{(-2\,i\,a)}}{8\,\sqrt{b}\left(\frac{i\,b}{|b|}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/8*sqrt(pi)*erf(-sqrt(b)*x*(-I*b/abs(b) + 1))*e^(2*I*a)/(sqrt(b)*(
-I*b/abs(b) + 1)) - 1/8*sqrt(pi)*erf(-sqrt(b)*x*(I*b/abs(b) + 1))*e^(-2*I*a
)/(sqrt(b)*(I*b/abs(b) + 1))
```

maple [A] time = 0.05, size = 45, normalized size = 0.64

$$\frac{x}{2}+\frac{\sqrt{\pi}\,\left(\cos\left(2a\right)\mathrm{FresnelC}\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right)-\sin\left(2a\right)\mathrm{S}\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right)\right)}{4\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)^2,x)
```

```
[Out] 1/2*x+1/4*Pi^(1/2)/b^(1/2)*(cos(2*a)*FresnelC(2*x*b^(1/2)/Pi^(1/2))-sin(2*a
)*FresnelS(2*x*b^(1/2)/Pi^(1/2)))
```

maxima [C] time = 1.45, size = 70, normalized size = 1.00

$$-\frac{4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\left(\left(\left(i-1\right)\cos\left(2a\right)+\left(i+1\right)\sin\left(2a\right)\right)\mathrm{erf}\left(\sqrt{2ib}\,x\right)+\left(-\left(i+1\right)\cos\left(2a\right)-\left(i-1\right)\sin\left(2a\right)\right)\mathrm{erf}\left(\sqrt{-2ib}\,x\right)\right)\cdots}{32\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(4^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*cos(2*a) + (I + 1)*sin(2*a))*erf(
sqrt(2*I*b)*x) + (-(I + 1)*cos(2*a) - (I - 1)*sin(2*a))*erf(sqrt(-2*I*b)*x)
)*b^(3/2) - 16*b^2*x)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(b\,x^2+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)^2,x)
```

```
[Out] int(cos(a + b*x^2)^2, x)
```

sympy [A] time = 0.80, size = 56, normalized size = 0.80

$$\frac{x}{2}+\frac{\sqrt{\pi}\left(-\sin\left(2a\right)S\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)+\cos\left(2a\right)C\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)**2,x)
```

```
[Out] x/2 + sqrt(pi)*(-sin(2*a)*fresnels(2*sqrt(b)*x/sqrt(pi)) + cos(2*a)*fresnel
c(2*sqrt(b)*x/sqrt(pi)))*sqrt(1/b)/4
```

3.12 $\int \frac{\cos^2(a+bx^2)}{x} \, dx$

Optimal. Leaf size=37

$$\frac{1}{4}\cos(2a)\text{Ci}\left(2bx^2\right) - \frac{1}{4}\sin(2a)\text{Si}\left(2bx^2\right) + \frac{\log(x)}{2}$$

```
[Out] 1/4*Ci(2*b*x^2)*cos(2*a)+1/2*ln(x)-1/4*Si(2*b*x^2)*sin(2*a)
```

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3404, 3378, 3376, 3375}

$$\frac{1}{4}\cos(2a)\text{CosIntegral}\left(2bx^2\right) - \frac{1}{4}\sin(2a)\text{Si}\left(2bx^2\right) + \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]^2/x,x]
```

```
[Out] (Cos[2*a]*CosIntegral[2*b*x^2])/4 + Log[x]/2 - (Sin[2*a]*SinIntegral[2*b*x^
2])/4
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3404

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2\left(a+bx^2\right)}{x}\,dx &= \int \left(\frac{1}{2x}+\frac{\cos\left(2a+2bx^2\right)}{2x}\right)dx\\
&= \frac{\log(x)}{2}+\frac{1}{2}\int \frac{\cos\left(2a+2bx^2\right)}{x}\,dx\\
&= \frac{\log(x)}{2}+\frac{1}{2}\cos(2a)\int \frac{\cos\left(2bx^2\right)}{x}\,dx-\frac{1}{2}\sin(2a)\int \frac{\sin\left(2bx^2\right)}{x}\,dx\\
&= \frac{1}{4}\cos(2a)\text{Ci}\left(2bx^2\right)+\frac{\log(x)}{2}-\frac{1}{4}\sin(2a)\text{Si}\left(2bx^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 34, normalized size = 0.92

$$\frac{1}{4}\left(\cos(2a)\text{Ci}\left(2bx^2\right)-\sin(2a)\text{Si}\left(2bx^2\right)+2\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^2/x,x]
```

```
[Out] (Cos[2*a]*CosIntegral[2*b*x^2] + 2*Log[x] - Sin[2*a]*SinIntegral[2*b*x^2])/
4
```

fricas [A] time = 0.90, size = 39, normalized size = 1.05

$$\frac{1}{8}\left(\text{Ci}\left(2\,bx^2\right)+\text{Ci}\left(-2\,bx^2\right)\right)\cos\left(2\,a\right)-\frac{1}{4}\,\sin\left(2\,a\right)\text{Si}\left(2\,bx^2\right)+\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*(cos_integral(2*b*x^2) + cos_integral(-2*b*x^2))*cos(2*a) - 1/4*sin(2*a
)*sin_integral(2*b*x^2) + 1/2*log(x)
```

giac [A] time = 0.41, size = 35, normalized size = 0.95

$$\frac{1}{4}\,\cos\left(2\,a\right)\text{Ci}\left(2\,bx^2\right)+\frac{1}{4}\,\sin\left(2\,a\right)\text{Si}\left(-2\,bx^2\right)+\frac{1}{4}\,\log\left(bx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/4*cos(2*a)*cos_integral(2*b*x^2) + 1/4*sin(2*a)*sin_integral(-2*b*x^2) +
1/4*log(b*x^2)
```

maple [C] time = 0.15, size = 68, normalized size = 1.84

$$\frac{\ln(x)}{2}+\frac{i\mathrm{e}^{-2ia}\mathrm{csgn}\left(b\,x^{2}\right)\pi}{8}-\frac{i\mathrm{e}^{-2ia}\,\mathrm{Si}\left(2b\,x^{2}\right)}{4}-\frac{\mathrm{e}^{-2ia}\,\mathrm{Ei}\left(1,-2ib\,x^{2}\right)}{8}-\frac{\mathrm{e}^{2ia}\,\mathrm{Ei}\left(1,-2ib\,x^{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)^2/x,x)
```

```
[Out] 1/2*ln(x)+1/8*I*exp(-2*I*a)*csgn(b*x^2)*Pi-1/4*I*exp(-2*I*a)*Si(2*b*x^2)-1/
8*exp(-2*I*a)*Ei(1,-2*I*b*x^2)-1/8*exp(2*I*a)*Ei(1,-2*I*b*x^2)
```

maxima [C] time = 1.31, size = 51, normalized size = 1.38

$$\frac{1}{8}\left(\mathrm{Ei}\left(2i\,bx^{2}\right)+\mathrm{Ei}\left(-2i\,bx^{2}\right)\right)\cos\left(2\,a\right)+\frac{1}{8}\left(i\,\mathrm{Ei}\left(2i\,bx^{2}\right)-i\,\mathrm{Ei}\left(-2i\,bx^{2}\right)\right)\sin\left(2\,a\right)+\frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*(Ei(2*I*b*x^2) + Ei(-2*I*b*x^2))*cos(2*a) + 1/8*(I*Ei(2*I*b*x^2) - I*Ei
(-2*I*b*x^2))*sin(2*a) + 1/2*log(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\cos\left(b\,x^{2}+a\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)^2/x,x)
```

```
[Out] int(cos(a + b*x^2)^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^{2}\left(a+bx^{2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)**2/x,x)
```

```
[Out] Integral(cos(a + b*x**2)**2/x, x)
```

3.13 $\int \frac{\cos^2(a+bx^2)}{x^2} dx$

Optimal. Leaf size=76

$$-\sqrt{\pi}\sqrt{b}\sin(2a)C\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)-\sqrt{\pi}\sqrt{b}\cos(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)-\frac{\cos^2\left(a+bx^2\right)}{x}$$

```
[Out] -cos(b*x^2+a)^2/x-cos(2*a)*FresnelS(2*x*b^(1/2)/Pi^(1/2))*b^(1/2)*Pi^(1/2)-
FresnelC(2*x*b^(1/2)/Pi^(1/2))*sin(2*a)*b^(1/2)*Pi^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3394, 4573, 3373, 3353, 3352, 3351}

$$-\sqrt{\pi}\sqrt{b}\sin(2a)\text{FresnelC}\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)-\sqrt{\pi}\sqrt{b}\cos(2a)S\left(\frac{2\sqrt{b}x}{\sqrt{\pi}}\right)-\frac{\cos^2\left(a+bx^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]^2/x^2,x]
```

```
[Out] -(Cos[a + b*x^2]^2/x) - Sqrt[b]*Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x)/Sq
rt[Pi]] - Sqrt[b]*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3373

```
Int[((a_.) + (b_.)*Sin[u_])^(p_.), x_Symbol] :> Int[(a + b*Sin[ExpandToSum[
u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ
```

```
[u, x]
```

Rule 3394

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_)]^(p_)*(x_)^(m_.), x_Symbol] :> Simp[(x^(m +
 1)*Cos[a + b*x^n]^p)/(m + 1), x] + Dist[(b*n*p)/(m + 1), Int[Cos[a + b*x^n
]^(p - 1)*Sin[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
 + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 4573

```
Int[Cos[w_]^(p_.)*(u_.)*Sin[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin[2
*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2\left(a+bx^2\right)}{x^2}\,dx &= -\frac{\cos^2\left(a+bx^2\right)}{x} - (4b)\int \cos\left(a+bx^2\right)\sin\left(a+bx^2\right)\,dx \\
&= -\frac{\cos^2\left(a+bx^2\right)}{x} - (2b)\int \sin\left(2\left(a+bx^2\right)\right)\,dx \\
&= -\frac{\cos^2\left(a+bx^2\right)}{x} - (2b)\int \sin\left(2a+2bx^2\right)\,dx \\
&= -\frac{\cos^2\left(a+bx^2\right)}{x} - (2b\cos(2a))\int \sin\left(2bx^2\right)\,dx - (2b\sin(2a))\int \cos\left(2bx^2\right)\,dx \\
&= -\frac{\cos^2\left(a+bx^2\right)}{x} - \sqrt{b}\,\sqrt{\pi}\,\cos(2a)S\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right) - \sqrt{b}\,\sqrt{\pi}\,C\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right)\sin(2a)
\end{aligned}$$

Mathematica [A] time = 0.16, size = 76, normalized size = 1.00

$$-\frac{\sqrt{\pi}\,\sqrt{b}\,x\sin(2a)C\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right) + \sqrt{\pi}\,\sqrt{b}\,x\cos(2a)S\left(\frac{2\sqrt{b}\,x}{\sqrt{\pi}}\right) + \cos^2\left(a+bx^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^2/x^2,x]
```

```
[Out] -((Cos[a + b*x^2]^2 + Sqrt[b]*Sqrt[Pi]*x*Cos[2*a]*FresnelS[(2*Sqrt[b]*x)/Sq
rt[Pi]] + Sqrt[b]*Sqrt[Pi]*x*FresnelC[(2*Sqrt[b]*x)/Sqrt[Pi]]*Sin[2*a])/x)
```

fricas [A] time = 0.89, size = 66, normalized size = 0.87

$$-\frac{\pi x\sqrt{\frac{b}{\pi}}\cos(2a)\,\mathrm{S}\left(2x\sqrt{\frac{b}{\pi}}\right)+\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2x\sqrt{\frac{b}{\pi}}\right)\sin(2a)+\cos\left(bx^2+a\right)^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(pi*x*sqrt(b/pi)*cos(2*a)*fresnel_sin(2*x*sqrt(b/pi)) + pi*x*sqrt(b/pi)*fr
esnel_cos(2*x*sqrt(b/pi))*sin(2*a) + cos(b*x^2 + a)^2)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx^2+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^2 + a)^2/x^2, x)
```

maple [A] time = 0.04, size = 62, normalized size = 0.82

$$-\frac{1}{2x}-\frac{\cos\left(2b\,x^2+2a\right)}{2x}-\sqrt{b}\,\sqrt{\pi}\,\left(\cos(2a)\,\mathrm{S}\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right)+\sin(2a)\,\mathrm{FresnelC}\left(\frac{2x\sqrt{b}}{\sqrt{\pi}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)^2/x^2,x)
```

```
[Out] -1/2/x-1/2/x*cos(2*b*x^2+2*a)-b^(1/2)*Pi^(1/2)*(cos(2*a)*FresnelS(2*x*b^(1/
2)/Pi^(1/2))+sin(2*a)*FresnelC(2*x*b^(1/2)/Pi^(1/2)))
```

maxima [C] time = 1.54, size = 83, normalized size = 1.09

$$\frac{\sqrt{2}\,\sqrt{bx^2}\left(\left(-(i+1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},2i\,bx^2\right)+(i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},-2i\,bx^2\right)\right)\cos(2a)+\left((i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},2i\,bx^2\right)-(i+1)\right.}{16\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/16*(sqrt(2)*sqrt(b*x^2)*((-(I + 1)*sqrt(2)*gamma(-1/2, 2*I*b*x^2) + (I -
1)*sqrt(2)*gamma(-1/2, -2*I*b*x^2))*cos(2*a) + ((I - 1)*sqrt(2)*gamma(-1/2,
 2*I*b*x^2) - (I + 1)*sqrt(2)*gamma(-1/2, -2*I*b*x^2))*sin(2*a)) - 8)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(b\,x^2+a\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)^2/x^2,x)
```

```
[Out] int(cos(a + b*x^2)^2/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(a+bx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)**2/x**2,x)
```

```
[Out] Integral(cos(a + b*x**2)**2/x**2, x)
```

3.14 $\int \frac{\cos^2(a+bx^2)}{x^3} \, dx$

Optimal. Leaf size=57

$$-\frac{1}{2}b\sin(2a)\text{Ci}\left(2bx^2\right)-\frac{1}{2}b\cos(2a)\text{Si}\left(2bx^2\right)-\frac{\cos\left(2\left(a+bx^2\right)\right)}{4x^2}-\frac{1}{4x^2}$$

```
[Out] -1/4/x^2-1/4*cos(2*b*x^2+2*a)/x^2-1/2*b*cos(2*a)*Si(2*b*x^2)-1/2*b*Ci(2*b*x
^2)*sin(2*a)
```

Rubi [A] time = 0.12, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3404, 3380, 3297, 3303, 3299, 3302}

$$-\frac{1}{2}b\sin(2a)\text{CosIntegral}\left(2bx^2\right)-\frac{1}{2}b\cos(2a)\text{Si}\left(2bx^2\right)-\frac{\cos\left(2\left(a+bx^2\right)\right)}{4x^2}-\frac{1}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]^2/x^3,x]
```

```
[Out] -1/(4*x^2) - Cos[2*(a + b*x^2)]/(4*x^2) - (b*CosIntegral[2*b*x^2]*Sin[2*a])
/2 - (b*Cos[2*a]*SinIntegral[2*b*x^2])/2
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3404

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2\left(a+bx^2\right)}{x^3}\,dx &= \int \left(\frac{1}{2x^3}+\frac{\cos\left(2a+2bx^2\right)}{2x^3}\right)dx \\
&= -\frac{1}{4x^2}+\frac{1}{2}\int \frac{\cos\left(2a+2bx^2\right)}{x^3}\,dx \\
&= -\frac{1}{4x^2}+\frac{1}{4}\,\text{Subst}\left(\int \frac{\cos(2a+2bx)}{x^2}\,dx,x,x^2\right) \\
&= -\frac{1}{4x^2}-\frac{\cos\left(2\left(a+bx^2\right)\right)}{4x^2}-\frac{1}{2}b\,\text{Subst}\left(\int \frac{\sin(2a+2bx)}{x}\,dx,x,x^2\right) \\
&= -\frac{1}{4x^2}-\frac{\cos\left(2\left(a+bx^2\right)\right)}{4x^2}-\frac{1}{2}(b\cos(2a))\,\text{Subst}\left(\int \frac{\sin(2bx)}{x}\,dx,x,x^2\right)-\frac{1}{2}(b\sin(2a))\,\text{Su} \\
&= -\frac{1}{4x^2}-\frac{\cos\left(2\left(a+bx^2\right)\right)}{4x^2}-\frac{1}{2}b\,\text{Ci}\left(2bx^2\right)\sin(2a)-\frac{1}{2}b\cos(2a)\,\text{Si}\left(2bx^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 50, normalized size = 0.88

$$
-\frac{bx^2\sin(2a)\,\text{Ci}\left(2bx^2\right)+bx^2\cos(2a)\,\text{Si}\left(2bx^2\right)+\cos^2\left(a+bx^2\right)}{2x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^2/x^3,x]
```

```
[Out] -1/2*(Cos[a + b*x^2]^2 + b*x^2*CosIntegral[2*b*x^2]*Sin[2*a] + b*x^2*Cos[2*
a]*SinIntegral[2*b*x^2])/x^2
```

fricas [A] time = 0.67, size = 65, normalized size = 1.14

$$-\frac{2\,bx^2\cos\left(2\,a\right)\mathrm{Si}\left(2\,bx^2\right)+2\,\cos\left(bx^2+a\right)^2+\left(bx^2\,\mathrm{Ci}\left(2\,bx^2\right)+bx^2\,\mathrm{Ci}\left(-2\,bx^2\right)\right)\sin\left(2\,a\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^2*cos(2*a)*sin_integral(2*b*x^2) + 2*cos(b*x^2 + a)^2 + (b*x^2*
cos_integral(2*b*x^2) + b*x^2*cos_integral(-2*b*x^2))*sin(2*a))/x^2
```

giac [B] time = 0.40, size = 107, normalized size = 1.88

$$-\frac{2\left(bx^2+a\right)b^2\,\mathrm{Ci}\left(2\,bx^2\right)\sin\left(2\,a\right)-2\,ab^2\,\mathrm{Ci}\left(2\,bx^2\right)\sin\left(2\,a\right)-2\left(bx^2+a\right)b^2\cos\left(2\,a\right)\mathrm{Si}\left(-2\,bx^2\right)+2\,ab^2\cos\left(2\,a\right.}{4\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*(b*x^2 + a)*b^2*cos_integral(2*b*x^2)*sin(2*a) - 2*a*b^2*cos_integr
al(2*b*x^2)*sin(2*a) - 2*(b*x^2 + a)*b^2*cos(2*a)*sin_integral(-2*b*x^2) +
2*a*b^2*cos(2*a)*sin_integral(-2*b*x^2) + b^2*cos(2*b*x^2 + 2*a) + b^2)/(b^
2*x^2)
```

maple [C] time = 0.16, size = 89, normalized size = 1.56

$$-\frac{1}{4x^2}+\frac{\pi\,\mathrm{e}^{-2ia}\mathrm{csgn}\left(b\,x^2\right)b}{4}-\frac{\mathrm{e}^{-2ia}\,\mathrm{Si}\left(2b\,x^2\right)b}{2}+\frac{i\mathrm{e}^{-2ia}\,\mathrm{Ei}\left(1,-2ib\,x^2\right)b}{4}-\frac{i\mathrm{e}^{2ia}b\,\mathrm{Ei}\left(1,-2ib\,x^2\right)}{4}-\frac{\cos\left(2b\,x^2+2a\right)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)^2/x^3,x)
```

```
[Out] -1/4/x^2+1/4*Pi*exp(-2*I*a)*csgn(b*x^2)*b-1/2*exp(-2*I*a)*Si(2*b*x^2)*b+1/4
*I*exp(-2*I*a)*Ei(1,-2*I*b*x^2)*b-1/4*I*exp(2*I*a)*b*Ei(1,-2*I*b*x^2)-1/4*c
os(2*b*x^2+2*a)/x^2
```

maxima [C] time = 1.74, size = 61, normalized size = 1.07

$$-\frac{\left(\left(i\,\Gamma\left(-1,2i\,bx^2\right)-i\,\Gamma\left(-1,-2i\,bx^2\right)\right)\cos\left(2\,a\right)+\left(\Gamma\left(-1,2i\,bx^2\right)+\Gamma\left(-1,-2i\,bx^2\right)\right)\sin\left(2\,a\right)\right)bx^2+1}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(((I*gamma(-1, 2*I*b*x^2) - I*gamma(-1, -2*I*b*x^2))*cos(2*a) + (gamma
(-1, 2*I*b*x^2) + gamma(-1, -2*I*b*x^2))*sin(2*a))*b*x^2 + 1)/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(b\,x^2+a\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)^2/x^3,x)
```

```
[Out] int(cos(a + b*x^2)^2/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(a+bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)**2/x**3,x)
```

```
[Out] Integral(cos(a + b*x**2)**2/x**3, x)
```

3.15 $\int x^3 \cos^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=79

$$\frac{\cos^3\left(a+bx^2\right)}{18b^2}+\frac{\cos\left(a+bx^2\right)}{3b^2}+\frac{x^2\sin\left(a+bx^2\right)}{3b}+\frac{x^2\sin\left(a+bx^2\right)\cos^2\left(a+bx^2\right)}{6b}$$

```
[Out] 1/3*cos(b*x^2+a)/b^2+1/18*cos(b*x^2+a)^3/b^2+1/3*x^2*sin(b*x^2+a)/b+1/6*x^2
*cos(b*x^2+a)^2*sin(b*x^2+a)/b
```

Rubi [A] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3380, 3310, 3296, 2638}

$$\frac{\cos^3\left(a+bx^2\right)}{18b^2}+\frac{\cos\left(a+bx^2\right)}{3b^2}+\frac{x^2\sin\left(a+bx^2\right)}{3b}+\frac{x^2\sin\left(a+bx^2\right)\cos^2\left(a+bx^2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[a + b*x^2]^3,x]
```

```
[Out] Cos[a + b*x^2]/(3*b^2) + Cos[a + b*x^2]^3/(18*b^2) + (x^2*Sin[a + b*x^2])/(
3*b) + (x^2*Cos[a + b*x^2]^2*Sin[a + b*x^2])/(6*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^3\cos^3\left(a+bx^2\right)dx &= \frac{1}{2}\text{Subst}\left(\int x\cos^3(a+bx)\,dx,x,x^2\right)\\ &= \frac{\cos^3\left(a+bx^2\right)}{18b^2}+\frac{x^2\cos^2\left(a+bx^2\right)\sin\left(a+bx^2\right)}{6b}+\frac{1}{3}\text{Subst}\left(\int x\cos(a+bx)\,dx,x,x^2\right.\\ &= \frac{\cos^3\left(a+bx^2\right)}{18b^2}+\frac{x^2\sin\left(a+bx^2\right)}{3b}+\frac{x^2\cos^2\left(a+bx^2\right)\sin\left(a+bx^2\right)}{6b}-\frac{\text{Subst}\left(\int\sin(a\right.}{3}\\ &= \frac{\cos\left(a+bx^2\right)}{3b^2}+\frac{\cos^3\left(a+bx^2\right)}{18b^2}+\frac{x^2\sin\left(a+bx^2\right)}{3b}+\frac{x^2\cos^2\left(a+bx^2\right)\sin\left(a+bx^2\right)}{6b}\end{aligned}$$

Mathematica [A] time = 0.14, size = 55, normalized size = 0.70

$$\frac{3bx^2\left(9\sin\left(a+bx^2\right)+\sin\left(3\left(a+bx^2\right)\right)\right)+27\cos\left(a+bx^2\right)+\cos\left(3\left(a+bx^2\right)\right)}{72b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[a + b*x^2]^3,x]
```

```
[Out] (27*Cos[a + b*x^2] + Cos[3*(a + b*x^2)] + 3*b*x^2*(9*Sin[a + b*x^2] + Sin[3
*(a + b*x^2)]))/(72*b^2)
```

fricas [A] time = 0.87, size = 58, normalized size = 0.73

$$\frac{\cos\left(bx^2+a\right)^3+3\left(bx^2\cos\left(bx^2+a\right)^2+2\,bx^2\right)\sin\left(bx^2+a\right)+6\,\cos\left(bx^2+a\right)}{18\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*(cos(b*x^2 + a)^3 + 3*(b*x^2*cos(b*x^2 + a)^2 + 2*b*x^2)*sin(b*x^2 + a
) + 6*cos(b*x^2 + a))/b^2
```

giac [A] time = 0.37, size = 58, normalized size = 0.73

$$\frac{3\,bx^2\sin\left(3\,bx^2+3\,a\right)+27\,bx^2\sin\left(bx^2+a\right)+\cos\left(3\,bx^2+3\,a\right)+27\,\cos\left(bx^2+a\right)}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/72*(3*b*x^2*sin(3*b*x^2 + 3*a) + 27*b*x^2*sin(b*x^2 + a) + cos(3*b*x^2 +
3*a) + 27*cos(b*x^2 + a))/b^2
```

maple [A] time = 0.04, size = 66, normalized size = 0.84

$$\frac{3x^2\sin\left(b\,x^2+a\right)}{8b}+\frac{3\cos\left(b\,x^2+a\right)}{8b^2}+\frac{x^2\sin\left(3b\,x^2+3a\right)}{24b}+\frac{\cos\left(3b\,x^2+3a\right)}{72b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(b*x^2+a)^3,x)
```

```
[Out] 3/8*x^2*sin(b*x^2+a)/b+3/8*cos(b*x^2+a)/b^2+1/24/b*x^2*sin(3*b*x^2+3*a)+1/7
2/b^2*cos(3*b*x^2+3*a)
```

maxima [A] time = 1.05, size = 58, normalized size = 0.73

$$\frac{3\,bx^2\sin\left(3\,bx^2+3\,a\right)+27\,bx^2\sin\left(bx^2+a\right)+\cos\left(3\,bx^2+3\,a\right)+27\,\cos\left(bx^2+a\right)}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/72*(3*b*x^2*sin(3*b*x^2 + 3*a) + 27*b*x^2*sin(b*x^2 + a) + cos(3*b*x^2 +
3*a) + 27*cos(b*x^2 + a))/b^2
```

mupad [B] time = 0.41, size = 66, normalized size = 0.84

$$\frac{\frac{\cos\left(b\,x^2+a\right)}{3}+\frac{{\cos\left(b\,x^2+a\right)}^3}{18}+b\,\left(\frac{x^2\,\sin\left(b\,x^2+a\right)}{3}+\frac{x^2\,{\cos\left(b\,x^2+a\right)}^2\,\sin\left(b\,x^2+a\right)}{6}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(a + b*x^2)^3,x)
```

```
[Out] (cos(a + b*x^2)/3 + cos(a + b*x^2)^3/18 + b*((x^2*sin(a + b*x^2))/3 + (x^2*
cos(a + b*x^2)^2*sin(a + b*x^2))/6))/b^2
```

sympy [A] time = 2.77, size = 92, normalized size = 1.16

$$\begin{cases} \frac{x^2 \sin^3\left(a+bx^2\right)}{3b} + \frac{x^2 \sin\left(a+bx^2\right)\cos^2\left(a+bx^2\right)}{2b} + \frac{\sin^2\left(a+bx^2\right)\cos\left(a+bx^2\right)}{3b^2} + \frac{7\cos^3\left(a+bx^2\right)}{18b^2} & \text{for } b \neq 0 \\ \frac{x^4 \cos^3(a)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(b*x**2+a)**3,x)
```

```
[Out] Piecewise((x**2*sin(a + b*x**2)**3/(3*b) + x**2*sin(a + b*x**2)*cos(a + b*x
**2)**2/(2*b) + sin(a + b*x**2)**2*cos(a + b*x**2)/(3*b**2) + 7*cos(a + b*x
**2)**3/(18*b**2), Ne(b, 0)), (x**4*cos(a)**3/4, True))
```

3.16 $\int x^2 \cos^3\left(a+bx^2\right) dx$

Optimal. Leaf size=188

$$-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sin(3a)C\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}}+\frac{3x}{}$$

```
[Out] 3/8*x*sin(b*x^2+a)/b+1/24*x*sin(3*b*x^2+3*a)/b-1/144*cos(3*a)*FresnelS(x*b^
(1/2)*6^(1/2)/Pi^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)-1/144*FresnelC(x*b^(1/2)*6
^(1/2)/Pi^(1/2))*sin(3*a)*6^(1/2)*Pi^(1/2)/b^(3/2)-3/16*cos(a)*FresnelS(x*b
^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)-3/16*FresnelC(x*b^(1/2)*2
^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(3/2)
```

Rubi [A] time = 0.18, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3404, 3386, 3353, 3352, 3351}

$$-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sin(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}x\right)}{24b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)S}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*x^2]^3,x]
```

```
[Out] (-3*Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x])/(8*b^(3/2)) - (Sqrt[P
i/6]*Cos[3*a]*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x])/(24*b^(3/2)) - (3*Sqrt[Pi/2]*
FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/(8*b^(3/2)) - (Sqrt[Pi/6]*FresnelC[S
qrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/(24*b^(3/2)) + (3*x*Sin[a + b*x^2])/(8*b) +
(x*Sin[3*a + 3*b*x^2])/(24*b)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3404

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \cos^3\left(a+bx^2\right) dx &= \int \left(\frac{3}{4}x^2 \cos\left(a+bx^2\right)+\frac{1}{4}x^2 \cos\left(3a+3bx^2\right)\right) dx \\
&= \frac{1}{4}\int x^2 \cos\left(3a+3bx^2\right) dx+\frac{3}{4}\int x^2 \cos\left(a+bx^2\right) dx \\
&= \frac{3x\sin\left(a+bx^2\right)}{8b}+\frac{x\sin\left(3a+3bx^2\right)}{24b}-\frac{\int \sin\left(3a+3bx^2\right) dx}{24b}-\frac{3\int \sin\left(a+bx^2\right) dx}{8b} \\
&= \frac{3x\sin\left(a+bx^2\right)}{8b}+\frac{x\sin\left(3a+3bx^2\right)}{24b}-\frac{(3\cos(a))\int \sin\left(bx^2\right) dx}{8b}-\frac{\cos(3a)\int \sin\left(3b\right.}{24b} \\
&= -\frac{3\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\cos(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{24b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{8b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 160, normalized size = 0.85

$$\frac{-27\sqrt{2\pi}\sin(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{6\pi}\sin(3a)C\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)-27\sqrt{2\pi}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\sqrt{6\pi}\cos(3a)S\left(\sqrt{b}\right.}{144b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*x^2]^3,x]
```

```
[Out] (-27*Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x] - Sqrt[6*Pi]*Cos[3*a]
*FresnelS[Sqrt[b]*Sqrt[6/Pi]*x] - 27*Sqrt[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]
```

```
*x]*Sin[a] - Sqrt[6*Pi]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a] + 54*Sqrt[b
]*x*Sin[a + b*x^2] + 6*Sqrt[b]*x*Sin[3*(a + b*x^2)])/(144*b^(3/2))
```

fricas [A] time = 0.66, size = 148, normalized size = 0.79

$$-\frac{\sqrt{6}\,\pi\sqrt{\frac{b}{\pi}}\,\cos(3\,a)\,S\left(\sqrt{6}\,x\sqrt{\frac{b}{\pi}}\right) + 27\,\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\cos(a)\,S\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right) + \sqrt{6}\,\pi\sqrt{\frac{b}{\pi}}\,C\left(\sqrt{6}\,x\sqrt{\frac{b}{\pi}}\right)\sin(3\,a) + 27\,\sqrt{2}\cdots}{144\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/144*(sqrt(6)*pi*sqrt(b/pi)*cos(3*a)*fresnel_sin(sqrt(6)*x*sqrt(b/pi)) +
27*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)) + sqrt(6)
*pi*sqrt(b/pi)*fresnel_cos(sqrt(6)*x*sqrt(b/pi))*sin(3*a) + 27*sqrt(2)*pi*s
qrt(b/pi)*fresnel_cos(sqrt(2)*x*sqrt(b/pi))*sin(a) - 24*(b*x*cos(b*x^2 + a)
^2 + 2*b*x)*sin(b*x^2 + a))/b^2
```

giac [C] time = 0.52, size = 259, normalized size = 1.38

$$-\frac{i\,xe^{\left(3i\,bx^2+3i\,a\right)}}{48\,b} - \frac{3i\,xe^{\left(i\,bx^2+i\,a\right)}}{16\,b} + \frac{3i\,xe^{\left(-i\,bx^2-i\,a\right)}}{16\,b} + \frac{i\,xe^{\left(-3i\,bx^2-3i\,a\right)}}{48\,b} - \frac{i\,\sqrt{6}\,\sqrt{\pi}\,\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{6}\,\sqrt{b}\,x\left(-\frac{i\,b}{|b|}+1\right)\right)e^{(3i\,a)}}{288\,b^{\frac{3}{2}}\left(-\frac{i\,b}{|b|}+1\right)} - \frac{3i\,\sqrt{2}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/48*I*x*e^(3*I*b*x^2 + 3*I*a)/b - 3/16*I*x*e^(I*b*x^2 + I*a)/b + 3/16*I*x
*e^(-I*b*x^2 - I*a)/b + 1/48*I*x*e^(-3*I*b*x^2 - 3*I*a)/b - 1/288*I*sqrt(6)
*sqrt(pi)*erf(-1/2*sqrt(6)*sqrt(b)*x*(-I*b/abs(b) + 1))*e^(3*I*a)/(b^(3/2)*
(-I*b/abs(b) + 1)) - 3/32*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(-I*b/abs(b
) + 1)*sqrt(abs(b)))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + 3/32*I*sq
rt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(
b*(I*b/abs(b) + 1)*sqrt(abs(b))) + 1/288*I*sqrt(6)*sqrt(pi)*erf(-1/2*sqrt(6
)*sqrt(b)*x*(I*b/abs(b) + 1))*e^(-3*I*a)/(b^(3/2)*(I*b/abs(b) + 1))
```

maple [A] time = 0.04, size = 130, normalized size = 0.69

$$\frac{3x\sin\left(b\,x^2+a\right)}{8b} - \frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(a)\,\mathrm{FresnelC}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{16b^{\frac{3}{2}}} + \frac{x\sin\left(3b\,x^2+3a\right)}{24b} - \frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(b*x^2+a)^3,x)
```

```
[Out] 3/8*x*sin(b*x^2+a)/b-3/16/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(x*b^(1/
2)*2^(1/2)/Pi^(1/2))+sin(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2)))+1/24*x*si
n(3*b*x^2+3*a)/b-1/144/b^(3/2)*2^(1/2)*Pi^(1/2)*3^(1/2)*(cos(3*a)*FresnelS(
2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x)+sin(3*a)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1
/2)*b^(1/2)*x))
```

maxima [C] time = 0.96, size = 143, normalized size = 0.76

$$\frac{72\,b^2x\sin\left(3\,bx^2+3\,a\right)+648\,b^2x\sin\left(bx^2+a\right)+3\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\left(\left(-\left(i+1\right)\,\cos\left(3\,a\right)+\left(i-1\right)\,\sin\left(3\,a\right)\right)\mathrm{erf}\left(\sqrt{3i\,b}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1728*(72*b^2*x*sin(3*b*x^2 + 3*a) + 648*b^2*x*sin(b*x^2 + a) + 3*9^(1/4)*
sqrt(2)*sqrt(pi)*((-(I + 1)*cos(3*a) + (I - 1)*sin(3*a))*erf(sqrt(3*I*b)*x)
 + ((I - 1)*cos(3*a) - (I + 1)*sin(3*a))*erf(sqrt(-3*I*b)*x))*b^(3/2) + sqr
t(2)*sqrt(pi)*((-(81*I + 81)*cos(a) + (81*I - 81)*sin(a))*erf(sqrt(I*b)*x)
+ ((81*I - 81)*cos(a) - (81*I + 81)*sin(a))*erf(sqrt(-I*b)*x))*b^(3/2))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\cos\left(b\,x^2+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a + b*x^2)^3,x)
```

```
[Out] int(x^2*cos(a + b*x^2)^3, x)
```

sympy [B] time = 4.38, size = 439, normalized size = 2.34

$$\frac{3b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\sin\left(a\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\right|-\frac{b^2x^4}{4}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}+\frac{3b^{\frac{3}{2}}x^5\sqrt{\frac{1}{b}}\sin\left(3a\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\right|-\frac{9b^2x^4}{4}\right)}{32\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}-\frac{3\sqrt{b}\,x^3\sqrt{\frac{1}{b}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(b*x**2+a)**3,x)
```

```
[Out] 3*b**(3/2)*x**5*sqrt(1/b)*sin(a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3
/2, 7/4, 9/4), -b**2*x**4/4)/(32*gamma(7/4)*gamma(9/4)) + 3*b**(3/2)*x**5*s
```

```
qrt(1/b)*sin(3*a)*gamma(3/4)*gamma(5/4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4),
-9*b**2*x**4/4)/(32*gamma(7/4)*gamma(9/4)) - 3*sqrt(b)*x**3*sqrt(1/b)*cos(a
)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*x**4/4)/(3
2*gamma(5/4)*gamma(7/4)) - sqrt(b)*x**3*sqrt(1/b)*cos(3*a)*gamma(1/4)*gamma
(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -9*b**2*x**4/4)/(32*gamma(5/4)*gam
ma(7/4)) - 3*sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*sin(a)*fresnels(sqrt(2)*sqrt(b
)*x/sqrt(pi))/8 - sqrt(6)*sqrt(pi)*x**2*sqrt(1/b)*sin(3*a)*fresnels(sqrt(6)
*sqrt(b)*x/sqrt(pi))/24 + 3*sqrt(2)*sqrt(pi)*x**2*sqrt(1/b)*cos(a)*fresnelc
(sqrt(2)*sqrt(b)*x/sqrt(pi))/8 + sqrt(6)*sqrt(pi)*x**2*sqrt(1/b)*cos(3*a)*f
resnelc(sqrt(6)*sqrt(b)*x/sqrt(pi))/24
```

3.17 $\int x\cos^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=33

$$\frac{\sin\left(a+bx^2\right)}{2b}-\frac{\sin^3\left(a+bx^2\right)}{6b}$$

```
[Out] 1/2*sin(b*x^2+a)/b-1/6*sin(b*x^2+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=$ 0.167, Rules used = {3380, 2633}

$$\frac{\sin\left(a+bx^2\right)}{2b}-\frac{\sin^3\left(a+bx^2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x^2]^3,x]
```

```
[Out] Sin[a + b*x^2]/(2*b) - Sin[a + b*x^2]^3/(6*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x\cos^3\left(a+bx^2\right)\,dx&=\frac{1}{2}\text{Subst}\left(\int\cos^3(a+bx)\,dx,x,x^2\right)\\&=-\frac{\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin\left(a+bx^2\right)\right)}{2b}\\&=\frac{\sin\left(a+bx^2\right)}{2b}-\frac{\sin^3\left(a+bx^2\right)}{6b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{\sin\left(a+bx^2\right)}{2b}-\frac{\sin^3\left(a+bx^2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x^2]^3,x]
[Out] Sin[a + b*x^2]/(2*b) - Sin[a + b*x^2]^3/(6*b)
```

fricas [A] time = 0.72, size = 25, normalized size = 0.76

$$\frac{\left(\cos\left(bx^2+a\right)^2+2\right)\sin\left(bx^2+a\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x^2+a)^3,x, algorithm="fricas")
[Out] 1/6*(cos(b*x^2 + a)^2 + 2)*sin(b*x^2 + a)/b
```

giac [A] time = 0.37, size = 26, normalized size = 0.79

$$-\frac{\sin\left(bx^2+a\right)^3-3\,\sin\left(bx^2+a\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x^2+a)^3,x, algorithm="giac")
[Out] -1/6*(sin(b*x^2 + a)^3 - 3*sin(b*x^2 + a))/b
```

maple [A] time = 0.03, size = 26, normalized size = 0.79

$$\frac{\left(2+\cos^2\left(b\,x^2+a\right)\right)\sin\left(b\,x^2+a\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x^2+a)^3,x)
[Out] 1/6/b*(2+cos(b*x^2+a)^2)*sin(b*x^2+a)
```

maxima [A] time = 0.47, size = 27, normalized size = 0.82

$$\frac{\sin\left(3\,bx^2+3\,a\right)+9\,\sin\left(bx^2+a\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(sin(3*b*x^2 + 3*a) + 9*sin(b*x^2 + a))/b
```

mupad [**B**] time = 0.29, size = 28, normalized size = 0.85

$$\frac{3\,\sin\left(b\,x^2+a\right)-\sin\left(b\,x^2+a\right)^3}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x^2)^3,x)
```

```
[Out] (3*sin(a + b*x^2) - sin(a + b*x^2)^3)/(6*b)
```

sympy [**A**] time = 0.77, size = 44, normalized size = 1.33

$$\begin{cases} \frac{\sin^{3}\left(a+bx^{2}\right)}{3b} + \frac{\sin\left(a+bx^{2}\right)\cos^{2}\left(a+bx^{2}\right)}{2b} & \text{for}\ b \neq 0 \\ \frac{x^{2}\cos^{3}\left(a\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x**2+a)**3,x)
```

```
[Out] Piecewise((sin(a + b*x**2)**3/(3*b) + sin(a + b*x**2)*cos(a + b*x**2)**2/(2
*b), Ne(b, 0)), (x**2*cos(a)**3/2, True))
```

3.18 $\int \cos^3\left(a+bx^2\right)\,dx$

Optimal. Leaf size=153

$$\frac{3\sqrt{\frac{\pi}{2}}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}}+\frac{\sqrt{\frac{\pi}{6}}\cos(3a)C\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}}-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\sin(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}}$$

```
[Out] 1/24*cos(3*a)*FresnelC(x*b^(1/2)*6^(1/2)/Pi^(1/2))*6^(1/2)*Pi^(1/2)/b^(1/2)
-1/24*FresnelS(x*b^(1/2)*6^(1/2)/Pi^(1/2))*sin(3*a)*6^(1/2)*Pi^(1/2)/b^(1/2
)+3/8*cos(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(1/2)-
3/8*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3358, 3354, 3352, 3351}

$$\frac{3\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)}{4\sqrt{b}}+\frac{\sqrt{\frac{\pi}{6}}\cos(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}x\right)}{4\sqrt{b}}-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}}-\frac{\sqrt{\frac{\pi}{6}}\sin(3a)S\left(\right.}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]^3,x]
```

```
[Out] (3*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x])/(4*Sqrt[b]) + (Sqrt[Pi
/6]*Cos[3*a]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x])/(4*Sqrt[b]) - (3*Sqrt[Pi/2]*Fr
esnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/(4*Sqrt[b]) - (Sqrt[Pi/6]*FresnelS[Sqr
t[b]*Sqrt[6/Pi]*x]*Sin[3*a])/(4*Sqrt[b])
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cos^3\left(a+bx^2\right)dx &= \int \left(\frac{3}{4}\cos\left(a+bx^2\right)+\frac{1}{4}\cos\left(3a+3bx^2\right)\right)dx\\ &= \frac{1}{4}\int \cos\left(3a+3bx^2\right)dx+\frac{3}{4}\int \cos\left(a+bx^2\right)dx\\ &= \frac{1}{4}(3\cos(a))\int \cos\left(bx^2\right)dx+\frac{1}{4}\cos(3a)\int \cos\left(3bx^2\right)dx-\frac{1}{4}(3\sin(a))\int \sin\left(bx^2\right)dx\\ &= \frac{3\sqrt{\frac{\pi}{2}}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)}{4\sqrt{b}}+\frac{\sqrt{\frac{\pi}{6}}\cos(3a)C\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)}{4\sqrt{b}}-\frac{3\sqrt{\frac{\pi}{2}}S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)\sin(a)}{4\sqrt{b}}-\end{aligned}$$

Mathematica [A] time = 0.23, size = 116, normalized size = 0.76

$$\frac{\sqrt{\frac{\pi}{6}}\left(3\sqrt{3}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)+\cos(3a)C\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)-3\sqrt{3}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\sin(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^3,x]
```

```
[Out] (Sqrt[Pi/6]*(3*Sqrt[3]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x] + Cos[3*a]*Fre
snelC[Sqrt[b]*Sqrt[6/Pi]*x] - 3*Sqrt[3]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[
a] - FresnelS[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a]))/(4*Sqrt[b])
```

fricas [A] time = 1.37, size = 121, normalized size = 0.79

$$\frac{\sqrt{6}\,\pi\sqrt{\frac{b}{\pi}}\cos(3\,a)\,\mathrm{C}\left(\sqrt{6}\,x\sqrt{\frac{b}{\pi}}\right)+9\,\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{C}\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)-\sqrt{6}\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{6}\,x\sqrt{\frac{b}{\pi}}\right)\sin(3\,a)-9\,\sqrt{2}\,\pi}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(6)*pi*sqrt(b/pi)*cos(3*a)*fresnel_cos(sqrt(6)*x*sqrt(b/pi)) + 9*
sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x*sqrt(b/pi)) - sqrt(6)*pi
```

```
*sqrt(b/pi)*fresnel_sin(sqrt(6)*x*sqrt(b/pi))*sin(3*a) - 9*sqrt(2)*pi*sqrt(
b/pi)*fresnel_sin(sqrt(2)*x*sqrt(b/pi))*sin(a))/b
```

giac [C] time = 0.60, size = 185, normalized size = 1.21

$$-\frac{\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{6}\sqrt{b}\,x\left(-\frac{ib}{|b|}+1\right)\right)e^{(3ia)}}{48\sqrt{b}\left(-\frac{ib}{|b|}+1\right)}-\frac{3\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\,x\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(ia)}}{16\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}}-\frac{3\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\,x\left(\frac{ib}{|b|}\right.\right.}{16\left(\frac{ib}{|b|}+1\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(6)*sqrt(pi)*erf(-1/2*sqrt(6)*sqrt(b)*x*(-I*b/abs(b) + 1))*e^(3*I
*a)/(sqrt(b)*(-I*b/abs(b) + 1)) - 3/16*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*
(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/((-I*b/abs(b) + 1)*sqrt(abs(b))) -
3/16*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(
-I*a)/((I*b/abs(b) + 1)*sqrt(abs(b))) - 1/48*sqrt(6)*sqrt(pi)*erf(-1/2*sqrt
(6)*sqrt(b)*x*(I*b/abs(b) + 1))*e^(-3*I*a)/(sqrt(b)*(I*b/abs(b) + 1))
```

maple [A] time = 0.04, size = 101, normalized size = 0.66

$$\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin(a)\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{8\sqrt{b}}+\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\left(\cos(3a)\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)-\sin(\right.}{24\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)^3,x)
```

```
[Out] 3/8*2^(1/2)*Pi^(1/2)/b^(1/2)*(cos(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))-s
in(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2)))+1/24*2^(1/2)*Pi^(1/2)*3^(1/2)/b
^(1/2)*(cos(3*a)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x)-sin(3*a)*Fres
nelS(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(1/2)*x))
```

maxima [C] time = 1.29, size = 112, normalized size = 0.73

$$-\frac{3\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\left(((i-1)\,\cos(3\,a)+(i+1)\,\sin(3\,a))\,\mathrm{erf}\left(\sqrt{3i\,b}\,x\right)+(-(i+1)\,\cos(3\,a)-(i-1)\,\sin(3\,a))\,\mathrm{erf}\left(\sqrt{-3}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/288*(3*9^(1/4)*sqrt(2)*sqrt(pi)*(((I - 1)*cos(3*a) + (I + 1)*sin(3*a))*e
rf(sqrt(3*I*b)*x) + (-(I + 1)*cos(3*a) - (I - 1)*sin(3*a))*erf(sqrt(-3*I*b)
```

```
*x))*b^(3/2) + sqrt(2)*sqrt(pi)*(((27*I - 27)*cos(a) + (27*I + 27)*sin(a))*
erf(sqrt(I*b)*x) + (-(27*I + 27)*cos(a) - (27*I - 27)*sin(a))*erf(sqrt(-I*b
)*x))*b^(3/2))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(b\,x^2+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)^3,x)
```

```
[Out] int(cos(a + b*x^2)^3, x)
```

sympy [A] time = 1.14, size = 129, normalized size = 0.84

$$\frac{3\sqrt{2}\,\sqrt{\pi}\left(-\sin\left(a\right)S\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)+\cos\left(a\right)C\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)\right)\sqrt{\frac{1}{b}}}{8}+\frac{\sqrt{6}\,\sqrt{\pi}\left(-\sin\left(3a\right)S\left(\frac{\sqrt{6}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)+\cos\left(3a\right)C\left(\frac{\sqrt{6}\,\sqrt{b}\,x}{\sqrt{\pi}}\right)\right)\cdots}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)**3,x)
```

```
[Out] 3*sqrt(2)*sqrt(pi)*(-sin(a)*fresnels(sqrt(2)*sqrt(b)*x/sqrt(pi)) + cos(a)*f
resnelc(sqrt(2)*sqrt(b)*x/sqrt(pi)))*sqrt(1/b)/8 + sqrt(6)*sqrt(pi)*(-sin(3
*a)*fresnels(sqrt(6)*sqrt(b)*x/sqrt(pi)) + cos(3*a)*fresnelc(sqrt(6)*sqrt(b
)*x/sqrt(pi)))*sqrt(1/b)/24
```

3.19 $\int \frac{\cos^3(a+bx^2)}{x} \, dx$

Optimal. Leaf size=55

$$\frac{3}{8}\cos(a)\text{Ci}\left(bx^2\right)+\frac{1}{8}\cos(3a)\text{Ci}\left(3bx^2\right)-\frac{3}{8}\sin(a)\text{Si}\left(bx^2\right)-\frac{1}{8}\sin(3a)\text{Si}\left(3bx^2\right)$$

```
[Out] 3/8*Ci(b*x^2)*cos(a)+1/8*Ci(3*b*x^2)*cos(3*a)-3/8*Si(b*x^2)*sin(a)-1/8*Si(3
*b*x^2)*sin(3*a)
```

Rubi [A] time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3404, 3378, 3376, 3375}

$$\frac{3}{8}\cos(a)\text{CosIntegral}\left(bx^2\right)+\frac{1}{8}\cos(3a)\text{CosIntegral}\left(3bx^2\right)-\frac{3}{8}\sin(a)\text{Si}\left(bx^2\right)-\frac{1}{8}\sin(3a)\text{Si}\left(3bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]^3/x,x]
```

```
[Out] (3*Cos[a]*CosIntegral[b*x^2])/8 + (Cos[3*a]*CosIntegral[3*b*x^2])/8 - (3*Si
n[a]*SinIntegral[b*x^2])/8 - (Sin[3*a]*SinIntegral[3*b*x^2])/8
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3404

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3\left(a+bx^2\right)}{x}\,dx = \int \left(\frac{3\cos\left(a+bx^2\right)}{4x}+\frac{\cos\left(3a+3bx^2\right)}{4x}\right)dx$$
$$= \frac{1}{4}\int \frac{\cos\left(3a+3bx^2\right)}{x}\,dx+\frac{3}{4}\int \frac{\cos\left(a+bx^2\right)}{x}\,dx$$
$$= \frac{1}{4}(3\cos(a))\int \frac{\cos\left(bx^2\right)}{x}\,dx+\frac{1}{4}\cos(3a)\int \frac{\cos\left(3bx^2\right)}{x}\,dx-\frac{1}{4}(3\sin(a))\int \frac{\sin\left(bx^2\right)}{x}\,dx$$
$$= \frac{3}{8}\cos(a)\mathrm{Ci}\left(bx^2\right)+\frac{1}{8}\cos(3a)\mathrm{Ci}\left(3bx^2\right)-\frac{3}{8}\sin(a)\mathrm{Si}\left(bx^2\right)-\frac{1}{8}\sin(3a)\mathrm{Si}\left(3bx^2\right)$$

Mathematica [A] time = 0.09, size = 50, normalized size = 0.91

$$\frac{1}{8}\left(3\cos(a)\mathrm{Ci}\left(bx^2\right)+\cos(3a)\mathrm{Ci}\left(3bx^2\right)-3\sin(a)\mathrm{Si}\left(bx^2\right)-\sin(3a)\mathrm{Si}\left(3bx^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^3/x,x]
```

```
[Out] (3*Cos[a]*CosIntegral[b*x^2] + Cos[3*a]*CosIntegral[3*b*x^2] - 3*Sin[a]*Sin
Integral[b*x^2] - Sin[3*a]*SinIntegral[3*b*x^2])/8
```

fricas [A] time = 1.52, size = 63, normalized size = 1.15

$$\frac{1}{16}\left(\mathrm{Ci}\left(3\,bx^2\right)+\mathrm{Ci}\left(-3\,bx^2\right)\right)\cos\left(3\,a\right)+\frac{3}{16}\left(\mathrm{Ci}\left(bx^2\right)+\mathrm{Ci}\left(-bx^2\right)\right)\cos(a)-\frac{1}{8}\,\sin\left(3\,a\right)\mathrm{Si}\left(3\,bx^2\right)-\frac{3}{8}\,\sin(a)\,\mathrm{Si}\left(bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/16*(cos_integral(3*b*x^2) + cos_integral(-3*b*x^2))*cos(3*a) + 3/16*(cos_
integral(b*x^2) + cos_integral(-b*x^2))*cos(a) - 1/8*sin(3*a)*sin_integral(
3*b*x^2) - 3/8*sin(a)*sin_integral(b*x^2)
```

giac [A] time = 0.61, size = 47, normalized size = 0.85

$$\frac{1}{8}\cos\left(3\,a\right)\mathrm{Ci}\left(3\,bx^2\right)+\frac{3}{8}\cos(a)\,\mathrm{Ci}\left(bx^2\right)-\frac{3}{8}\sin(a)\,\mathrm{Si}\left(bx^2\right)+\frac{1}{8}\sin\left(3\,a\right)\mathrm{Si}\left(-3\,bx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/8*cos(3*a)*cos_integral(3*b*x^2) + 3/8*cos(a)*cos_integral(b*x^2) - 3/8*s
in(a)*sin_integral(b*x^2) + 1/8*sin(3*a)*sin_integral(-3*b*x^2)
```

maple [C] time = 0.17, size = 125, normalized size = 2.27

$$\frac{i e^{-3ia}\pi \operatorname{csgn}\left(b x^2\right)}{16}-\frac{i e^{-3ia} \operatorname{Si}\left(3b x^2\right)}{8}-\frac{e^{-3ia} \operatorname{Ei}\left(1,-3ib x^2\right)}{16}+\frac{3i\pi \operatorname{csgn}\left(b x^2\right) e^{-ia}}{16}-\frac{3i e^{-ia} \operatorname{Si}\left(b x^2\right)}{8}-\frac{3 e^{-ia} \operatorname{Ei}\left(1,-ib\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)^3/x,x)
```

```
[Out] 1/16*I*exp(-3*I*a)*Pi*csgn(b*x^2)-1/8*I*exp(-3*I*a)*Si(3*b*x^2)-1/16*exp(-3
*I*a)*Ei(1,-3*I*b*x^2)+3/16*I*Pi*csgn(b*x^2)*exp(-I*a)-3/8*I*exp(-I*a)*Si(b
*x^2)-3/16*exp(-I*a)*Ei(1,-I*b*x^2)-3/16*exp(I*a)*Ei(1,-I*b*x^2)-1/16*exp(3
*I*a)*Ei(1,-3*I*b*x^2)
```

maxima [C] time = 1.51, size = 89, normalized size = 1.62

$$\frac{1}{16}\left(\operatorname{Ei}\left(3i\,bx^2\right)+\operatorname{Ei}\left(-3i\,bx^2\right)\right)\cos\left(3\,a\right)+\frac{3}{16}\left(\operatorname{Ei}\left(i\,bx^2\right)+\operatorname{Ei}\left(-i\,bx^2\right)\right)\cos(a)+\frac{1}{16}\left(i\operatorname{Ei}\left(3i\,bx^2\right)-i\operatorname{Ei}\left(-3i\,bx^2\right)\right)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*(Ei(3*I*b*x^2) + Ei(-3*I*b*x^2))*cos(3*a) + 3/16*(Ei(I*b*x^2) + Ei(-I*
b*x^2))*cos(a) + 1/16*(I*Ei(3*I*b*x^2) - I*Ei(-3*I*b*x^2))*sin(3*a) + 1/16*
(3*I*Ei(I*b*x^2) - 3*I*Ei(-I*b*x^2))*sin(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(b\,x^2+a\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)^3/x,x)
```

```
[Out] int(cos(a + b*x^2)^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3\left(a+bx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)**3/x,x)
```

```
[Out] Integral(cos(a + b*x**2)**3/x, x)
```

3.20 $\int \frac{\cos^3(a+bx^2)}{x^2} dx$

Optimal. Leaf size=168

$$-\frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\frac{1}{2}\sqrt{\frac{3\pi}{2}}\sqrt{b}\sin(3a)C\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)-\frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\frac{1}{2}\sqrt{\frac{3\pi}{2}}\sqrt{b}$$

```
[Out] -cos(b*x^2+a)^3/x-3/4*cos(a)*FresnelS(x*b^(1/2)*2^(1/2)/Pi^(1/2))*b^(1/2)*2
^(1/2)*Pi^(1/2)-3/4*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*b^(1/2)*2^(
1/2)*Pi^(1/2)-1/4*cos(3*a)*FresnelS(x*b^(1/2)*6^(1/2)/Pi^(1/2))*b^(1/2)*6^(
1/2)*Pi^(1/2)-1/4*FresnelC(x*b^(1/2)*6^(1/2)/Pi^(1/2))*sin(3*a)*b^(1/2)*6^(
1/2)*Pi^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3394, 4574, 3353, 3352, 3351}

$$-\frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}x\right)-\frac{1}{2}\sqrt{\frac{3\pi}{2}}\sqrt{b}\sin(3a)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{b}x\right)-\frac{3}{2}\sqrt{\frac{\pi}{2}}\sqrt{b}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]^3/x^2,x]
```

```
[Out] -(Cos[a + b*x^2]^3/x) - (3*Sqrt[b]*Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[
2/Pi]*x])/2 - (Sqrt[b]*Sqrt[(3*Pi)/2]*Cos[3*a]*FresnelS[Sqrt[b]*Sqrt[6/Pi]*
x])/2 - (3*Sqrt[b]*Sqrt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]*Sin[a])/2 - (S
qrt[b]*Sqrt[(3*Pi)/2]*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/2
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
```

```
; FreeQ[{c, d, e, f}, x]
```

Rule 3394

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_)]^(p_)*(x_)^(m_.), x_Symbol] :> Simp[(x^(m +
 1)*Cos[a + b*x^n]^p)/(m + 1), x] + Dist[(b*n*p)/(m + 1), Int[Cos[a + b*x^n
]^(p - 1)*Sin[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
 + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3\left(a+bx^2\right)}{x^2}\,dx &= -\frac{\cos^3\left(a+bx^2\right)}{x}-(6b)\int \cos^2\left(a+bx^2\right)\sin\left(a+bx^2\right)\,dx\\
&= -\frac{\cos^3\left(a+bx^2\right)}{x}-(6b)\int \left(\frac{1}{4}\sin\left(a+bx^2\right)+\frac{1}{4}\sin\left(3a+3bx^2\right)\right)dx\\
&= -\frac{\cos^3\left(a+bx^2\right)}{x}-\frac{1}{2}(3b)\int \sin\left(a+bx^2\right)\,dx-\frac{1}{2}(3b)\int \sin\left(3a+3bx^2\right)\,dx\\
&= -\frac{\cos^3\left(a+bx^2\right)}{x}-\frac{1}{2}(3b\cos(a))\int \sin\left(bx^2\right)\,dx-\frac{1}{2}(3b\cos(3a))\int \sin\left(3bx^2\right)\,dx-\frac{1}{2}(3b\\
&= -\frac{\cos^3\left(a+bx^2\right)}{x}-\frac{3}{2}\sqrt{b}\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)-\frac{1}{2}\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos(3a)S\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)-
\end{aligned}$$

Mathematica [A] time = 0.64, size = 166, normalized size = 0.99

$$-\frac{3\sqrt{2\pi}\sqrt{b}\,x\sin(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)+\sqrt{6\pi}\sqrt{b}\,x\sin(3a)C\left(\sqrt{b}\sqrt{\frac{6}{\pi}}x\right)+3\sqrt{2\pi}\sqrt{b}\,x\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}x\right)+\sqrt{6\pi}\sqrt{b}}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^3/x^2,x]
```

```
[Out] -1/4*(3*Cos[a + b*x^2] + Cos[3*(a + b*x^2)] + 3*Sqrt[b]*Sqrt[2*Pi]*x*Cos[a]
*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x] + Sqrt[b]*Sqrt[6*Pi]*x*Cos[3*a]*FresnelS[Sq
```

```
rt[b]*Sqrt[6/Pi]*x] + 3*Sqrt[b]*Sqrt[2*Pi]*x*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x]
*Sin[a] + Sqrt[b]*Sqrt[6*Pi]*x*FresnelC[Sqrt[b]*Sqrt[6/Pi]*x]*Sin[3*a])/x
```

fricas [A] time = 0.86, size = 136, normalized size = 0.81

$$-\frac{\sqrt{6}\,\pi x\sqrt{\frac{b}{\pi}}\,\cos(3\,a)\,S\left(\sqrt{6}\,x\sqrt{\frac{b}{\pi}}\right)+3\,\sqrt{2}\,\pi x\sqrt{\frac{b}{\pi}}\,\cos(a)\,S\left(\sqrt{2}\,x\sqrt{\frac{b}{\pi}}\right)+\sqrt{6}\,\pi x\sqrt{\frac{b}{\pi}}\,C\left(\sqrt{6}\,x\sqrt{\frac{b}{\pi}}\right)\sin(3\,a)+3\,\sqrt{2}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(6)*pi*x*sqrt(b/pi)*cos(3*a)*fresnel_sin(sqrt(6)*x*sqrt(b/pi)) +
3*sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x*sqrt(b/pi)) + sqrt(6
)*pi*x*sqrt(b/pi)*fresnel_cos(sqrt(6)*x*sqrt(b/pi))*sin(3*a) + 3*sqrt(2)*pi
*x*sqrt(b/pi)*fresnel_cos(sqrt(2)*x*sqrt(b/pi))*sin(a) + 4*cos(b*x^2 + a)^3
)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx^2+a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^2 + a)^3/x^2, x)
```

maple [A] time = 0.04, size = 128, normalized size = 0.76

$$-\frac{3\cos\left(b\,x^2+a\right)}{4x}-\frac{3\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\mathrm{S}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(a)\,\mathrm{FresnelC}\left(\frac{x\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{4}-\frac{\cos\left(3b\,x^2+3a\right)}{4x}-\frac{\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)^3/x^2,x)
```

```
[Out] -3/4*cos(b*x^2+a)/x-3/4*b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(x*b^(1/2)
*2^(1/2)/Pi^(1/2))+sin(a)*FresnelC(x*b^(1/2)*2^(1/2)/Pi^(1/2)))-1/4*cos(3*b
*x^2+3*a)/x-1/4*b^(1/2)*2^(1/2)*Pi^(1/2)*3^(1/2)*(cos(3*a)*FresnelS(2^(1/2)
/Pi^(1/2)*3^(1/2)*b^(1/2)*x)+sin(3*a)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*b^(
1/2)*x))
```

maxima [C] time = 3.09, size = 151, normalized size = 0.90

$$\frac{\sqrt{3}\,\sqrt{bx^2}\left(\left(-(i+1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},3i\,bx^2\right)+(i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},-3i\,bx^2\right)\right)\cos(3\,a)+\left((i-1)\,\sqrt{2}\,\Gamma\left(-\frac{1}{2},3i\,bx^2\right)-(i+1)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/32*(sqrt(3)*sqrt(b*x^2)*((-(I + 1)*sqrt(2)*gamma(-1/2, 3*I*b*x^2) + (I -
1)*sqrt(2)*gamma(-1/2, -3*I*b*x^2))*cos(3*a) + ((I - 1)*sqrt(2)*gamma(-1/2,
 3*I*b*x^2) - (I + 1)*sqrt(2)*gamma(-1/2, -3*I*b*x^2))*sin(3*a)) + sqrt(b*x
^2)*((-(3*I + 3)*sqrt(2)*gamma(-1/2, I*b*x^2) + (3*I - 3)*sqrt(2)*gamma(-1/
2, -I*b*x^2))*cos(a) + ((3*I - 3)*sqrt(2)*gamma(-1/2, I*b*x^2) - (3*I + 3)*
sqrt(2)*gamma(-1/2, -I*b*x^2))*sin(a)))/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(b\,x^2+a\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)^3/x^2,x)
```

```
[Out] int(cos(a + b*x^2)^3/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3\left(a+bx^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)**3/x**2,x)
```

```
[Out] Integral(cos(a + b*x**2)**3/x**2, x)
```

3.21 $\int \frac{\cos^3(a+bx^2)}{x^3}\,dx$

Optimal. Leaf size=91

$$-\frac{3}{8}b\sin(a)\text{Ci}\left(bx^2\right)-\frac{3}{8}b\sin(3a)\text{Ci}\left(3bx^2\right)-\frac{3}{8}b\cos(a)\text{Si}\left(bx^2\right)-\frac{3}{8}b\cos(3a)\text{Si}\left(3bx^2\right)-\frac{3\cos\left(a+bx^2\right)}{8x^2}-\frac{\cos\left(3\left(a+b\right.\right.}{8x^2}$$

```
[Out] -3/8*cos(b*x^2+a)/x^2-1/8*cos(3*b*x^2+3*a)/x^2-3/8*b*cos(a)*Si(b*x^2)-3/8*b
*cos(3*a)*Si(3*b*x^2)-3/8*b*Ci(b*x^2)*sin(a)-3/8*b*Ci(3*b*x^2)*sin(3*a)
```

Rubi [A] time = 0.20, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3404, 3380, 3297, 3303, 3299, 3302}

$$-\frac{3}{8}b\sin(a)\text{CosIntegral}\left(bx^2\right)-\frac{3}{8}b\sin(3a)\text{CosIntegral}\left(3bx^2\right)-\frac{3}{8}b\cos(a)\text{Si}\left(bx^2\right)-\frac{3}{8}b\cos(3a)\text{Si}\left(3bx^2\right)-\frac{3\cos\left(a\right.}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]^3/x^3,x]
```

```
[Out] (-3*Cos[a + b*x^2])/(8*x^2) - Cos[3*(a + b*x^2)]/(8*x^2) - (3*b*CosIntegral
[b*x^2]*Sin[a])/8 - (3*b*CosIntegral[3*b*x^2]*Sin[3*a])/8 - (3*b*Cos[a]*Sin
Integral[b*x^2])/8 - (3*b*Cos[3*a]*SinIntegral[3*b*x^2])/8
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3404

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3\left(a+bx^2\right)}{x^3}\,dx &= \int \left(\frac{3\cos\left(a+bx^2\right)}{4x^3}+\frac{\cos\left(3a+3bx^2\right)}{4x^3}\right)dx\\
&= \frac{1}{4}\int \frac{\cos\left(3a+3bx^2\right)}{x^3}\,dx+\frac{3}{4}\int \frac{\cos\left(a+bx^2\right)}{x^3}\,dx\\
&= \frac{1}{8}\operatorname{Subst}\left(\int \frac{\cos(3a+3bx)}{x^2}\,dx,x,x^2\right)+\frac{3}{8}\operatorname{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx,x,x^2\right)\\
&= -\frac{3\cos\left(a+bx^2\right)}{8x^2}-\frac{\cos\left(3\left(a+bx^2\right)\right)}{8x^2}-\frac{1}{8}(3b)\operatorname{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx,x,x^2\right)-\frac{1}{8}(3b)\,\mathrm{S}\\
&= -\frac{3\cos\left(a+bx^2\right)}{8x^2}-\frac{\cos\left(3\left(a+bx^2\right)\right)}{8x^2}-\frac{1}{8}(3b\cos(a))\operatorname{Subst}\left(\int \frac{\sin(bx)}{x}\,dx,x,x^2\right)-\frac{1}{8}(3b\\
&= -\frac{3\cos\left(a+bx^2\right)}{8x^2}-\frac{\cos\left(3\left(a+bx^2\right)\right)}{8x^2}-\frac{3}{8}b\,\mathrm{Ci}\left(bx^2\right)\sin(a)-\frac{3}{8}b\,\mathrm{Ci}\left(3bx^2\right)\sin(3a)-\frac{3}{8}b\,\mathrm{c}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 90, normalized size = 0.99

$$
-\frac{3bx^2\sin(a)\mathrm{Ci}\left(bx^2\right)+3bx^2\sin(3a)\mathrm{Ci}\left(3bx^2\right)+3bx^2\cos(a)\mathrm{Si}\left(bx^2\right)+3bx^2\cos(3a)\mathrm{Si}\left(3bx^2\right)+3\cos\left(a+bx^2\right)+}{8x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^3/x^3,x]
```

```
[Out] -1/8*(3*Cos[a + b*x^2] + Cos[3*(a + b*x^2)] + 3*b*x^2*CosIntegral[b*x^2]*Si
n[a] + 3*b*x^2*CosIntegral[3*b*x^2]*Sin[3*a] + 3*b*x^2*Cos[a]*SinIntegral[b
*x^2] + 3*b*x^2*Cos[3*a]*SinIntegral[3*b*x^2])/x^2
```

fricas [A] time = 0.68, size = 108, normalized size = 1.19

$$-\frac{6\,bx^2\cos(3\,a)\,\mathrm{Si}\left(3\,bx^2\right)+6\,bx^2\cos(a)\,\mathrm{Si}\left(bx^2\right)+8\,\cos\left(bx^2+a\right)^3+3\left(bx^2\,\mathrm{Ci}\left(3\,bx^2\right)+bx^2\,\mathrm{Ci}\left(-3\,bx^2\right)\right)\sin(3\,a)}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*b*x^2*cos(3*a)*sin_integral(3*b*x^2) + 6*b*x^2*cos(a)*sin_integral
(b*x^2) + 8*cos(b*x^2 + a)^3 + 3*(b*x^2*cos_integral(3*b*x^2) + b*x^2*cos_i
ntegral(-3*b*x^2))*sin(3*a) + 3*(b*x^2*cos_integral(b*x^2) + b*x^2*cos_inte
gral(-b*x^2))*sin(a))/x^2
```

giac [B] time = 0.46, size = 185, normalized size = 2.03

$$-\frac{3\left(bx^2+a\right)b^2\,\mathrm{Ci}\left(3\,bx^2\right)\sin(3\,a)-3\,ab^2\,\mathrm{Ci}\left(3\,bx^2\right)\sin(3\,a)+3\left(bx^2+a\right)b^2\,\mathrm{Ci}\left(bx^2\right)\sin(a)-3\,ab^2\,\mathrm{Ci}\left(bx^2\right)\sin(a)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(b*x^2 + a)*b^2*cos_integral(3*b*x^2)*sin(3*a) - 3*a*b^2*cos_integr
al(3*b*x^2)*sin(3*a) + 3*(b*x^2 + a)*b^2*cos_integral(b*x^2)*sin(a) - 3*a*b
^2*cos_integral(b*x^2)*sin(a) + 3*(b*x^2 + a)*b^2*cos(a)*sin_integral(b*x^2
) - 3*a*b^2*cos(a)*sin_integral(b*x^2) - 3*(b*x^2 + a)*b^2*cos(3*a)*sin_int
egral(-3*b*x^2) + 3*a*b^2*cos(3*a)*sin_integral(-3*b*x^2) + b^2*cos(3*b*x^2
 + 3*a) + 3*b^2*cos(b*x^2 + a))/(b^2*x^2)
```

maple [C] time = 0.20, size = 162, normalized size = 1.78

$$\frac{3\,\mathrm{e}^{-3ia}\mathrm{csgn}\left(b\,x^2\right)\pi b}{16}-\frac{3\,\mathrm{e}^{-3ia}\,\mathrm{Si}\left(3b\,x^2\right)b}{8}+\frac{3i\mathrm{e}^{-3ia}\,\mathrm{Ei}\left(1,-3ib\,x^2\right)b}{16}+\frac{3\,\mathrm{csgn}\left(b\,x^2\right)\mathrm{e}^{-ia}\pi b}{16}-\frac{3\,\mathrm{e}^{-ia}\,\mathrm{Si}\left(b\,x^2\right)b}{8}+\frac{3i\mathrm{e}^{-ia}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)^3/x^3,x)
```

```
[Out] 3/16*exp(-3*I*a)*csgn(b*x^2)*Pi*b-3/8*exp(-3*I*a)*Si(3*b*x^2)*b+3/16*I*exp(
-3*I*a)*Ei(1,-3*I*b*x^2)*b+3/16*csgn(b*x^2)*exp(-I*a)*Pi*b-3/8*exp(-I*a)*Si
(b*x^2)*b+3/16*I*exp(-I*a)*Ei(1,-I*b*x^2)*b-3/16*I*exp(I*a)*b*Ei(1,-I*b*x^2
)-3/16*I*exp(3*I*a)*b*Ei(1,-3*I*b*x^2)-3/8*cos(b*x^2+a)/x^2-1/8*cos(3*b*x^2
+3*a)/x^2
```

maxima [C] time = 1.06, size = 98, normalized size = 1.08

$$-\frac{1}{16}\left(\left(3i\,\Gamma\left(-1,3i\,bx^2\right)-3i\,\Gamma\left(-1,-3i\,bx^2\right)\right)\cos\left(3\,a\right)+\left(3i\,\Gamma\left(-1,i\,bx^2\right)-3i\,\Gamma\left(-1,-i\,bx^2\right)\right)\cos(a)+3\left(\Gamma\left(-1,3i\,b\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*((3*I*gamma(-1, 3*I*b*x^2) - 3*I*gamma(-1, -3*I*b*x^2))*cos(3*a) + (3
*I*gamma(-1, I*b*x^2) - 3*I*gamma(-1, -I*b*x^2))*cos(a) + 3*(gamma(-1, 3*I*
b*x^2) + gamma(-1, -3*I*b*x^2))*sin(3*a) + 3*(gamma(-1, I*b*x^2) + gamma(-1
, -I*b*x^2))*sin(a))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(b\,x^2+a\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)^3/x^3,x)
```

```
[Out] int(cos(a + b*x^2)^3/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3\left(a+bx^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)**3/x**3,x)
```

```
[Out] Integral(cos(a + b*x**2)**3/x**3, x)
```

3.22 $\int x \cos^7\left(a+bx^2\right) dx$

Optimal. Leaf size=67

$$-\frac{\sin^7\left(a+bx^2\right)}{14b}+\frac{3\sin^5\left(a+bx^2\right)}{10b}-\frac{\sin^3\left(a+bx^2\right)}{2b}+\frac{\sin\left(a+bx^2\right)}{2b}$$

```
[Out] 1/2*sin(b*x^2+a)/b-1/2*sin(b*x^2+a)^3/b+3/10*sin(b*x^2+a)^5/b-1/14*sin(b*x^
2+a)^7/b
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3380, 2633}

$$-\frac{\sin^7\left(a+bx^2\right)}{14b}+\frac{3\sin^5\left(a+bx^2\right)}{10b}-\frac{\sin^3\left(a+bx^2\right)}{2b}+\frac{\sin\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x^2]^7,x]
```

```
[Out] Sin[a + b*x^2]/(2*b) - Sin[a + b*x^2]^3/(2*b) + (3*Sin[a + b*x^2]^5)/(10*b)
 - Sin[a + b*x^2]^7/(14*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int x\cos^7\left(a+bx^2\right)\,dx = \frac{1}{2}\text{Subst}\left(\int \cos^7(a+bx)\,dx, x, x^2\right)$$
$$= -\frac{\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx, x, -\sin\left(a+bx^2\right)\right)}{2b}$$
$$= \frac{\sin\left(a+bx^2\right)}{2b} - \frac{\sin^3\left(a+bx^2\right)}{2b} + \frac{3\sin^5\left(a+bx^2\right)}{10b} - \frac{\sin^7\left(a+bx^2\right)}{14b}$$

Mathematica [A] time = 0.09, size = 54, normalized size = 0.81

$$\frac{-5\sin^7\left(a+bx^2\right)+21\sin^5\left(a+bx^2\right)-35\sin^3\left(a+bx^2\right)+35\sin\left(a+bx^2\right)}{70b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x^2]^7,x]
```

```
[Out] (35*Sin[a + b*x^2] - 35*Sin[a + b*x^2]^3 + 21*Sin[a + b*x^2]^5 - 5*Sin[a +
b*x^2]^7)/(70*b)
```

fricas [A] time = 0.94, size = 51, normalized size = 0.76

$$\frac{\left(5\,\cos\left(bx^2+a\right)^6+6\,\cos\left(bx^2+a\right)^4+8\,\cos\left(bx^2+a\right)^2+16\right)\sin\left(bx^2+a\right)}{70\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/70*(5*cos(b*x^2 + a)^6 + 6*cos(b*x^2 + a)^4 + 8*cos(b*x^2 + a)^2 + 16)*si
n(b*x^2 + a)/b
```

giac [A] time = 0.37, size = 52, normalized size = 0.78

$$-\frac{5\,\sin\left(bx^2+a\right)^7-21\,\sin\left(bx^2+a\right)^5+35\,\sin\left(bx^2+a\right)^3-35\,\sin\left(bx^2+a\right)}{70\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x^2+a)^7,x, algorithm="giac")
```

```
[Out] -1/70*(5*sin(b*x^2 + a)^7 - 21*sin(b*x^2 + a)^5 + 35*sin(b*x^2 + a)^3 - 35*
sin(b*x^2 + a))/b
```

maple [A] time = 0.03, size = 50, normalized size = 0.75

$$\frac{\left(\frac{16}{5}+\cos^{6}\left(b\,x^{2}+a\right)+\frac{6\left(\cos^{4}\left(b\,x^{2}+a\right)\right)}{5}+\frac{8\left(\cos^{2}\left(b\,x^{2}+a\right)\right)}{5}\right)\sin\left(b\,x^{2}+a\right)}{14b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x^2+a)^7,x)
```

```
[Out] 1/14/b*(16/5+cos(b*x^2+a)^6+6/5*cos(b*x^2+a)^4+8/5*cos(b*x^2+a)^2)*sin(b*x^
2+a)
```

maxima [A] time = 1.10, size = 55, normalized size = 0.82

$$\frac{5\,\sin\left(7\,bx^{2}+7\,a\right)+49\,\sin\left(5\,bx^{2}+5\,a\right)+245\,\sin\left(3\,bx^{2}+3\,a\right)+1225\,\sin\left(bx^{2}+a\right)}{4480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/4480*(5*sin(7*b*x^2 + 7*a) + 49*sin(5*b*x^2 + 5*a) + 245*sin(3*b*x^2 + 3*
a) + 1225*sin(b*x^2 + a))/b
```

mupad [B] time = 0.75, size = 55, normalized size = 0.82

$$\frac{245\,\sin\left(3\,b\,x^{2}+3\,a\right)+49\,\sin\left(5\,b\,x^{2}+5\,a\right)+5\,\sin\left(7\,b\,x^{2}+7\,a\right)+1225\,\sin\left(b\,x^{2}+a\right)}{4480\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x^2)^7,x)
```

```
[Out] (245*sin(3*a + 3*b*x^2) + 49*sin(5*a + 5*b*x^2) + 5*sin(7*a + 7*b*x^2) + 12
25*sin(a + b*x^2))/(4480*b)
```

sympy [A] time = 7.82, size = 94, normalized size = 1.40

$$\begin{cases} \frac{8\sin^{7}\left(a+bx^{2}\right)}{35b}+\frac{4\sin^{5}\left(a+bx^{2}\right)\cos^{2}\left(a+bx^{2}\right)}{5b}+\frac{\sin^{3}\left(a+bx^{2}\right)\cos^{4}\left(a+bx^{2}\right)}{b}+\frac{\sin\left(a+bx^{2}\right)\cos^{6}\left(a+bx^{2}\right)}{2b} & \text{for}\ b\neq 0 \\ \frac{x^{2}\cos^{7}(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x**2+a)**7,x)
```

```
[Out] Piecewise((8*sin(a + b*x**2)**7/(35*b) + 4*sin(a + b*x**2)**5*cos(a + b*x**
2)**2/(5*b) + sin(a + b*x**2)**3*cos(a + b*x**2)**4/b + sin(a + b*x**2)*cos
(a + b*x**2)**6/(2*b), Ne(b, 0)), (x**2*cos(a)**7/2, True))
```

3.23 $\int x^{5/2} \cos\left(a+bx^2\right)\,dx$

Optimal. Leaf size=111

$$\frac{x^{3/2}\sin\left(a+bx^2\right)}{2b}-\frac{3ie^{ia}x^{3/2}\Gamma\left(\frac{3}{4},-ibx^2\right)}{16b\left(-ibx^2\right)^{3/4}}+\frac{3ie^{-ia}x^{3/2}\Gamma\left(\frac{3}{4},ibx^2\right)}{16b\left(ibx^2\right)^{3/4}}$$

```
[Out] -3/16*I*exp(I*a)*x^(3/2)*GAMMA(3/4,-I*b*x^2)/b/(-I*b*x^2)^(3/4)+3/16*I*x^(3
/2)*GAMMA(3/4,I*b*x^2)/b/exp(I*a)/(I*b*x^2)^(3/4)+1/2*x^(3/2)*sin(b*x^2+a)/
b
```

Rubi [A] time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3386, 3389, 2218}

$$-\frac{3ie^{ia}x^{3/2}\text{Gamma}\left(\frac{3}{4},-ibx^2\right)}{16b\left(-ibx^2\right)^{3/4}}+\frac{3ie^{-ia}x^{3/2}\text{Gamma}\left(\frac{3}{4},ibx^2\right)}{16b\left(ibx^2\right)^{3/4}}+\frac{x^{3/2}\sin\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Cos[a + b*x^2],x]
```

```
[Out] (((-3*I)/16)*E^(I*a)*x^(3/2)*Gamma[3/4, (-I)*b*x^2])/(b*((-I)*b*x^2)^(3/4))
 + (((3*I)/16)*x^(3/2)*Gamma[3/4, I*b*x^2])/(b*E^(I*a)*(I*b*x^2)^(3/4)) + (
x^(3/2)*Sin[a + b*x^2])/(2*b)
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
```

```
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\cos\left(a+bx^2\right)\,dx &= \frac{x^{3/2}\sin\left(a+bx^2\right)}{2b}-\frac{3\int\sqrt{x}\,\sin\left(a+bx^2\right)\,dx}{4b}\\ &= \frac{x^{3/2}\sin\left(a+bx^2\right)}{2b}-\frac{(3i)\int e^{-ia-ibx^2}\sqrt{x}\,dx}{8b}+\frac{(3i)\int e^{ia+ibx^2}\sqrt{x}\,dx}{8b}\\ &= -\frac{3ie^{ia}x^{3/2}\Gamma\left(\frac{3}{4},-ibx^2\right)}{16b\left(-ibx^2\right)^{3/4}}+\frac{3ie^{-ia}x^{3/2}\Gamma\left(\frac{3}{4},ibx^2\right)}{16b\left(ibx^2\right)^{3/4}}+\frac{x^{3/2}\sin\left(a+bx^2\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.20, size = 113, normalized size = 1.02

$$\frac{bx^{11/2}\left(8\left(b^2x^4\right)^{3/4}\sin\left(a+bx^2\right)+3\left(ibx^2\right)^{3/4}(\sin(a)-i\cos(a))\Gamma\left(\frac{3}{4},-ibx^2\right)+3\left(-ibx^2\right)^{3/4}(\sin(a)+i\cos(a))\Gamma\left(\frac{3}{4}\right.\right.}{16\left(b^2x^4\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Cos[a + b*x^2],x]
```

```
[Out] (b*x^(11/2)*(3*(I*b*x^2)^(3/4)*Gamma[3/4, (-I)*b*x^2]*((-I)*Cos[a] + Sin[a]
) + 3*((-I)*b*x^2)^(3/4)*Gamma[3/4, I*b*x^2]*(I*Cos[a] + Sin[a]) + 8*(b^2*x
^4)^(3/4)*Sin[a + b*x^2]))/(16*(b^2*x^4)^(7/4))
```

fricas [A] time = 0.88, size = 58, normalized size = 0.52

$$\frac{8\,bx^{\frac{3}{2}}\sin\left(bx^2+a\right)+3\,(i\,b)^{\frac{1}{4}}\,e^{(-i\,a)}\Gamma\left(\frac{3}{4},i\,bx^2\right)+3\,(-i\,b)^{\frac{1}{4}}\,e^{(i\,a)}\Gamma\left(\frac{3}{4},-i\,bx^2\right)}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*cos(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*x^(3/2)*sin(b*x^2 + a) + 3*(I*b)^(1/4)*e^(-I*a)*gamma(3/4, I*b*x^
2) + 3*(-I*b)^(1/4)*e^(I*a)*gamma(3/4, -I*b*x^2))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}}\cos\left(bx^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*cos(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)*cos(b*x^2 + a), x)
```

maple [C] time = 0.11, size = 229, normalized size = 2.06

$$\frac{2^{\frac{3}{4}} \cos(a)\sqrt{\pi}\left(\frac{2x^{\frac{3}{2}} 2^{\frac{1}{4}} \left(b^{2}\right)^{\frac{7}{8}} \sin\left(b\,x^{2}\right)}{7\sqrt{\pi}\, b}+\frac{3x^{\frac{7}{2}} \left(b^{2}\right)^{\frac{7}{8}} 2^{\frac{1}{4}} \sin\left(b\,x^{2}\right) \mathrm{LommelS1}\left(\frac{5}{4},\frac{3}{2},b\,x^{2}\right)}{14\sqrt{\pi}\left(b\,x^{2}\right)^{\frac{5}{4}}}+\frac{3x^{\frac{7}{2}} \left(b^{2}\right)^{\frac{7}{8}} 2^{\frac{1}{4}} \left(\cos\left(b\,x^{2}\right)x^{2} b-\sin\left(b\,x^{2}\right)\right) \mathrm{LommelS1}\left(\frac{1}{4},\right.}{8\sqrt{\pi}\left(b\,x^{2}\right)^{\frac{9}{4}}}}{2\left(b^{2}\right)^{\frac{7}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*cos(b*x^2+a),x)
```

```
[Out] 1/2*2^(3/4)/(b^2)^(7/8)*cos(a)*Pi^(1/2)*(2/7/Pi^(1/2)*x^(3/2)*2^(1/4)*(b^2)
^(7/8)/b*sin(b*x^2)+3/14/Pi^(1/2)*x^(7/2)*(b^2)^(7/8)*2^(1/4)/(b*x^2)^(5/4)
*sin(b*x^2)*LommelS1(5/4,3/2,b*x^2)+3/8/Pi^(1/2)*x^(7/2)*(b^2)^(7/8)*2^(1/4
)/(b*x^2)^(9/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(1/4,1/2,b*x^2))-1/2*
2^(3/4)/b^(7/4)*sin(a)*Pi^(1/2)*(-1/8/Pi^(1/2)*x^(7/2)*b^(7/4)*2^(1/4)/(b*x
^2)^(5/4)*sin(b*x^2)*LommelS1(1/4,3/2,b*x^2)-1/2/Pi^(1/2)*x^(7/2)*b^(7/4)*2
^(1/4)/(b*x^2)^(9/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(5/4,1/2,b*x^2))
```

maxima [B] time = 2.19, size = 156, normalized size = 1.41

$$\frac{16\,bx^{2}\sin\left(bx^{2}+a\right)+\left(bx^{2}\right)^{\frac{1}{4}}\left(\left(3\sqrt{\sqrt{2}+2}\left(\Gamma\left(\frac{3}{4},i\,bx^{2}\right)+\Gamma\left(\frac{3}{4},-i\,bx^{2}\right)\right)-\sqrt{-\sqrt{2}+2}\left(-3i\,\Gamma\left(\frac{3}{4},i\,bx^{2}\right)+3i\,\Gamma\left(\frac{3}{4},\right.\right.\right.\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*cos(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/32*(16*b*x^2*sin(b*x^2 + a) + (b*x^2)^(1/4)*((3*sqrt(sqrt(2) + 2)*(gamma(
3/4, I*b*x^2) + gamma(3/4, -I*b*x^2)) - sqrt(-sqrt(2) + 2)*(-3*I*gamma(3/4,
 I*b*x^2) + 3*I*gamma(3/4, -I*b*x^2)))*cos(a) + (3*sqrt(-sqrt(2) + 2)*(gamm
a(3/4, I*b*x^2) + gamma(3/4, -I*b*x^2)) - sqrt(sqrt(2) + 2)*(3*I*gamma(3/4,
 I*b*x^2) - 3*I*gamma(3/4, -I*b*x^2)))*sin(a)))/(b^2*sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\cos\left(b\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*cos(a + b*x^2),x)
[Out] int(x^(5/2)*cos(a + b*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}} \cos \left(a + bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*cos(b*x**2+a),x)
[Out] Integral(x**(5/2)*cos(a + b*x**2), x)
```

3.24 $\int x^{3/2} \cos\left(a + bx^2\right) dx$

Optimal. Leaf size=111

$$\frac{\sqrt{x}\sin\left(a+bx^2\right)}{2b}-\frac{ie^{ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},-ibx^2\right)}{16b\sqrt[4]{-ibx^2}}+\frac{ie^{-ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},ibx^2\right)}{16b\sqrt[4]{ibx^2}}$$

```
[Out] -1/16*I*exp(I*a)*GAMMA(1/4,-I*b*x^2)*x^(1/2)/b/(-I*b*x^2)^(1/4)+1/16*I*GAMM
A(1/4,I*b*x^2)*x^(1/2)/b/exp(I*a)/(I*b*x^2)^(1/4)+1/2*sin(b*x^2+a)*x^(1/2)/
b
```

Rubi [A] time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3386, 3389, 2218}

$$-\frac{ie^{ia}\sqrt{x}\,\text{Gamma}\left(\frac{1}{4},-ibx^2\right)}{16b\sqrt[4]{-ibx^2}}+\frac{ie^{-ia}\sqrt{x}\,\text{Gamma}\left(\frac{1}{4},ibx^2\right)}{16b\sqrt[4]{ibx^2}}+\frac{\sqrt{x}\sin\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Cos[a + b*x^2],x]
```

```
[Out] ((-I/16)*E^(I*a)*Sqrt[x]*Gamma[1/4, (-I)*b*x^2])/(b*((-I)*b*x^2)^(1/4)) + (
(I/16)*Sqrt[x]*Gamma[1/4, I*b*x^2])/(b*E^(I*a)*(I*b*x^2)^(1/4)) + (Sqrt[x]*
Sin[a + b*x^2])/(2*b)
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
```

```
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\cos\left(a+bx^2\right)dx &= \frac{\sqrt{x}\sin\left(a+bx^2\right)}{2b}-\frac{\int\frac{\sin\left(a+bx^2\right)}{\sqrt{x}}\,dx}{4b}\\ &= \frac{\sqrt{x}\sin\left(a+bx^2\right)}{2b}-\frac{i\int\frac{e^{-ia-ibx^2}}{\sqrt{x}}\,dx}{8b}+\frac{i\int\frac{e^{ia+ibx^2}}{\sqrt{x}}\,dx}{8b}\\ &= -\frac{ie^{ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},-ibx^2\right)}{16b\sqrt[4]{-ibx^2}}+\frac{ie^{-ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},ibx^2\right)}{16b\sqrt[4]{ibx^2}}+\frac{\sqrt{x}\sin\left(a+bx^2\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.15, size = 111, normalized size = 1.00

$$\frac{bx^{9/2}\left(8\sqrt[4]{b^2x^4}\sin\left(a+bx^2\right)+\sqrt[4]{ibx^2}(\sin(a)-i\cos(a))\Gamma\left(\frac{1}{4},-ibx^2\right)+\sqrt[4]{-ibx^2}(\sin(a)+i\cos(a))\Gamma\left(\frac{1}{4},ibx^2\right)\right)}{16\left(b^2x^4\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Cos[a + b*x^2],x]
```

```
[Out] (b*x^(9/2)*((I*b*x^2)^(1/4)*Gamma[1/4, (-I)*b*x^2]*((-I)*Cos[a] + Sin[a]) +
 ((-I)*b*x^2)^(1/4)*Gamma[1/4, I*b*x^2]*(I*Cos[a] + Sin[a]) + 8*(b^2*x^4)^(
1/4)*Sin[a + b*x^2]))/(16*(b^2*x^4)^(5/4))
```

fricas [A] time = 0.60, size = 56, normalized size = 0.50

$$\frac{(i\,b)^{\frac{3}{4}}\,e^{(-i\,a)}\Gamma\left(\frac{1}{4},i\,bx^2\right)+(-i\,b)^{\frac{3}{4}}\,e^{(i\,a)}\Gamma\left(\frac{1}{4},-i\,bx^2\right)+8\,b\sqrt{x}\sin\left(bx^2+a\right)}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/16*((I*b)^(3/4)*e^(-I*a)*gamma(1/4, I*b*x^2) + (-I*b)^(3/4)*e^(I*a)*gamma
(1/4, -I*b*x^2) + 8*b*sqrt(x)*sin(b*x^2 + a))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\cos\left(bx^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)*cos(b*x^2 + a), x)
```

maple [C] time = 0.08, size = 290, normalized size = 2.61

$$\frac{2^{\frac{1}{4}}\cos(a)\sqrt{\pi}\left(\frac{2\sqrt{x}\,2^{\frac{3}{4}}\left(b^{2}\right)^{\frac{5}{8}}\sin\left(b\,x^{2}\right)}{5\sqrt{\pi}\,b}+\frac{2\sqrt{x}\,2^{\frac{3}{4}}\left(b^{2}\right)^{\frac{5}{8}}\left(\cos\left(b\,x^{2}\right)x^{2}b-\sin\left(b\,x^{2}\right)\right)}{5\sqrt{\pi}\,b}+\frac{x^{\frac{9}{2}}\left(b^{2}\right)^{\frac{5}{8}}2^{\frac{3}{4}}b\sin\left(b\,x^{2}\right)\mathrm{LommelS1}\left(\frac{3}{4},\frac{3}{2},b\,x^{2}\right)}{10\sqrt{\pi}\left(b\,x^{2}\right)^{\frac{7}{4}}}-\frac{2x^{\frac{9}{2}}\left(b^{2}\right)^{\frac{5}{8}}}{}\right.}{2\left(b^{2}\right)^{\frac{5}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*cos(b*x^2+a),x)
```

```
[Out] 1/2*2^(1/4)/(b^2)^(5/8)*cos(a)*Pi^(1/2)*(2/5/Pi^(1/2)*x^(1/2)*2^(3/4)*(b^2)
^(5/8)/b*sin(b*x^2)+2/5/Pi^(1/2)*x^(1/2)*2^(3/4)*(b^2)^(5/8)/b*(cos(b*x^2)*
x^2*b-sin(b*x^2))+1/10/Pi^(1/2)*x^(9/2)*(b^2)^(5/8)*2^(3/4)*b/(b*x^2)^(7/4)
*sin(b*x^2)*LommelS1(3/4,3/2,b*x^2)-2/5/Pi^(1/2)*x^(9/2)*(b^2)^(5/8)*2^(3/4
)*b/(b*x^2)^(11/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(7/4,1/2,b*x^2))-1
/2*2^(1/4)/b^(5/4)*sin(a)*Pi^(1/2)*(2/9/Pi^(1/2)*x^(5/2)*2^(3/4)*b^(5/4)*si
n(b*x^2)-2/9/Pi^(1/2)*x^(9/2)*b^(9/4)*2^(3/4)/(b*x^2)^(7/4)*sin(b*x^2)*Lomm
elS1(7/4,3/2,b*x^2)-1/2/Pi^(1/2)*x^(9/2)*b^(9/4)*2^(3/4)/(b*x^2)^(11/4)*(co
s(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(3/4,1/2,b*x^2))
```

maxima [B] time = 1.34, size = 158, normalized size = 1.42

$$\frac{16\left(bx^{2}\right)^{\frac{1}{4}}\sqrt{x}\sin\left(bx^{2}+a\right)+\left(\left(\sqrt{-\sqrt{2}+2}\left(\Gamma\left(\frac{1}{4},i\,bx^{2}\right)+\Gamma\left(\frac{1}{4},-i\,bx^{2}\right)\right)+\sqrt{\sqrt{2}+2}\left(i\,\Gamma\left(\frac{1}{4},i\,bx^{2}\right)-i\,\Gamma\left(\frac{1}{4},-i\,bx^{2}\right.\right.\right.}{32\left(bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/32*(16*(b*x^2)^(1/4)*sqrt(x)*sin(b*x^2 + a) + ((sqrt(-sqrt(2) + 2)*(gamma
(1/4, I*b*x^2) + gamma(1/4, -I*b*x^2)) + sqrt(sqrt(2) + 2)*(I*gamma(1/4, I*
b*x^2) - I*gamma(1/4, -I*b*x^2)))*cos(a) + (sqrt(sqrt(2) + 2)*(gamma(1/4, I
*b*x^2) + gamma(1/4, -I*b*x^2)) + sqrt(-sqrt(2) + 2)*(-I*gamma(1/4, I*b*x^2
) + I*gamma(1/4, -I*b*x^2)))*sin(a))*sqrt(x))/((b*x^2)^(1/4)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\cos\left(b\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*cos(a + b*x^2),x)
```

```
[Out] int(x^(3/2)*cos(a + b*x^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \cos \left(a + bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*cos(b*x**2+a),x)
```

```
[Out] Integral(x**(3/2)*cos(a + b*x**2), x)
```

3.25 $\int \sqrt{x} \cos\left(a+bx^2\right) dx$

Optimal. Leaf size=81

$$-\frac{e^{ia}x^{3/2}\Gamma\left(\frac{3}{4},-ibx^2\right)}{4\left(-ibx^2\right)^{3/4}}-\frac{e^{-ia}x^{3/2}\Gamma\left(\frac{3}{4},ibx^2\right)}{4\left(ibx^2\right)^{3/4}}$$

```
[Out] -1/4*exp(I*a)*x^(3/2)*GAMMA(3/4,-I*b*x^2)/(-I*b*x^2)^(3/4)-1/4*x^(3/2)*GAMM
A(3/4,I*b*x^2)/exp(I*a)/(I*b*x^2)^(3/4)
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3390, 2218}

$$-\frac{e^{ia}x^{3/2}\text{Gamma}\left(\frac{3}{4},-ibx^2\right)}{4\left(-ibx^2\right)^{3/4}}-\frac{e^{-ia}x^{3/2}\text{Gamma}\left(\frac{3}{4},ibx^2\right)}{4\left(ibx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Cos[a + b*x^2],x]
```

```
[Out] -(E^(I*a)*x^(3/2)*Gamma[3/4, (-I)*b*x^2])/(4*((-I)*b*x^2)^(3/4)) - (x^(3/2)
*Gamma[3/4, I*b*x^2])/(4*E^(I*a)*(I*b*x^2)^(3/4))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3390

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \sqrt{x} \cos\left(a+bx^2\right) dx = \frac{1}{2}\int e^{-ia-ibx^2}\sqrt{x}\, dx + \frac{1}{2}\int e^{ia+ibx^2}\sqrt{x}\, dx$$
$$= -\frac{e^{ia}x^{3/2}\Gamma\left(\frac{3}{4},-ibx^2\right)}{4\left(-ibx^2\right)^{3/4}} - \frac{e^{-ia}x^{3/2}\Gamma\left(\frac{3}{4},ibx^2\right)}{4\left(ibx^2\right)^{3/4}}$$

Mathematica [A] time = 0.08, size = 89, normalized size = 1.10

$$-\frac{x^{3/2}\left(\left(-ibx^2\right)^{3/4}(\cos(a)-i\sin(a))\Gamma\left(\frac{3}{4},ibx^2\right)+\left(ibx^2\right)^{3/4}(\cos(a)+i\sin(a))\Gamma\left(\frac{3}{4},-ibx^2\right)\right)}{4\left(b^2x^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Cos[a + b*x^2],x]
```

```
[Out] -1/4*(x^(3/2)*(((-I)*b*x^2)^(3/4)*Gamma[3/4, I*b*x^2]*(Cos[a] - I*Sin[a]) +
 (I*b*x^2)^(3/4)*Gamma[3/4, (-I)*b*x^2]*(Cos[a] + I*Sin[a])))/(b^2*x^4)^(3/
4)
```

fricas [A] time = 0.99, size = 44, normalized size = 0.54

$$\frac{i\,(i\,b)^{\frac{1}{4}}\,e^{(-i\,a)}\Gamma\left(\frac{3}{4},i\,bx^2\right)-i\,(-i\,b)^{\frac{1}{4}}\,e^{(i\,a)}\Gamma\left(\frac{3}{4},-i\,bx^2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(I*(I*b)^(1/4)*e^(-I*a)*gamma(3/4, I*b*x^2) - I*(-I*b)^(1/4)*e^(I*a)*ga
mma(3/4, -I*b*x^2))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \cos\left(bx^2+a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)*cos(b*x^2 + a), x)
```

maple [C] time = 0.06, size = 290, normalized size = 3.58

$$\frac{2^{\frac{3}{4}}\cos(a)\sqrt{\pi}\left(\frac{4\,2^{\frac{1}{4}}\left(b^{2}\right)^{\frac{3}{8}}\sin\left(b\,x^{2}\right)}{3\sqrt{\pi}\,\sqrt{x}\,b}+\frac{4\,2^{\frac{1}{4}}\left(b^{2}\right)^{\frac{3}{8}}\left(\cos\left(b\,x^{2}\right)x^{2}b-\sin\left(b\,x^{2}\right)\right)}{3\sqrt{\pi}\,\sqrt{x}\,b}-\frac{x^{\frac{7}{2}}\left(b^{2}\right)^{\frac{3}{8}}2^{\frac{1}{4}}b\sin\left(b\,x^{2}\right)\mathrm{LommelS1}\left(\frac{1}{4},\frac{3}{2},b\,x^{2}\right)}{3\sqrt{\pi}\,\left(b\,x^{2}\right)^{\frac{5}{4}}}-\frac{4x^{\frac{7}{2}}\left(b^{2}\right)^{\frac{3}{8}}2^{\frac{1}{4}}b(\mathrm{co}\cdots}{\cdots}\right.}{4\left(b^{2}\right)^{\frac{3}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*cos(b*x^2+a),x)
```

```
[Out] 1/4*2^(3/4)/(b^2)^(3/8)*cos(a)*Pi^(1/2)*(4/3/Pi^(1/2)/x^(1/2)*2^(1/4)*(b^2)
^(3/8)/b*sin(b*x^2)+4/3/Pi^(1/2)/x^(1/2)*2^(1/4)*(b^2)^(3/8)/b*(cos(b*x^2)*
x^2*b-sin(b*x^2))-1/3/Pi^(1/2)*x^(7/2)*(b^2)^(3/8)*2^(1/4)*b/(b*x^2)^(5/4)*
sin(b*x^2)*LommelS1(1/4,3/2,b*x^2)-4/3/Pi^(1/2)*x^(7/2)*(b^2)^(3/8)*2^(1/4)
*b/(b*x^2)^(9/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(5/4,1/2,b*x^2))-1/4
*2^(3/4)/b^(3/4)*sin(a)*Pi^(1/2)*(4/7/Pi^(1/2)*x^(3/2)*2^(1/4)*b^(3/4)*sin(
b*x^2)-4/7/Pi^(1/2)*x^(7/2)*b^(7/4)*2^(1/4)/(b*x^2)^(5/4)*sin(b*x^2)*Lommel
S1(5/4,3/2,b*x^2)-1/Pi^(1/2)*x^(7/2)*b^(7/4)*2^(1/4)/(b*x^2)^(9/4)*(cos(b*x
^2)*x^2*b-sin(b*x^2))*LommelS1(1/4,1/2,b*x^2))
```

maxima [B] time = 1.16, size = 138, normalized size = 1.70

$$-\frac{\left(bx^{2}\right)^{\frac{1}{4}}\left(\left(\sqrt{-\sqrt{2}+2}\left(\Gamma\left(\frac{3}{4},i\,bx^{2}\right)+\Gamma\left(\frac{3}{4},-i\,bx^{2}\right)\right)-\sqrt{\sqrt{2}+2}\left(i\,\Gamma\left(\frac{3}{4},i\,bx^{2}\right)-i\,\Gamma\left(\frac{3}{4},-i\,bx^{2}\right)\right)\right)\cos(a)-\left(\sqrt{\sqrt{2}+\cdots}\right.\right.}{8\,b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/8*(b*x^2)^(1/4)*((sqrt(-sqrt(2) + 2)*(gamma(3/4, I*b*x^2) + gamma(3/4, -
I*b*x^2)) - sqrt(sqrt(2) + 2)*(I*gamma(3/4, I*b*x^2) - I*gamma(3/4, -I*b*x^
2)))*cos(a) - (sqrt(sqrt(2) + 2)*(gamma(3/4, I*b*x^2) + gamma(3/4, -I*b*x^2
)) + sqrt(-sqrt(2) + 2)*(I*gamma(3/4, I*b*x^2) - I*gamma(3/4, -I*b*x^2)))*s
in(a))/(b*sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\cos\left(b\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*cos(a + b*x^2),x)
```

```
[Out] int(x^(1/2)*cos(a + b*x^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \cos \left(a + bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*cos(b*x**2+a),x)
```

```
[Out] Integral(sqrt(x)*cos(a + b*x**2), x)
```

3.26 $\int \frac{\cos(a+bx^2)}{\sqrt{x}}\,dx$

Optimal. Leaf size=81

$$-\frac{e^{ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},-ibx^2\right)}{4\sqrt[4]{-ibx^2}}-\frac{e^{-ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},ibx^2\right)}{4\sqrt[4]{ibx^2}}$$

```
[Out] -1/4*exp(I*a)*GAMMA(1/4,-I*b*x^2)*x^(1/2)/(-I*b*x^2)^(1/4)-1/4*GAMMA(1/4,I*
b*x^2)*x^(1/2)/exp(I*a)/(I*b*x^2)^(1/4)
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3390, 2218}

$$-\frac{e^{ia}\sqrt{x}\,\text{Gamma}\left(\frac{1}{4},-ibx^2\right)}{4\sqrt[4]{-ibx^2}}-\frac{e^{-ia}\sqrt{x}\,\text{Gamma}\left(\frac{1}{4},ibx^2\right)}{4\sqrt[4]{ibx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]/Sqrt[x],x]
```

```
[Out] -(E^(I*a)*Sqrt[x]*Gamma[1/4, (-I)*b*x^2])/(4*((-I)*b*x^2)^(1/4)) - (Sqrt[x]
*Gamma[1/4, I*b*x^2])/(4*E^(I*a)*(I*b*x^2)^(1/4))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3390

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos\left(a+bx^2\right)}{\sqrt{x}}\,dx = \frac{1}{2}\int \frac{e^{-ia-ibx^2}}{\sqrt{x}}\,dx + \frac{1}{2}\int \frac{e^{ia+ibx^2}}{\sqrt{x}}\,dx$$
$$= -\frac{e^{ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},-ibx^2\right)}{4\sqrt[4]{-ibx^2}} - \frac{e^{-ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},ibx^2\right)}{4\sqrt[4]{ibx^2}}$$

Mathematica [A] time = 0.07, size = 89, normalized size = 1.10

$$-\frac{\sqrt{x}\left(\sqrt[4]{-ibx^2}\,(\cos(a)-i\sin(a))\Gamma\left(\frac{1}{4},ibx^2\right)+\sqrt[4]{ibx^2}\,(\cos(a)+i\sin(a))\Gamma\left(\frac{1}{4},-ibx^2\right)\right)}{4\sqrt[4]{b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]/Sqrt[x],x]
```

```
[Out] -1/4*(Sqrt[x]*(((-I)*b*x^2)^(1/4)*Gamma[1/4, I*b*x^2]*(Cos[a] - I*Sin[a]) +
 (I*b*x^2)^(1/4)*Gamma[1/4, (-I)*b*x^2]*(Cos[a] + I*Sin[a])))/(b^2*x^4)^(1/
4)
```

fricas [A] time = 0.95, size = 44, normalized size = 0.54

$$\frac{i\,(i\,b)^{\frac{3}{4}}\,e^{(-i\,a)}\Gamma\left(\frac{1}{4},i\,bx^2\right)-i\,(-i\,b)^{\frac{3}{4}}\,e^{(i\,a)}\Gamma\left(\frac{1}{4},-i\,bx^2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(I*(I*b)^(3/4)*e^(-I*a)*gamma(1/4, I*b*x^2) - I*(-I*b)^(3/4)*e^(I*a)*ga
mma(1/4, -I*b*x^2))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx^2+a\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^2 + a)/sqrt(x), x)
```

maple [C] time = 0.08, size = 338, normalized size = 4.17

$$\frac{\cos(a)\sqrt{\pi}\,2^{\frac{1}{4}}\left(\frac{6\,2^{\frac{3}{4}}\left(b^{2}\right)^{\frac{1}{8}}\left(\frac{8x^{4}b^{2}}{27}+\frac{2}{3}\right)\sin\left(b\,x^{2}\right)}{\sqrt{\pi}\,x^{\frac{3}{2}}b}+\frac{4\,2^{\frac{3}{4}}\left(b^{2}\right)^{\frac{1}{8}}\left(\cos\left(b\,x^{2}\right)x^{2}b-\sin\left(b\,x^{2}\right)\right)}{\sqrt{\pi}\,x^{\frac{3}{2}}b}-\frac{16x^{\frac{9}{2}}\left(b^{2}\right)^{\frac{1}{8}}b^{2}2^{\frac{3}{4}}\sin\left(b\,x^{2}\right)\mathrm{LommelS1}\left(\frac{7}{4},\frac{3}{2},b\,x^{2}\right)}{9\sqrt{\pi}\,\left(b\,x^{2}\right)^{\frac{7}{4}}}-\frac{4x}{}\right.}{4\left(b^{2}\right)^{\frac{1}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)/x^(1/2),x)
```

```
[Out] 1/4*cos(a)*Pi^(1/2)*2^(1/4)/(b^2)^(1/8)*(6/Pi^(1/2)/x^(3/2)*2^(3/4)*(b^2)^(
1/8)*(8/27*x^4*b^2+2/3)/b*sin(b*x^2)+4/Pi^(1/2)/x^(3/2)*2^(3/4)*(b^2)^(1/8)
/b*(cos(b*x^2)*x^2*b-sin(b*x^2))-16/9/Pi^(1/2)*x^(9/2)*(b^2)^(1/8)*b^2*2^(3
/4)/(b*x^2)^(7/4)*sin(b*x^2)*LommelS1(7/4,3/2,b*x^2)-4/Pi^(1/2)*x^(9/2)*(b^
2)^(1/8)*b^2*2^(3/4)/(b*x^2)^(11/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(
3/4,1/2,b*x^2))-1/4*sin(a)*Pi^(1/2)*2^(1/4)/b^(1/4)*(4/5/Pi^(1/2)*x^(1/2)*2
^(3/4)*b^(1/4)*sin(b*x^2)-16/5/Pi^(1/2)*x^(1/2)*2^(3/4)*b^(1/4)*(cos(b*x^2)
*x^2*b-sin(b*x^2))-4/5/Pi^(1/2)*x^(9/2)*b^(9/4)*2^(3/4)/(b*x^2)^(7/4)*sin(b
*x^2)*LommelS1(3/4,3/2,b*x^2)+16/5/Pi^(1/2)*x^(9/2)*b^(9/4)*2^(3/4)/(b*x^2)
^(11/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(7/4,1/2,b*x^2))
```

maxima [B] time = 2.84, size = 135, normalized size = 1.67

$$-\frac{\left(\left(\sqrt{\sqrt{2}+2}\left(\Gamma\left(\frac{1}{4},i\,bx^{2}\right)+\Gamma\left(\frac{1}{4},-i\,bx^{2}\right)\right)-\sqrt{-\sqrt{2}+2}\left(i\,\Gamma\left(\frac{1}{4},i\,bx^{2}\right)-i\,\Gamma\left(\frac{1}{4},-i\,bx^{2}\right)\right)\right)\cos(a)-\left(\sqrt{-\sqrt{2}+2}\left(\Gamma\right.\right.\right.}{8\left(bx^{2}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*((sqrt(sqrt(2) + 2)*(gamma(1/4, I*b*x^2) + gamma(1/4, -I*b*x^2)) - sqr
t(-sqrt(2) + 2)*(I*gamma(1/4, I*b*x^2) - I*gamma(1/4, -I*b*x^2)))*cos(a) -
(sqrt(-sqrt(2) + 2)*(gamma(1/4, I*b*x^2) + gamma(1/4, -I*b*x^2)) + sqrt(sqr
t(2) + 2)*(I*gamma(1/4, I*b*x^2) - I*gamma(1/4, -I*b*x^2)))*sin(a))*sqrt(x)
/(b*x^2)^(1/4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(b\,x^{2}+a\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)/x^(1/2),x)
```

```
[Out] int(cos(a + b*x^2)/x^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a + bx^2\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)/x**(1/2),x)
```

```
[Out] Integral(cos(a + b*x**2)/sqrt(x), x)
```

3.27 $\int \frac{\cos(a+bx^2)}{x^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\cos\left(a+bx^2\right)}{\sqrt{x}}-\frac{ie^{ia}bx^{3/2}\Gamma\left(\frac{3}{4},-ibx^2\right)}{\left(-ibx^2\right)^{3/4}}+\frac{ie^{-ia}bx^{3/2}\Gamma\left(\frac{3}{4},ibx^2\right)}{\left(ibx^2\right)^{3/4}}$$

```
[Out] -I*b*exp(I*a)*x^(3/2)*GAMMA(3/4,-I*b*x^2)/(-I*b*x^2)^(3/4)+I*b*x^(3/2)*GAMM
A(3/4,I*b*x^2)/exp(I*a)/(I*b*x^2)^(3/4)-2*cos(b*x^2+a)/x^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3388, 3389, 2218}

$$-\frac{ie^{ia}bx^{3/2}\text{Gamma}\left(\frac{3}{4},-ibx^2\right)}{\left(-ibx^2\right)^{3/4}}+\frac{ie^{-ia}bx^{3/2}\text{Gamma}\left(\frac{3}{4},ibx^2\right)}{\left(ibx^2\right)^{3/4}}-\frac{2\cos\left(a+bx^2\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]/x^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x^2])/Sqrt[x] - (I*b*E^(I*a)*x^(3/2)*Gamma[3/4, (-I)*b*x^2])/
((-I)*b*x^2)^(3/4) + (I*b*x^(3/2)*Gamma[3/4, I*b*x^2])/(E^(I*a)*(I*b*x^2)^(
3/4))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
```

```
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos\left(a+bx^2\right)}{x^{3/2}}\,dx &= -\frac{2\cos\left(a+bx^2\right)}{\sqrt{x}}-(4b)\int \sqrt{x}\sin\left(a+bx^2\right)dx\\
&= -\frac{2\cos\left(a+bx^2\right)}{\sqrt{x}}-(2ib)\int e^{-ia-ibx^2}\sqrt{x}\,dx+(2ib)\int e^{ia+ibx^2}\sqrt{x}\,dx\\
&= -\frac{2\cos\left(a+bx^2\right)}{\sqrt{x}}-\frac{ibe^{ia}x^{3/2}\Gamma\left(\frac{3}{4},-ibx^2\right)}{\left(-ibx^2\right)^{3/4}}+\frac{ibe^{-ia}x^{3/2}\Gamma\left(\frac{3}{4},ibx^2\right)}{\left(ibx^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 114, normalized size = 1.16

$$
\frac{-2\left(b^2x^4\right)^{3/4}\cos\left(a+bx^2\right)+bx^2\left(ibx^2\right)^{3/4}(\sin(a)-i\cos(a))\Gamma\left(\frac{3}{4},-ibx^2\right)+i\left(-ibx^2\right)^{7/4}(\sin(a)+i\cos(a))\Gamma\left(\frac{3}{4},ib\right.}{\sqrt{x}\left(b^2x^4\right)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]/x^(3/2),x]
```

```
[Out] (-2*(b^2*x^4)^(3/4)*Cos[a + b*x^2] + b*x^2*(I*b*x^2)^(3/4)*Gamma[3/4, (-I)*
b*x^2]*((-I)*Cos[a] + Sin[a]) + I*((-I)*b*x^2)^(7/4)*Gamma[3/4, I*b*x^2]*(I
*Cos[a] + Sin[a]))/(Sqrt[x]*(b^2*x^4)^(3/4))
```

fricas [A] time = 0.80, size = 56, normalized size = 0.57

$$
\frac{(i\,b)^{\frac{1}{4}}\,xe^{(-i\,a)}\Gamma\left(\frac{3}{4},i\,bx^2\right)+(-i\,b)^{\frac{1}{4}}\,xe^{(i\,a)}\Gamma\left(\frac{3}{4},-i\,bx^2\right)-2\sqrt{x}\cos\left(bx^2+a\right)}{x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^(3/2),x, algorithm="fricas")
```

```
[Out] ((I*b)^(1/4)*x*e^(-I*a)*gamma(3/4, I*b*x^2) + (-I*b)^(1/4)*x*e^(I*a)*gamma(
3/4, -I*b*x^2) - 2*sqrt(x)*cos(b*x^2 + a))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\cos\left(bx^2+a\right)}{x^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^2 + a)/x^(3/2), x)
```

maple [C] time = 0.07, size = 338, normalized size = 3.45

$$\frac{\cos(a)\sqrt{\pi}\,2^{\frac{3}{4}}\left(b^{2}\right)^{\frac{1}{8}}\left(-\frac{12\,2^{\frac{1}{4}}\left(\frac{8x^{4}b^{2}}{21}+\frac{2}{3}\right)\sin\left(b\,x^{2}\right)}{\sqrt{\pi}\,x^{\frac{5}{2}}\left(b^{2}\right)^{\frac{1}{8}}b}-\frac{8\,2^{\frac{1}{4}}\left(\cos\left(b\,x^{2}\right)x^{2}b-\sin\left(b\,x^{2}\right)\right)}{\sqrt{\pi}\,x^{\frac{5}{2}}\left(b^{2}\right)^{\frac{1}{8}}b}+\frac{32x^{\frac{7}{2}}b^{2}2^{\frac{1}{4}}\sin\left(b\,x^{2}\right)\mathrm{LommelS1}\left(\frac{5}{4},\frac{3}{2},b\,x^{2}\right)}{7\sqrt{\pi}\left(b^{2}\right)^{\frac{1}{8}}\left(b\,x^{2}\right)^{\frac{5}{4}}}+\frac{8x^{\frac{7}{2}}b^{2}2^{\frac{1}{4}}}{\cdots}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)/x^(3/2),x)
```

```
[Out] 1/8*cos(a)*Pi^(1/2)*2^(3/4)*(b^2)^(1/8)*(-12/Pi^(1/2)/x^(5/2)*2^(1/4)/(b^2)
^(1/8)*(8/21*x^4*b^2+2/3)/b*sin(b*x^2)-8/Pi^(1/2)/x^(5/2)*2^(1/4)/(b^2)^(1/
8)/b*(cos(b*x^2)*x^2*b-sin(b*x^2))+32/7/Pi^(1/2)*x^(7/2)/(b^2)^(1/8)*b^2*2^
(1/4)/(b*x^2)^(5/4)*sin(b*x^2)*LommelS1(5/4,3/2,b*x^2)+8/Pi^(1/2)*x^(7/2)/(
b^2)^(1/8)*b^2*2^(1/4)/(b*x^2)^(9/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1
(1/4,1/2,b*x^2))-1/8*sin(a)*Pi^(1/2)*2^(3/4)*b^(1/4)*(8/3/Pi^(1/2)/x^(1/2)*
2^(1/4)/b^(1/4)*sin(b*x^2)+32/3/Pi^(1/2)/x^(1/2)*2^(1/4)/b^(1/4)*(cos(b*x^2
)*x^2*b-sin(b*x^2))-8/3/Pi^(1/2)*x^(7/2)*b^(7/4)*2^(1/4)/(b*x^2)^(5/4)*sin(
b*x^2)*LommelS1(1/4,3/2,b*x^2)-32/3/Pi^(1/2)*x^(7/2)*b^(7/4)*2^(1/4)/(b*x^2
)^(9/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))*LommelS1(5/4,1/2,b*x^2))
```

maxima [B] time = 1.34, size = 133, normalized size = 1.36

$$-\frac{\left(bx^{2}\right)^{\frac{1}{4}}\left(\left(\sqrt{\sqrt{2}+2}\left(\Gamma\left(-\frac{1}{4},i\,bx^{2}\right)+\Gamma\left(-\frac{1}{4},-i\,bx^{2}\right)\right)+\sqrt{-\sqrt{2}+2}\left(i\,\Gamma\left(-\frac{1}{4},i\,bx^{2}\right)-i\,\Gamma\left(-\frac{1}{4},-i\,bx^{2}\right)\right)\right)\cos(a)+\left(\cdots\right.\right.}{8\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(b*x^2)^(1/4)*((sqrt(sqrt(2) + 2)*(gamma(-1/4, I*b*x^2) + gamma(-1/4,
-I*b*x^2)) + sqrt(-sqrt(2) + 2)*(I*gamma(-1/4, I*b*x^2) - I*gamma(-1/4, -I*
b*x^2)))*cos(a) + (sqrt(-sqrt(2) + 2)*(gamma(-1/4, I*b*x^2) + gamma(-1/4, -
I*b*x^2)) + sqrt(sqrt(2) + 2)*(-I*gamma(-1/4, I*b*x^2) + I*gamma(-1/4, -I*b
*x^2)))*sin(a))/sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(b\,x^2+a\right)}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)/x^(3/2),x)
```

```
[Out] int(cos(a + b*x^2)/x^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a + bx^2\right)}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)/x**(3/2),x)
```

```
[Out] Integral(cos(a + b*x**2)/x**(3/2), x)
```

3.28 $\int \frac{\cos\left(a+bx^2\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{ie^{ia}b\sqrt{x}\,\Gamma\left(\frac{1}{4},-ibx^2\right)}{3\sqrt[4]{-ibx^2}}+\frac{ie^{-ia}b\sqrt{x}\,\Gamma\left(\frac{1}{4},ibx^2\right)}{3\sqrt[4]{ibx^2}}-\frac{2\cos\left(a+bx^2\right)}{3x^{3/2}}$$

```
[Out] -2/3*cos(b*x^2+a)/x^(3/2)-1/3*I*b*exp(I*a)*GAMMA(1/4,-I*b*x^2)*x^(1/2)/(-I*
b*x^2)^(1/4)+1/3*I*b*GAMMA(1/4,I*b*x^2)*x^(1/2)/exp(I*a)/(I*b*x^2)^(1/4)
```

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3388, 3389, 2218}

$$-\frac{ie^{ia}b\sqrt{x}\,\text{Gamma}\left(\frac{1}{4},-ibx^2\right)}{3\sqrt[4]{-ibx^2}}+\frac{ie^{-ia}b\sqrt{x}\,\text{Gamma}\left(\frac{1}{4},ibx^2\right)}{3\sqrt[4]{ibx^2}}-\frac{2\cos\left(a+bx^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]/x^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x^2])/(3*x^(3/2)) - ((I/3)*b*E^(I*a)*Sqrt[x]*Gamma[1/4, (-I)*
b*x^2])/((-I)*b*x^2)^(1/4) + ((I/3)*b*Sqrt[x]*Gamma[1/4, I*b*x^2])/(E^(I*a)
*(I*b*x^2)^(1/4))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
```

```
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos\left(a+bx^2\right)}{x^{5/2}}\,dx &= -\frac{2\cos\left(a+bx^2\right)}{3x^{3/2}}-\frac{1}{3}(4b)\int \frac{\sin\left(a+bx^2\right)}{\sqrt{x}}\,dx\\
&= -\frac{2\cos\left(a+bx^2\right)}{3x^{3/2}}-\frac{1}{3}(2ib)\int \frac{e^{-ia-ibx^2}}{\sqrt{x}}\,dx+\frac{1}{3}(2ib)\int \frac{e^{ia+ibx^2}}{\sqrt{x}}\,dx\\
&= -\frac{2\cos\left(a+bx^2\right)}{3x^{3/2}}-\frac{ibe^{ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},-ibx^2\right)}{3\sqrt[4]{-ibx^2}}+\frac{ibe^{-ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},ibx^2\right)}{3\sqrt[4]{ibx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 117, normalized size = 1.12

$$
\frac{-2\sqrt[4]{b^2x^4}\,\cos\left(a+bx^2\right)+bx^2\sqrt[4]{ibx^2}\,(\sin(a)-i\cos(a))\Gamma\left(\frac{1}{4},-ibx^2\right)+i\left(-ibx^2\right)^{5/4}(\sin(a)+i\cos(a))\Gamma\left(\frac{1}{4},ibx^2\right)}{3x^{3/2}\sqrt[4]{b^2x^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]/x^(5/2),x]
```

```
[Out] (-2*(b^2*x^4)^(1/4)*Cos[a + b*x^2] + b*x^2*(I*b*x^2)^(1/4)*Gamma[1/4, (-I)*
b*x^2]*((-I)*Cos[a] + Sin[a]) + I*((-I)*b*x^2)^(5/4)*Gamma[1/4, I*b*x^2]*(I
*Cos[a] + Sin[a]))/(3*x^(3/2)*(b^2*x^4)^(1/4))
```

fricas [A] time = 0.91, size = 61, normalized size = 0.59

$$
\frac{(i\,b)^{\frac{3}{4}}\,x^2e^{(-i\,a)}\Gamma\left(\frac{1}{4},i\,bx^2\right)+(-i\,b)^{\frac{3}{4}}\,x^2e^{(i\,a)}\Gamma\left(\frac{1}{4},-i\,bx^2\right)-2\sqrt{x}\,\cos\left(bx^2+a\right)}{3\,x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((I*b)^(3/4)*x^2*e^(-I*a)*gamma(1/4, I*b*x^2) + (-I*b)^(3/4)*x^2*e^(I*a
)*gamma(1/4, -I*b*x^2) - 2*sqrt(x)*cos(b*x^2 + a))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\cos\left(bx^2+a\right)}{x^{\frac{5}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^2 + a)/x^(5/2), x)
```

maple [C] time = 0.06, size = 358, normalized size = 3.44

$$\frac{\cos(a)\sqrt{\pi}\,2^{\frac{1}{4}}\left(b^2\right)^{\frac{3}{8}}\left(-\frac{4\,2^{\frac{3}{4}}\left(\frac{8x^4b^2}{15}+\frac{2}{3}\right)\sin\left(b\,x^2\right)}{\sqrt{\pi}\,x^{\frac{7}{2}}\left(b^2\right)^{\frac{3}{8}}b}-\frac{8\,2^{\frac{3}{4}}\left(-16x^4b^2+5\right)\left(\cos\left(b\,x^2\right)x^2b-\sin\left(b\,x^2\right)\right)}{15\sqrt{\pi}\,x^{\frac{7}{2}}\left(b^2\right)^{\frac{3}{8}}b}+\frac{32x^{\frac{9}{2}}\,2^{\frac{3}{4}}b^3\sin\left(b\,x^2\right)\mathrm{LommelS1}\left(\frac{3}{4},\frac{3}{2},b\,x^2\right)}{15\sqrt{\pi}\,\left(b^2\right)^{\frac{3}{8}}\left(b\,x^2\right)^{\frac{7}{4}}}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)/x^(5/2),x)
```

```
[Out] 1/8*cos(a)*Pi^(1/2)*2^(1/4)*(b^2)^(3/8)*(-4/Pi^(1/2)/x^(7/2)*2^(3/4)/(b^2)^
(3/8)*(8/15*x^4*b^2+2/3)/b*sin(b*x^2)-8/15/Pi^(1/2)/x^(7/2)*2^(3/4)/(b^2)^(
3/8)/b*(-16*b^2*x^4+5)*(cos(b*x^2)*x^2*b-sin(b*x^2))+32/15/Pi^(1/2)*x^(9/2)
/(b^2)^(3/8)*2^(3/4)*b^3/(b*x^2)^(7/4)*sin(b*x^2)*LommelS1(3/4,3/2,b*x^2)-1
28/15/Pi^(1/2)*x^(9/2)/(b^2)^(3/8)*2^(3/4)*b^3/(b*x^2)^(11/4)*(cos(b*x^2)*x
^2*b-sin(b*x^2))*LommelS1(7/4,1/2,b*x^2))-1/8*sin(a)*Pi^(1/2)*2^(1/4)*b^(3/
4)*(12/Pi^(1/2)/x^(3/2)*2^(3/4)/b^(3/4)*(32/81*x^4*b^2+2/3)*sin(b*x^2)+32/3
/Pi^(1/2)/x^(3/2)*2^(3/4)/b^(3/4)*(cos(b*x^2)*x^2*b-sin(b*x^2))-128/27/Pi^(
1/2)*x^(9/2)*b^(9/4)*2^(3/4)/(b*x^2)^(7/4)*sin(b*x^2)*LommelS1(7/4,3/2,b*x^
2)-32/3/Pi^(1/2)*x^(9/2)*b^(9/4)*2^(3/4)/(b*x^2)^(11/4)*(cos(b*x^2)*x^2*b-s
in(b*x^2))*LommelS1(3/4,1/2,b*x^2))
```

maxima [B] time = 1.80, size = 133, normalized size = 1.28

$$-\frac{\left(bx^2\right)^{\frac{3}{4}}\left(\left(\sqrt{-\sqrt{2}+2}\left(\Gamma\left(-\frac{3}{4},i\,bx^2\right)+\Gamma\left(-\frac{3}{4},-i\,bx^2\right)\right)+\sqrt{\sqrt{2}+2}\left(i\,\Gamma\left(-\frac{3}{4},i\,bx^2\right)-i\,\Gamma\left(-\frac{3}{4},-i\,bx^2\right)\right)\right)\cos(a)+\left(\right.\right.}{8\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*(b*x^2)^(3/4)*((sqrt(-sqrt(2) + 2)*(gamma(-3/4, I*b*x^2) + gamma(-3/4,
 -I*b*x^2)) + sqrt(sqrt(2) + 2)*(I*gamma(-3/4, I*b*x^2) - I*gamma(-3/4, -I*
b*x^2)))*cos(a) + (sqrt(sqrt(2) + 2)*(gamma(-3/4, I*b*x^2) + gamma(-3/4, -I
*b*x^2)) + sqrt(-sqrt(2) + 2)*(-I*gamma(-3/4, I*b*x^2) + I*gamma(-3/4, -I*b
*x^2)))*sin(a))/x^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(b\,x^2+a\right)}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)/x^(5/2),x)
[Out] int(cos(a + b*x^2)/x^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+bx^2\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)/x**(5/2),x)
[Out] Integral(cos(a + b*x**2)/x**(5/2), x)
```

3.29 $\int x^{5/2} \cos^2\left(a + bx^2\right) dx$

Optimal. Leaf size=132

$$\frac{x^{3/2}\sin\left(2\left(a+bx^2\right)\right)}{8b}-\frac{3ie^{2ia}x^{3/2}\Gamma\left(\frac{3}{4},-2ibx^2\right)}{64\,2^{3/4}b\left(-ibx^2\right)^{3/4}}+\frac{3ie^{-2ia}x^{3/2}\Gamma\left(\frac{3}{4},2ibx^2\right)}{64\,2^{3/4}b\left(ibx^2\right)^{3/4}}+\frac{x^{7/2}}{7}$$

```
[Out] 1/7*x^(7/2)-3/128*I*exp(2*I*a)*x^(3/2)*GAMMA(3/4,-2*I*b*x^2)*2^(1/4)/b/(-I*
b*x^2)^(3/4)+3/128*I*x^(3/2)*GAMMA(3/4,2*I*b*x^2)*2^(1/4)/b/exp(2*I*a)/(I*b
*x^2)^(3/4)+1/8*x^(3/2)*sin(2*b*x^2+2*a)/b
```

Rubi [A] time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3402, 3404, 3386, 3389, 2218}

$$-\frac{3ie^{2ia}x^{3/2}\text{Gamma}\left(\frac{3}{4},-2ibx^2\right)}{64\,2^{3/4}b\left(-ibx^2\right)^{3/4}}+\frac{3ie^{-2ia}x^{3/2}\text{Gamma}\left(\frac{3}{4},2ibx^2\right)}{64\,2^{3/4}b\left(ibx^2\right)^{3/4}}+\frac{x^{3/2}\sin\left(2\left(a+bx^2\right)\right)}{8b}+\frac{x^{7/2}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Cos[a + b*x^2]^2,x]
```

```
[Out] x^(7/2)/7 - (((3*I)/64)*E^((2*I)*a)*x^(3/2)*Gamma[3/4, (-2*I)*b*x^2])/(2^(3
/4)*b*((-I)*b*x^2)^(3/4)) + (((3*I)/64)*x^(3/2)*Gamma[3/4, (2*I)*b*x^2])/(2
^(3/4)*b*E^((2*I)*a)*(I*b*x^2)^(3/4)) + (x^(3/2)*Sin[2*(a + b*x^2)])/(8*b)
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
```

```
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3402

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_), x
_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m + 1) - 1
)*(a + b*Cos[c + (d*x^(k*n))/e^n])^p, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]
```

Rule 3404

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\cos^2\left(a+bx^2\right)dx &= 2\,\text{Subst}\left(\int x^6\cos^2\left(a+bx^4\right)dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int\left(\frac{x^6}{2}+\frac{1}{2}x^6\cos\left(2a+2bx^4\right)\right)dx, x, \sqrt{x}\right)\\ &= \frac{x^{7/2}}{7}+\text{Subst}\left(\int x^6\cos\left(2a+2bx^4\right)dx, x, \sqrt{x}\right)\\ &= \frac{x^{7/2}}{7}+\frac{x^{3/2}\sin\left(2\left(a+bx^2\right)\right)}{8b}-\frac{3\,\text{Subst}\left(\int x^2\sin\left(2a+2bx^4\right)dx, x, \sqrt{x}\right)}{8b}\\ &= \frac{x^{7/2}}{7}+\frac{x^{3/2}\sin\left(2\left(a+bx^2\right)\right)}{8b}-\frac{(3i)\,\text{Subst}\left(\int e^{-2ia-2ibx^4}x^2\,dx, x, \sqrt{x}\right)}{16b}+\frac{(3i)\,\text{Subst}\left(\int\right.}{}\\ &= \frac{x^{7/2}}{7}-\frac{3ie^{2ia}x^{3/2}\Gamma\left(\frac{3}{4},-2ibx^2\right)}{64\;2^{3/4}b\left(-ibx^2\right)^{3/4}}+\frac{3ie^{-2ia}x^{3/2}\Gamma\left(\frac{3}{4},2ibx^2\right)}{64\;2^{3/4}b\left(ibx^2\right)^{3/4}}+\frac{x^{3/2}\sin\left(2\left(a+bx^2\right)\right)}{8b}\end{aligned}$$

Mathematica [A] time = 0.40, size = 142, normalized size = 1.08

$$\frac{bx^{11/2}\left(16\left(b^2x^4\right)^{3/4}\left(7\sin\left(2\left(a+bx^2\right)\right)+8bx^2\right)+21\sqrt[4]{2}\left(ibx^2\right)^{3/4}(\sin(2a)-i\cos(2a))\Gamma\left(\frac{3}{4},-2ibx^2\right)+21\sqrt[4]{2}\left(-i\right.}{896\left(b^2x^4\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Cos[a + b*x^2]^2,x]
```

```
[Out] (b*x^(11/2)*(21*2^(1/4)*(I*b*x^2)^(3/4)*Gamma[3/4, (-2*I)*b*x^2]*((-I)*Cos[
2*a] + Sin[2*a]) + 21*2^(1/4)*((-I)*b*x^2)^(3/4)*Gamma[3/4, (2*I)*b*x^2]*(I
*Cos[2*a] + Sin[2*a]) + 16*(b^2*x^4)^(3/4)*(8*b*x^2 + 7*Sin[2*(a + b*x^2)])
))/(896*(b^2*x^4)^(7/4))
```

fricas [A] time = 1.05, size = 78, normalized size = 0.59

$$\frac{21\,(2i\,b)^{\frac{1}{4}}\,e^{(-2i\,a)}\Gamma\left(\frac{3}{4},2i\,bx^2\right)+21\,(-2i\,b)^{\frac{1}{4}}\,e^{(2i\,a)}\Gamma\left(\frac{3}{4},-2i\,bx^2\right)+32\left(4\,b^2x^3+7\,bx\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)\right)\sqrt{x}}{896\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*cos(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/896*(21*(2*I*b)^(1/4)*e^(-2*I*a)*gamma(3/4, 2*I*b*x^2) + 21*(-2*I*b)^(1/4
)*e^(2*I*a)*gamma(3/4, -2*I*b*x^2) + 32*(4*b^2*x^3 + 7*b*x*cos(b*x^2 + a)*s
in(b*x^2 + a))*sqrt(x))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}}\cos\left(bx^2+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*cos(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)*cos(b*x^2 + a)^2, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}}\left(\cos^2\left(b\,x^2+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*cos(b*x^2+a)^2,x)
```

```
[Out] int(x^(5/2)*cos(b*x^2+a)^2,x)
```

maxima [A] time = 1.75, size = 174, normalized size = 1.32

$$\frac{256\,b^2x^4+224\,bx^2\sin\left(2\,bx^2+2\,a\right)+2^{\frac{1}{4}}\left(bx^2\right)^{\frac{1}{4}}\left(\left(21\,\sqrt{\sqrt{2}+2}\left(\Gamma\left(\frac{3}{4},2i\,bx^2\right)+\Gamma\left(\frac{3}{4},-2i\,bx^2\right)\right)-\sqrt{-\sqrt{2}+2}\left(-2\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*cos(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1792*(256*b^2*x^4 + 224*b*x^2*sin(2*b*x^2 + 2*a) + 2^(1/4)*(b*x^2)^(1/4)*
((21*sqrt(sqrt(2) + 2)*(gamma(3/4, 2*I*b*x^2) + gamma(3/4, -2*I*b*x^2)) - s
qrt(-sqrt(2) + 2)*(-21*I*gamma(3/4, 2*I*b*x^2) + 21*I*gamma(3/4, -2*I*b*x^2
)))*cos(2*a) + (21*sqrt(-sqrt(2) + 2)*(gamma(3/4, 2*I*b*x^2) + gamma(3/4, -
2*I*b*x^2)) - sqrt(sqrt(2) + 2)*(21*I*gamma(3/4, 2*I*b*x^2) - 21*I*gamma(3/
4, -2*I*b*x^2)))*sin(2*a)))/(b^2*sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\cos\left(b\,x^2+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*cos(a + b*x^2)^2,x)
```

```
[Out] int(x^(5/2)*cos(a + b*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}}\cos^2\left(a+bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*cos(b*x**2+a)**2,x)
```

```
[Out] Integral(x**(5/2)*cos(a + b*x**2)**2, x)
```

3.30 $\int x^{3/2}\cos^2\left(a+bx^2\right)\,dx$

Optimal. Leaf size=132

$$\frac{\sqrt{x}\sin\left(2\left(a+bx^2\right)\right)}{8b}-\frac{ie^{2ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},-2ibx^2\right)}{64\sqrt[4]{2}\,b\sqrt[4]{-ibx^2}}+\frac{ie^{-2ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},2ibx^2\right)}{64\sqrt[4]{2}\,b\sqrt[4]{ibx^2}}+\frac{x^{5/2}}{5}$$

```
[Out] 1/5*x^(5/2)-1/128*I*exp(2*I*a)*GAMMA(1/4,-2*I*b*x^2)*x^(1/2)*2^(3/4)/b/(-I*
b*x^2)^(1/4)+1/128*I*GAMMA(1/4,2*I*b*x^2)*x^(1/2)*2^(3/4)/b/exp(2*I*a)/(I*b
*x^2)^(1/4)+1/8*sin(2*b*x^2+2*a)*x^(1/2)/b
```

Rubi [A] time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3402, 3404, 3386, 3355, 2208}

$$-\frac{ie^{2ia}\sqrt{x}\,\text{Gamma}\left(\frac{1}{4},-2ibx^2\right)}{64\sqrt[4]{2}\,b\sqrt[4]{-ibx^2}}+\frac{ie^{-2ia}\sqrt{x}\,\text{Gamma}\left(\frac{1}{4},2ibx^2\right)}{64\sqrt[4]{2}\,b\sqrt[4]{ibx^2}}+\frac{\sqrt{x}\sin\left(2\left(a+bx^2\right)\right)}{8b}+\frac{x^{5/2}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Cos[a + b*x^2]^2,x]
```

```
[Out] x^(5/2)/5 - ((I/64)*E^((2*I)*a)*Sqrt[x]*Gamma[1/4, (-2*I)*b*x^2])/(2^(1/4)*
b*((-I)*b*x^2)^(1/4)) + ((I/64)*Sqrt[x]*Gamma[1/4, (2*I)*b*x^2])/(2^(1/4)*b
*E^((2*I)*a)*(I*b*x^2)^(1/4)) + (Sqrt[x]*Sin[2*(a + b*x^2)])/(8*b)
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3355

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
```

```
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3402

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_), x
_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m + 1) - 1
)*(a + b*Cos[c + (d*x^(k*n))/e^n])^p, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]
```

Rule 3404

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}\cos^2\left(a+bx^2\right)\,dx &= 2\,\text{Subst}\left(\int x^4\cos^2\left(a+bx^4\right)\,dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int\left(\frac{x^4}{2}+\frac{1}{2}x^4\cos\left(2a+2bx^4\right)\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{x^{5/2}}{5}+\text{Subst}\left(\int x^4\cos\left(2a+2bx^4\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{x^{5/2}}{5}+\frac{\sqrt{x}\,\sin\left(2\left(a+bx^2\right)\right)}{8b}-\frac{\text{Subst}\left(\int\sin\left(2a+2bx^4\right)\,dx, x, \sqrt{x}\right)}{8b}\\
&= \frac{x^{5/2}}{5}+\frac{\sqrt{x}\,\sin\left(2\left(a+bx^2\right)\right)}{8b}-\frac{i\,\text{Subst}\left(\int e^{-2ia-2ibx^4}\,dx, x, \sqrt{x}\right)}{16b}+\frac{i\,\text{Subst}\left(\int e^{2ia+2ibx
\end{aligned}$$

$$\phantom{\int x^{3/2}\cos^2\left(a+bx^2\right)\,dx} = \frac{x^{5/2}}{5}-\frac{ie^{2ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},-2ibx^2\right)}{64\sqrt[4]{2}\,b\sqrt[4]{-ibx^2}}+\frac{ie^{-2ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},2ibx^2\right)}{64\sqrt[4]{2}\,b\sqrt[4]{ibx^2}}+\frac{\sqrt{x}\,\sin\left(2\left(a+bx^2\right)\right)}{8b}$$

Mathematica [A] time = 0.33, size = 142, normalized size = 1.08

$$\frac{bx^{9/2}\left(16\sqrt[4]{b^2x^4}\left(5\sin\left(2\left(a+bx^2\right)\right)+8bx^2\right)+5\,2^{3/4}\sqrt[4]{ibx^2}\,(\sin(2a)-i\cos(2a))\Gamma\left(\frac{1}{4},-2ibx^2\right)+5\,2^{3/4}\sqrt[4]{-ibx^2}\,(\text{si}}{640\left(b^2x^4\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Cos[a + b*x^2]^2,x]
```

```
[Out] (b*x^(9/2)*(5*2^(3/4)*(I*b*x^2)^(1/4)*Gamma[1/4, (-2*I)*b*x^2]*((-I)*Cos[2*
a] + Sin[2*a]) + 5*2^(3/4)*((-I)*b*x^2)^(1/4)*Gamma[1/4, (2*I)*b*x^2]*(I*Co
s[2*a] + Sin[2*a]) + 16*(b^2*x^4)^(1/4)*(8*b*x^2 + 5*Sin[2*(a + b*x^2)])))/
(640*(b^2*x^4)^(5/4))
```

fricas [A] time = 0.87, size = 77, normalized size = 0.58

$$\frac{5\,(2i\,b)^{\frac{3}{4}}\,e^{(-2i\,a)}\Gamma\left(\frac{1}{4},2i\,bx^2\right)+5\,(-2i\,b)^{\frac{3}{4}}\,e^{(2i\,a)}\Gamma\left(\frac{1}{4},-2i\,bx^2\right)+32\left(4\,b^2x^2+5\,b\cos\left(bx^2+a\right)\sin\left(bx^2+a\right)\right)\sqrt{x}}{640\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/640*(5*(2*I*b)^(3/4)*e^(-2*I*a)*gamma(1/4, 2*I*b*x^2) + 5*(-2*I*b)^(3/4)*
e^(2*I*a)*gamma(1/4, -2*I*b*x^2) + 32*(4*b^2*x^2 + 5*b*cos(b*x^2 + a)*sin(b
*x^2 + a))*sqrt(x))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\cos\left(bx^2+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)*cos(b*x^2 + a)^2, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\left(\cos^2\left(b\,x^2+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*cos(b*x^2+a)^2,x)
```

```
[Out] int(x^(3/2)*cos(b*x^2+a)^2,x)
```

maxima [B] time = 1.97, size = 182, normalized size = 1.38

$$2^{\frac{3}{4}}\left(16\cdot 2^{\frac{1}{4}}\left(8\,bx^{\frac{5}{2}}+5\,\sqrt{x}\,\sin\left(2\,bx^2+2\,a\right)\right)\left(bx^2\right)^{\frac{1}{4}}+\left(\left(5\,\sqrt{-\sqrt{2}+2}\left(\Gamma\left(\frac{1}{4},2i\,bx^2\right)+\Gamma\left(\frac{1}{4},-2i\,bx^2\right)\right)+\sqrt{\sqrt{2}+2}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1280*2^(3/4)*(16*2^(1/4)*(8*b*x^(5/2) + 5*sqrt(x)*sin(2*b*x^2 + 2*a))*(b*
x^2)^(1/4) + ((5*sqrt(-sqrt(2) + 2)*(gamma(1/4, 2*I*b*x^2) + gamma(1/4, -2*
I*b*x^2)) + sqrt(sqrt(2) + 2)*(5*I*gamma(1/4, 2*I*b*x^2) - 5*I*gamma(1/4, -
2*I*b*x^2)))*cos(2*a) + (5*sqrt(sqrt(2) + 2)*(gamma(1/4, 2*I*b*x^2) + gamma
(1/4, -2*I*b*x^2)) + sqrt(-sqrt(2) + 2)*(-5*I*gamma(1/4, 2*I*b*x^2) + 5*I*g
amma(1/4, -2*I*b*x^2)))*sin(2*a))*sqrt(x))/((b*x^2)^(1/4)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2} \cos\left(b\,x^2+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*cos(a + b*x^2)^2,x)
```

```
[Out] int(x^(3/2)*cos(a + b*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \cos^2\left(a+bx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*cos(b*x**2+a)**2,x)
```

```
[Out] Integral(x**(3/2)*cos(a + b*x**2)**2, x)
```

3.31 $\int \sqrt{x} \cos^2\left(a+bx^2\right)\,dx$

Optimal. Leaf size=100

$$-\frac{e^{2ia}x^{3/2}\Gamma\left(\frac{3}{4},-2ibx^2\right)}{8\,2^{3/4}\left(-ibx^2\right)^{3/4}}-\frac{e^{-2ia}x^{3/2}\Gamma\left(\frac{3}{4},2ibx^2\right)}{8\,2^{3/4}\left(ibx^2\right)^{3/4}}+\frac{x^{3/2}}{3}$$

```
[Out] 1/3*x^(3/2)-1/16*exp(2*I*a)*x^(3/2)*GAMMA(3/4,-2*I*b*x^2)*2^(1/4)/(-I*b*x^2
)^(3/4)-1/16*x^(3/2)*GAMMA(3/4,2*I*b*x^2)*2^(1/4)/exp(2*I*a)/(I*b*x^2)^(3/4
)
```

Rubi [A] time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3402, 3404, 3390, 2218}

$$-\frac{e^{2ia}x^{3/2}\text{Gamma}\left(\frac{3}{4},-2ibx^2\right)}{8\,2^{3/4}\left(-ibx^2\right)^{3/4}}-\frac{e^{-2ia}x^{3/2}\text{Gamma}\left(\frac{3}{4},2ibx^2\right)}{8\,2^{3/4}\left(ibx^2\right)^{3/4}}+\frac{x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Cos[a + b*x^2]^2,x]
```

```
[Out] x^(3/2)/3 - (E^((2*I)*a)*x^(3/2)*Gamma[3/4, (-2*I)*b*x^2])/(8*2^(3/4)*((-I)
*b*x^2)^(3/4)) - (x^(3/2)*Gamma[3/4, (2*I)*b*x^2])/(8*2^(3/4)*E^((2*I)*a)*(
I*b*x^2)^(3/4))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3390

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3402

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_), x
_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m + 1) - 1
```

```
)*(a + b*Cos[c + (d*x^(k*n))/e^n])^p, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]
```

Rule 3404

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\cos^2\left(a+bx^2\right)dx &= 2\,\text{Subst}\left(\int x^2\cos^2\left(a+bx^4\right)dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int\left(\frac{x^2}{2}+\frac{1}{2}x^2\cos\left(2a+2bx^4\right)\right)dx, x, \sqrt{x}\right)\\
&= \frac{x^{3/2}}{3}+\text{Subst}\left(\int x^2\cos\left(2a+2bx^4\right)dx, x, \sqrt{x}\right)\\
&= \frac{x^{3/2}}{3}+\frac{1}{2}\text{Subst}\left(\int e^{-2ia-2ibx^4}x^2\,dx, x, \sqrt{x}\right)+\frac{1}{2}\text{Subst}\left(\int e^{2ia+2ibx^4}x^2\,dx, x, \sqrt{x}\right)\\
&= \frac{x^{3/2}}{3}-\frac{e^{2ia}x^{3/2}\Gamma\left(\frac{3}{4},-2ibx^2\right)}{8\,2^{3/4}\left(-ibx^2\right)^{3/4}}-\frac{e^{-2ia}x^{3/2}\Gamma\left(\frac{3}{4},2ibx^2\right)}{8\,2^{3/4}\left(ibx^2\right)^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 122, normalized size = 1.22

$$\frac{x^{3/2}\left(-3\sqrt[4]{2}\left(-ibx^2\right)^{3/4}(\cos(2a)-i\sin(2a))\Gamma\left(\frac{3}{4},2ibx^2\right)-3\sqrt[4]{2}\left(ibx^2\right)^{3/4}(\cos(2a)+i\sin(2a))\Gamma\left(\frac{3}{4},-2ibx^2\right)+16\left(\right.\right.}{48\left(b^2x^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Cos[a + b*x^2]^2,x]
```

```
[Out] (x^(3/2)*(16*(b^2*x^4)^(3/4) - 3*2^(1/4)*((-I)*b*x^2)^(3/4)*Gamma[3/4, (2*I
)*b*x^2]*(Cos[2*a] - I*Sin[2*a]) - 3*2^(1/4)*(I*b*x^2)^(3/4)*Gamma[3/4, (-2
*I)*b*x^2]*(Cos[2*a] + I*Sin[2*a])))/(48*(b^2*x^4)^(3/4))
```

fricas [A] time = 0.73, size = 50, normalized size = 0.50

$$\frac{16\,bx^{\frac{3}{2}}+3i\,(2i\,b)^{\frac{1}{4}}e^{(-2i\,a)}\Gamma\left(\frac{3}{4},2i\,bx^2\right)-3i\,(-2i\,b)^{\frac{1}{4}}e^{(2i\,a)}\Gamma\left(\frac{3}{4},-2i\,bx^2\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(16*b*x^(3/2) + 3*I*(2*I*b)^(1/4)*e^(-2*I*a)*gamma(3/4, 2*I*b*x^2) - 3
*I*(-2*I*b)^(1/4)*e^(2*I*a)*gamma(3/4, -2*I*b*x^2))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\cos\left(bx^2+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)*cos(b*x^2 + a)^2, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\left(\cos^2\left(b\,x^2+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*cos(b*x^2+a)^2,x)
```

```
[Out] int(x^(1/2)*cos(b*x^2+a)^2,x)
```

maxima [B] time = 2.60, size = 156, normalized size = 1.56

$$\frac{32\,bx^2-2^{\frac{1}{4}}\left(bx^2\right)^{\frac{1}{4}}\left(\left(\left(3\sqrt{-\sqrt{2}+2}\left(\Gamma\left(\frac{3}{4},2i\,bx^2\right)+\Gamma\left(\frac{3}{4},-2i\,bx^2\right)\right)-\sqrt{\sqrt{2}+2}\left(3i\,\Gamma\left(\frac{3}{4},2i\,bx^2\right)-3i\,\Gamma\left(\frac{3}{4},-2i\,bx^2\right)\right.\right.\right.\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(32*b*x^2 - 2^(1/4)*(b*x^2)^(1/4)*((3*sqrt(-sqrt(2) + 2)*(gamma(3/4, 2
*I*b*x^2) + gamma(3/4, -2*I*b*x^2)) - sqrt(sqrt(2) + 2)*(3*I*gamma(3/4, 2*I
*b*x^2) - 3*I*gamma(3/4, -2*I*b*x^2)))*cos(2*a) - (3*sqrt(sqrt(2) + 2)*(gam
ma(3/4, 2*I*b*x^2) + gamma(3/4, -2*I*b*x^2)) + sqrt(-sqrt(2) + 2)*(3*I*gamm
a(3/4, 2*I*b*x^2) - 3*I*gamma(3/4, -2*I*b*x^2)))*sin(2*a)))/(b*sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\cos\left(b\,x^2+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*cos(a + b*x^2)^2,x)
```

```
[Out] int(x^(1/2)*cos(a + b*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \cos^2\left(a + bx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*cos(b*x**2+a)**2,x)
```

```
[Out] Integral(sqrt(x)*cos(a + b*x**2)**2, x)
```

3.32 $\int \frac{\cos^2(a+bx^2)}{\sqrt{x}}\,dx$

Optimal. Leaf size=96

$$-\frac{e^{2ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},-2ibx^2\right)}{8\sqrt[4]{2}\sqrt[4]{-ibx^2}}-\frac{e^{-2ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},2ibx^2\right)}{8\sqrt[4]{2}\sqrt[4]{ibx^2}}+\sqrt{x}$$

```
[Out] x^(1/2)-1/16*exp(2*I*a)*GAMMA(1/4,-2*I*b*x^2)*x^(1/2)*2^(3/4)/(-I*b*x^2)^(1
/4)-1/16*GAMMA(1/4,2*I*b*x^2)*x^(1/2)*2^(3/4)/exp(2*I*a)/(I*b*x^2)^(1/4)
```

Rubi [A] time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3402, 3358, 3356, 2208}

$$-\frac{e^{2ia}\sqrt{x}\,\text{Gamma}\left(\frac{1}{4},-2ibx^2\right)}{8\sqrt[4]{2}\sqrt[4]{-ibx^2}}-\frac{e^{-2ia}\sqrt{x}\,\text{Gamma}\left(\frac{1}{4},2ibx^2\right)}{8\sqrt[4]{2}\sqrt[4]{ibx^2}}+\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]^2/Sqrt[x],x]
```

```
[Out] Sqrt[x] - (E^((2*I)*a)*Sqrt[x]*Gamma[1/4, (-2*I)*b*x^2])/(8*2^(1/4)*((-I)*b
*x^2)^(1/4)) - (Sqrt[x]*Gamma[1/4, (2*I)*b*x^2])/(8*2^(1/4)*E^((2*I)*a)*(I*
b*x^2)^(1/4))
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3356

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3358

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 1] && IGtQ[n, 1]
```

Rule 3402

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_), x
_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m + 1) - 1
)*(a + b*Cos[c + (d*x^(k*n))/e^n])^p, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2\left(a+bx^2\right)}{\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \cos^2\left(a+bx^4\right)\,dx, x, \sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(\frac{1}{2}+\frac{1}{2}\cos\left(2a+2bx^4\right)\right)dx, x, \sqrt{x}\right)\\
&= \sqrt{x}+\text{Subst}\left(\int \cos\left(2a+2bx^4\right)dx, x, \sqrt{x}\right)\\
&= \sqrt{x}+\frac{1}{2}\text{Subst}\left(\int e^{-2ia-2ibx^4}\,dx, x, \sqrt{x}\right)+\frac{1}{2}\text{Subst}\left(\int e^{2ia+2ibx^4}\,dx, x, \sqrt{x}\right)\\
&= \sqrt{x}-\frac{e^{2ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},-2ibx^2\right)}{8\sqrt[4]{2}\,\sqrt[4]{-ibx^2}}-\frac{e^{-2ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},2ibx^2\right)}{8\sqrt[4]{2}\,\sqrt[4]{ibx^2}}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 120, normalized size = 1.25

$$-\frac{\sqrt{x}\left(2^{3/4}\sqrt[4]{-ibx^2}\left(\cos(2a)-i\sin(2a)\right)\Gamma\left(\frac{1}{4},2ibx^2\right)+2^{3/4}\sqrt[4]{ibx^2}\left(\cos(2a)+i\sin(2a)\right)\Gamma\left(\frac{1}{4},-2ibx^2\right)-16\sqrt[4]{b^2x^4}\right)}{16\sqrt[4]{b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^2/Sqrt[x],x]
```

```
[Out] -1/16*(Sqrt[x]*(-16*(b^2*x^4)^(1/4) + 2^(3/4)*((-I)*b*x^2)^(1/4)*Gamma[1/4,
 (2*I)*b*x^2]*(Cos[2*a] - I*Sin[2*a]) + 2^(3/4)*(I*b*x^2)^(1/4)*Gamma[1/4,
(-2*I)*b*x^2]*(Cos[2*a] + I*Sin[2*a])))/(b^2*x^4)^(1/4)
```

fricas [A] time = 0.89, size = 50, normalized size = 0.52

$$\frac{i\,(2i\,b)^{\frac{3}{4}}\,e^{(-2i\,a)}\Gamma\left(\frac{1}{4},2i\,bx^2\right)-i\,(-2i\,b)^{\frac{3}{4}}\,e^{(2i\,a)}\Gamma\left(\frac{1}{4},-2i\,bx^2\right)+16\,b\sqrt{x}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(I*(2*I*b)^(3/4)*e^(-2*I*a)*gamma(1/4, 2*I*b*x^2) - I*(-2*I*b)^(3/4)*e
^(2*I*a)*gamma(1/4, -2*I*b*x^2) + 16*b*sqrt(x))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx^2+a\right)^2}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^2 + a)^2/sqrt(x), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(b\,x^2+a\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)^2/x^(1/2),x)
```

```
[Out] int(cos(b*x^2+a)^2/x^(1/2),x)
```

maxima [B] time = 2.45, size = 159, normalized size = 1.66

$$-\frac{2^{\frac{3}{4}}\left(\left(\left(\sqrt{\sqrt{2}+2}\left(\Gamma\left(\frac{1}{4},2i\,bx^2\right)+\Gamma\left(\frac{1}{4},-2i\,bx^2\right)\right)-\sqrt{-\sqrt{2}+2}\left(i\,\Gamma\left(\frac{1}{4},2i\,bx^2\right)-i\,\Gamma\left(\frac{1}{4},-2i\,bx^2\right)\right)\right)\cos\left(2\,a\right)-\left(\sqrt{-\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/32*2^(3/4)*(((sqrt(sqrt(2) + 2)*(gamma(1/4, 2*I*b*x^2) + gamma(1/4, -2*I
*b*x^2)) - sqrt(-sqrt(2) + 2)*(I*gamma(1/4, 2*I*b*x^2) - I*gamma(1/4, -2*I*
b*x^2)))*cos(2*a) - (sqrt(-sqrt(2) + 2)*(gamma(1/4, 2*I*b*x^2) + gamma(1/4,
 -2*I*b*x^2)) + sqrt(sqrt(2) + 2)*(I*gamma(1/4, 2*I*b*x^2) - I*gamma(1/4, -
2*I*b*x^2)))*sin(2*a))*sqrt(x) - 16*2^(1/4)*(b*x^2)^(1/4)*sqrt(x))/(b*x^2)^
(1/4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(b\,x^2+a\right)^2}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)^2/x^(1/2),x)
```

```
[Out] int(cos(a + b*x^2)^2/x^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(a+bx^2\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)**2/x**(1/2),x)
```

```
[Out] Integral(cos(a + b*x**2)**2/sqrt(x), x)
```

3.33 $\int \frac{\cos^2(a+bx^2)}{x^{3/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{\cos\left(2\left(a+bx^2\right)\right)}{\sqrt{x}}-\frac{ie^{2ia}bx^{3/2}\Gamma\left(\frac{3}{4},-2ibx^2\right)}{2^{3/4}\left(-ibx^2\right)^{3/4}}+\frac{ie^{-2ia}bx^{3/2}\Gamma\left(\frac{3}{4},2ibx^2\right)}{2^{3/4}\left(ibx^2\right)^{3/4}}-\frac{1}{\sqrt{x}}$$

```
[Out] -1/2*I*b*exp(2*I*a)*x^(3/2)*GAMMA(3/4,-2*I*b*x^2)*2^(1/4)/(-I*b*x^2)^(3/4)+
1/2*I*b*x^(3/2)*GAMMA(3/4,2*I*b*x^2)*2^(1/4)/exp(2*I*a)/(I*b*x^2)^(3/4)-1/x
^(1/2)-cos(2*b*x^2+2*a)/x^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3402, 3404, 3388, 3389, 2218}

$$-\frac{ie^{2ia}bx^{3/2}\text{Gamma}\left(\frac{3}{4},-2ibx^2\right)}{2^{3/4}\left(-ibx^2\right)^{3/4}}+\frac{ie^{-2ia}bx^{3/2}\text{Gamma}\left(\frac{3}{4},2ibx^2\right)}{2^{3/4}\left(ibx^2\right)^{3/4}}-\frac{\cos\left(2\left(a+bx^2\right)\right)}{\sqrt{x}}-\frac{1}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]^2/x^(3/2),x]
```

```
[Out] -(1/Sqrt[x]) - Cos[2*(a + b*x^2)]/Sqrt[x] - (I*b*E^((2*I)*a)*x^(3/2)*Gamma[
3/4, (-2*I)*b*x^2])/(2^(3/4)*((-I)*b*x^2)^(3/4)) + (I*b*x^(3/2)*Gamma[3/4,
(2*I)*b*x^2])/(2^(3/4)*E^((2*I)*a)*(I*b*x^2)^(3/4))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rule 3389

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m}, x] && IGtQ[n, 0]
```

Rule 3402

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_), x
_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m + 1) - 1
)*(a + b*Cos[c + (d*x^(k*n))/e^n])^p, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]
```

Rule 3404

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2\left(a+bx^2\right)}{x^{3/2}}\,dx = 2\,\text{Subst}\left(\int \frac{\cos^2\left(a+bx^4\right)}{x^2}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \left(\frac{1}{2x^2}+\frac{\cos\left(2a+2bx^4\right)}{2x^2}\right)dx, x, \sqrt{x}\right)$$
$$= -\frac{1}{\sqrt{x}} + \text{Subst}\left(\int \frac{\cos\left(2a+2bx^4\right)}{x^2}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{1}{\sqrt{x}} - \frac{\cos\left(2\left(a+bx^2\right)\right)}{\sqrt{x}} - (8b)\,\text{Subst}\left(\int x^2\sin\left(2a+2bx^4\right)dx, x, \sqrt{x}\right)$$
$$= -\frac{1}{\sqrt{x}} - \frac{\cos\left(2\left(a+bx^2\right)\right)}{\sqrt{x}} - (4ib)\,\text{Subst}\left(\int e^{-2ia-2ibx^4}x^2\,dx, x, \sqrt{x}\right) + (4ib)\,\text{Subst}\left(\int e^{2i
$$
$$= -\frac{1}{\sqrt{x}} - \frac{\cos\left(2\left(a+bx^2\right)\right)}{\sqrt{x}} - \frac{ibe^{2ia}x^{3/2}\Gamma\left(\frac{3}{4},-2ibx^2\right)}{2^{3/4}\left(-ibx^2\right)^{3/4}} + \frac{ibe^{-2ia}x^{3/2}\Gamma\left(\frac{3}{4},2ibx^2\right)}{2^{3/4}\left(ibx^2\right)^{3/4}}$$

Mathematica [A] time = 0.34, size = 137, normalized size = 1.17

$$\frac{-4\left(b^2x^4\right)^{3/4}\cos^2\left(a+bx^2\right)+\sqrt[4]{2}\,bx^2\left(ibx^2\right)^{3/4}(\sin(2a)-i\cos(2a))\Gamma\left(\frac{3}{4},-2ibx^2\right)+i\sqrt[4]{2}\left(-ibx^2\right)^{7/4}(\sin(2a)+i\,c}{2\sqrt{x}\left(b^2x^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^2/x^(3/2),x]
```

```
[Out] (-4*(b^2*x^4)^(3/4)*Cos[a + b*x^2]^2 + 2^(1/4)*b*x^2*(I*b*x^2)^(3/4)*Gamma[
3/4, (-2*I)*b*x^2]*((-I)*Cos[2*a] + Sin[2*a]) + I*2^(1/4)*((-I)*b*x^2)^(7/4
)*Gamma[3/4, (2*I)*b*x^2]*(I*Cos[2*a] + Sin[2*a]))/(2*Sqrt[x]*(b^2*x^4)^(3/
4))
```

fricas [A] time = 0.98, size = 59, normalized size = 0.50

$$\frac{(2i\,b)^{\frac{1}{4}}\,xe^{(-2i\,a)}\Gamma\left(\frac{3}{4},2i\,bx^2\right)+(-2i\,b)^{\frac{1}{4}}\,xe^{(2i\,a)}\Gamma\left(\frac{3}{4},-2i\,bx^2\right)-4\sqrt{x}\cos\left(bx^2+a\right)^2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((2*I*b)^(1/4)*x*e^(-2*I*a)*gamma(3/4, 2*I*b*x^2) + (-2*I*b)^(1/4)*x*e^
(2*I*a)*gamma(3/4, -2*I*b*x^2) - 4*sqrt(x)*cos(b*x^2 + a)^2)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(bx^2+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^2 + a)^2/x^(3/2), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int\frac{\cos^2\left(b\,x^2+a\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)^2/x^(3/2),x)
```

```
[Out] int(cos(b*x^2+a)^2/x^(3/2),x)
```

maxima [A] time = 1.87, size = 143, normalized size = 1.22

$$-\frac{2^{\frac{1}{4}}\left(bx^2\right)^{\frac{1}{4}}\left(\left(\sqrt{\sqrt{2}+2}\left(\Gamma\left(-\frac{1}{4},2i\,bx^2\right)+\Gamma\left(-\frac{1}{4},-2i\,bx^2\right)\right)+\sqrt{-\sqrt{2}+2}\left(i\,\Gamma\left(-\frac{1}{4},2i\,bx^2\right)-i\,\Gamma\left(-\frac{1}{4},-2i\,bx^2\right)\right)\right)\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(2^(1/4)*(b*x^2)^(1/4)*((sqrt(sqrt(2) + 2)*(gamma(-1/4, 2*I*b*x^2) +
gamma(-1/4, -2*I*b*x^2)) + sqrt(-sqrt(2) + 2)*(I*gamma(-1/4, 2*I*b*x^2) - I
*gamma(-1/4, -2*I*b*x^2)))*cos(2*a) + (sqrt(-sqrt(2) + 2)*(gamma(-1/4, 2*I*
b*x^2) + gamma(-1/4, -2*I*b*x^2)) + sqrt(sqrt(2) + 2)*(-I*gamma(-1/4, 2*I*b
*x^2) + I*gamma(-1/4, -2*I*b*x^2)))*sin(2*a)) + 16)/sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(b\,x^2+a\right)^2}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)^2/x^(3/2),x)
```

```
[Out] int(cos(a + b*x^2)^2/x^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(a+bx^2\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)**2/x**(3/2),x)
```

```
[Out] Integral(cos(a + b*x**2)**2/x**(3/2), x)
```

3.34 $\int \frac{\cos^2(a+bx^2)}{x^{5/2}} \, dx$

Optimal. Leaf size=116

$$-\frac{ie^{2ia}b\sqrt{x}\,\Gamma\left(\frac{1}{4},-2ibx^2\right)}{3\sqrt[4]{2}\sqrt[4]{-ibx^2}}+\frac{ie^{-2ia}b\sqrt{x}\,\Gamma\left(\frac{1}{4},2ibx^2\right)}{3\sqrt[4]{2}\sqrt[4]{ibx^2}}-\frac{2\cos^2\left(a+bx^2\right)}{3x^{3/2}}$$

```
[Out] -2/3*cos(b*x^2+a)^2/x^(3/2)-1/6*I*b*exp(2*I*a)*GAMMA(1/4,-2*I*b*x^2)*x^(1/2
)*2^(3/4)/(-I*b*x^2)^(1/4)+1/6*I*b*GAMMA(1/4,2*I*b*x^2)*x^(1/2)*2^(3/4)/exp
(2*I*a)/(I*b*x^2)^(1/4)
```

Rubi [A] time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3402, 3394, 4573, 3373, 3355, 2208}

$$-\frac{ie^{2ia}b\sqrt{x}\,\text{Gamma}\left(\frac{1}{4},-2ibx^2\right)}{3\sqrt[4]{2}\sqrt[4]{-ibx^2}}+\frac{ie^{-2ia}b\sqrt{x}\,\text{Gamma}\left(\frac{1}{4},2ibx^2\right)}{3\sqrt[4]{2}\sqrt[4]{ibx^2}}-\frac{2\cos^2\left(a+bx^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^2]^2/x^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x^2]^2)/(3*x^(3/2)) - ((I/3)*b*E^((2*I)*a)*Sqrt[x]*Gamma[1/4,
 (-2*I)*b*x^2])/(2^(1/4)*((-I)*b*x^2)^(1/4)) + ((I/3)*b*Sqrt[x]*Gamma[1/4,
(2*I)*b*x^2])/(2^(1/4)*E^((2*I)*a)*(I*b*x^2)^(1/4))
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3355

```
Int[Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[I/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] - Dist[I/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ[n, 2]
```

Rule 3373

```
Int[((a_.) + (b_.)*Sin[u_])^(p_.), x_Symbol] :> Int[(a + b*Sin[ExpandToSum[
u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ
[u, x]
```

Rule 3394

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_)]^(p_)*(x_)^(m_.), x_Symbol] :> Simp[(x^(m +
 1)*Cos[a + b*x^n]^p)/(m + 1), x] + Dist[(b*n*p)/(m + 1), Int[Cos[a + b*x^n
]^(p - 1)*Sin[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 1] && EqQ[m
 + n, 0] && NeQ[n, 1] && IntegerQ[n]
```

Rule 3402

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_.)*(x_))^(m_), x
_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m + 1) - 1
)*(a + b*Cos[c + (d*x^(k*n))/e^n])^p, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e}, x] && IntegerQ[p] && IGtQ[n, 0] && FractionQ[m]
```

Rule 4573

```
Int[Cos[w_]^(p_.)*(u_.)*Sin[v_]^(p_.), x_Symbol] :> Dist[1/2^p, Int[u*Sin[2
*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2\left(a+bx^2\right)}{x^{5/2}}\,dx &= 2\,\text{Subst}\left(\int \frac{\cos^2\left(a+bx^4\right)}{x^4}\,dx, x, \sqrt{x}\right)\\
&= -\frac{2\cos^2\left(a+bx^2\right)}{3x^{3/2}} - \frac{1}{3}(16b)\,\text{Subst}\left(\int \cos\left(a+bx^4\right)\sin\left(a+bx^4\right)\,dx, x, \sqrt{x}\right)\\
&= -\frac{2\cos^2\left(a+bx^2\right)}{3x^{3/2}} - \frac{1}{3}(8b)\,\text{Subst}\left(\int \sin\left(2\left(a+bx^4\right)\right)\,dx, x, \sqrt{x}\right)\\
&= -\frac{2\cos^2\left(a+bx^2\right)}{3x^{3/2}} - \frac{1}{3}(8b)\,\text{Subst}\left(\int \sin\left(2a+2bx^4\right)\,dx, x, \sqrt{x}\right)\\
&= -\frac{2\cos^2\left(a+bx^2\right)}{3x^{3/2}} - \frac{1}{3}(4ib)\,\text{Subst}\left(\int e^{-2ia-2ibx^4}\,dx, x, \sqrt{x}\right) + \frac{1}{3}(4ib)\,\text{Subst}\left(\int e^{2ia+2ibx^4}\right.\\
&= -\frac{2\cos^2\left(a+bx^2\right)}{3x^{3/2}} - \frac{ibe^{2ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},-2ibx^2\right)}{3\sqrt[4]{2}\,\sqrt[4]{-ibx^2}} + \frac{ibe^{-2ia}\sqrt{x}\,\Gamma\left(\frac{1}{4},2ibx^2\right)}{3\sqrt[4]{2}\,\sqrt[4]{ibx^2}}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 137, normalized size = 1.18

$$\frac{-4\sqrt[4]{b^2x^4}\,\cos^2\left(a+bx^2\right) + 2^{3/4}bx^2\sqrt[4]{ibx^2}\,(\sin(2a)-i\cos(2a))\Gamma\left(\frac{1}{4},-2ibx^2\right) + i2^{3/4}\left(-ibx^2\right)^{5/4}(\sin(2a)+i\cos(2a}{6x^{3/2}\sqrt[4]{b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^2]^2/x^(5/2),x]
```

```
[Out] (-4*(b^2*x^4)^(1/4)*Cos[a + b*x^2]^2 + 2^(3/4)*b*x^2*(I*b*x^2)^(1/4)*Gamma[
1/4, (-2*I)*b*x^2]*((-I)*Cos[2*a] + Sin[2*a]) + I*2^(3/4)*((-I)*b*x^2)^(5/4
)*Gamma[1/4, (2*I)*b*x^2]*(I*Cos[2*a] + Sin[2*a]))/(6*x^(3/2)*(b^2*x^4)^(1/
4))
```

fricas [A] time = 0.66, size = 63, normalized size = 0.54

$$\frac{(2i\,b)^{\frac{3}{4}}\,x^2 e^{(-2i\,a)}\Gamma\left(\frac{1}{4}, 2i\,bx^2\right) + (-2i\,b)^{\frac{3}{4}}\,x^2 e^{(2i\,a)}\Gamma\left(\frac{1}{4}, -2i\,bx^2\right) - 4\sqrt{x}\cos\left(bx^2+a\right)^2}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*((2*I*b)^(3/4)*x^2*e^(-2*I*a)*gamma(1/4, 2*I*b*x^2) + (-2*I*b)^(3/4)*x^
2*e^(2*I*a)*gamma(1/4, -2*I*b*x^2) - 4*sqrt(x)*cos(b*x^2 + a)^2)/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx^2+a\right)^2}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^2 + a)^2/x^(5/2), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(b\,x^2+a\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x^2+a)^2/x^(5/2),x)
```

```
[Out] int(cos(b*x^2+a)^2/x^(5/2),x)
```

maxima [A] time = 0.91, size = 145, normalized size = 1.25

$$-\frac{2^{\frac{3}{4}}\left(bx^2\right)^{\frac{3}{4}}\left(\left(3\sqrt{-\sqrt{2}+2}\left(\Gamma\left(-\frac{3}{4}, 2i\,bx^2\right)+\Gamma\left(-\frac{3}{4}, -2i\,bx^2\right)\right)+\sqrt{\sqrt{2}+2}\left(3i\,\Gamma\left(-\frac{3}{4}, 2i\,bx^2\right)-3i\,\Gamma\left(-\frac{3}{4}, -2i\,bx^2\right)\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x^2+a)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] -1/48*(2^(3/4)*(b*x^2)^(3/4)*((3*sqrt(-sqrt(2) + 2)*(gamma(-3/4, 2*I*b*x^2)
 + gamma(-3/4, -2*I*b*x^2)) + sqrt(sqrt(2) + 2)*(3*I*gamma(-3/4, 2*I*b*x^2)
 - 3*I*gamma(-3/4, -2*I*b*x^2)))*cos(2*a) + (3*sqrt(sqrt(2) + 2)*(gamma(-3/
4, 2*I*b*x^2) + gamma(-3/4, -2*I*b*x^2)) + sqrt(-sqrt(2) + 2)*(-3*I*gamma(-
3/4, 2*I*b*x^2) + 3*I*gamma(-3/4, -2*I*b*x^2)))*sin(2*a)) + 16)/x^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos \left(b\,x^2+a\right)^2}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^2)^2/x^(5/2),x)
```

```
[Out] int(cos(a + b*x^2)^2/x^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(a+bx^2\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x**2+a)**2/x**(5/2),x)
```

```
[Out] Integral(cos(a + b*x**2)**2/x**(5/2), x)
```

3.35 $\int \cos\left(a+\frac{b}{x}\right) dx$

Optimal. Leaf size=31

$$b\sin(a)\mathrm{Ci}\left(\frac{b}{x}\right)+b\cos(a)\mathrm{Si}\left(\frac{b}{x}\right)+x\cos\left(a+\frac{b}{x}\right)$$

```
[Out] x*cos(a+b/x)+b*cos(a)*Si(b/x)+b*Ci(b/x)*sin(a)
```

Rubi [A] time = 0.07, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3362, 3297, 3303, 3299, 3302}

$$b\sin(a)\mathrm{CosIntegral}\left(\frac{b}{x}\right)+b\cos(a)\mathrm{Si}\left(\frac{b}{x}\right)+x\cos\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b/x],x]
```

```
[Out] x*Cos[a + b/x] + b*CosIntegral[b/x]*Sin[a] + b*Cos[a]*SinIntegral[b/x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3362

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\int \cos\left(a+\frac{b}{x}\right)dx &= -\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= x\cos\left(a+\frac{b}{x}\right)+b\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx, x, \frac{1}{x}\right)\\ &= x\cos\left(a+\frac{b}{x}\right)+(b\cos(a))\,\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx, x, \frac{1}{x}\right)+(b\sin(a))\,\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x,\right.\\ &= x\cos\left(a+\frac{b}{x}\right)+b\text{Ci}\left(\frac{b}{x}\right)\sin(a)+b\cos(a)\text{Si}\left(\frac{b}{x}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 1.00

$$b\sin(a)\text{Ci}\left(\frac{b}{x}\right)+b\cos(a)\text{Si}\left(\frac{b}{x}\right)+x\cos\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b/x],x]
```

```
[Out] x*Cos[a + b/x] + b*CosIntegral[b/x]*Sin[a] + b*Cos[a]*SinIntegral[b/x]
```

fricas [A] time = 1.15, size = 45, normalized size = 1.45

$$b\cos(a)\,\text{Si}\left(\frac{b}{x}\right)+x\cos\left(\frac{ax+b}{x}\right)+\frac{1}{2}\left(b\,\text{Ci}\left(\frac{b}{x}\right)+b\,\text{Ci}\left(-\frac{b}{x}\right)\right)\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x),x, algorithm="fricas")
```

```
[Out] b*cos(a)*sin_integral(b/x) + x*cos((a*x + b)/x) + 1/2*(b*cos_integral(b/x)
+ b*cos_integral(-b/x))*sin(a)
```

giac [B] time = 0.34, size = 132, normalized size = 4.26

$$\frac{ab^2\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)\sin(a)-ab^2\cos(a)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)-\frac{(ax+b)b^2\operatorname{Ci}\left(-a+\frac{ax+b}{x}\right)\sin(a)}{x}+\frac{(ax+b)b^2\cos(a)\operatorname{Si}\left(a-\frac{ax+b}{x}\right)}{x}-b^2\cos\left(\frac{ax+b}{x}\right)}{\left(a-\frac{ax+b}{x}\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x),x, algorithm="giac")
```

```
[Out] (a*b^2*cos_integral(-a + (a*x + b)/x)*sin(a) - a*b^2*cos(a)*sin_integral(a
- (a*x + b)/x) - (a*x + b)*b^2*cos_integral(-a + (a*x + b)/x)*sin(a)/x + (a
*x + b)*b^2*cos(a)*sin_integral(a - (a*x + b)/x)/x - b^2*cos((a*x + b)/x))/
((a - (a*x + b)/x)*b)
```

maple [A] time = 0.05, size = 39, normalized size = 1.26

$$-b\left(-\frac{\cos\left(a+\frac{b}{x}\right)x}{b}-\operatorname{Si}\left(\frac{b}{x}\right)\cos(a)-\operatorname{Ci}\left(\frac{b}{x}\right)\sin(a)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b/x),x)
```

```
[Out] -b*(-cos(a+b/x)*x/b-Si(b/x)*cos(a)-Ci(b/x)*sin(a))
```

maxima [C] time = 1.90, size = 57, normalized size = 1.84

$$\frac{1}{2}\left(\left(-i\operatorname{Ei}\left(\frac{ib}{x}\right)+i\operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\cos(a)+\left(\operatorname{Ei}\left(\frac{ib}{x}\right)+\operatorname{Ei}\left(-\frac{ib}{x}\right)\right)\sin(a)\right)b+x\cos\left(\frac{ax+b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x),x, algorithm="maxima")
```

```
[Out] 1/2*((-I*Ei(I*b/x) + I*Ei(-I*b/x))*cos(a) + (Ei(I*b/x) + Ei(-I*b/x))*sin(a)
)*b + x*cos((a*x + b)/x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\cos\left(a+\frac{b}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b/x),x)
```

```
[Out] int(cos(a + b/x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos \left(a + \frac{b}{x}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x),x)
```

```
[Out] Integral(cos(a + b/x), x)
```

3.36 $\int \frac{\cos\left(a+\frac{b}{x}\right)}{x}\,dx$

Optimal. Leaf size=20

$$\sin(a)\text{Si}\left(\frac{b}{x}\right)-\cos(a)\text{Ci}\left(\frac{b}{x}\right)$$

```
[Out] -Ci(b/x)*cos(a)+Si(b/x)*sin(a)
```

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3378, 3376, 3375}

$$\sin(a)\text{Si}\left(\frac{b}{x}\right)-\cos(a)\text{CosIntegral}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b/x]/x,x]
```

```
[Out] -(Cos[a]*CosIntegral[b/x]) + Sin[a]*SinIntegral[b/x]
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x}\,dx = \cos(a)\int \frac{\cos\left(\frac{b}{x}\right)}{x}\,dx - \sin(a)\int \frac{\sin\left(\frac{b}{x}\right)}{x}\,dx$$
$$= -\cos(a)\mathrm{Ci}\left(\frac{b}{x}\right) + \sin(a)\mathrm{Si}\left(\frac{b}{x}\right)$$

Mathematica [A] time = 0.04, size = 20, normalized size = 1.00

$$\sin(a)\mathrm{Si}\left(\frac{b}{x}\right) - \cos(a)\mathrm{Ci}\left(\frac{b}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b/x]/x,x]
```

```
[Out] -(Cos[a]*CosIntegral[b/x]) + Sin[a]*SinIntegral[b/x]
```

fricas [A] time = 1.43, size = 28, normalized size = 1.40

$$-\frac{1}{2}\left(\mathrm{Ci}\left(\frac{b}{x}\right) + \mathrm{Ci}\left(-\frac{b}{x}\right)\right)\cos(a) + \sin(a)\,\mathrm{Si}\left(\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(cos_integral(b/x) + cos_integral(-b/x))*cos(a) + sin(a)*sin_integral(
b/x)
```

giac [B] time = 0.41, size = 41, normalized size = 2.05

$$-\frac{b\cos(a)\,\mathrm{Ci}\left(-a+\frac{ax+b}{x}\right) + b\sin(a)\,\mathrm{Si}\left(a-\frac{ax+b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x,x, algorithm="giac")
```

```
[Out] -(b*cos(a)*cos_integral(-a + (a*x + b)/x) + b*sin(a)*sin_integral(a - (a*x
+ b)/x))/b
```

maple [A] time = 0.04, size = 21, normalized size = 1.05

$$-\mathrm{Ci}\left(\frac{b}{x}\right)\cos(a) + \mathrm{Si}\left(\frac{b}{x}\right)\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b/x)/x,x)
```

```
[Out] -Ci(b/x)*cos(a)+Si(b/x)*sin(a)
```

maxima [C] time = 2.01, size = 43, normalized size = 2.15

$$-\frac{1}{2}\left(\mathrm{Ei}\left(\frac{i\,b}{x}\right)+\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\cos(a)-\frac{1}{2}\left(i\,\mathrm{Ei}\left(\frac{i\,b}{x}\right)-i\,\mathrm{Ei}\left(-\frac{i\,b}{x}\right)\right)\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(Ei(I*b/x) + Ei(-I*b/x))*cos(a) - 1/2*(I*Ei(I*b/x) - I*Ei(-I*b/x))*sin
(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\sin(a)\,\mathrm{sinint}\left(\frac{b}{x}\right)-\cos(a)\,\mathrm{cosint}\left(\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b/x)/x,x)
```

```
[Out] sin(a)*sinint(b/x) - cos(a)*cosint(b/x)
```

sympy [A] time = 0.97, size = 15, normalized size = 0.75

$$\sin\left(a\right)\mathrm{Si}\left(\frac{b}{x}\right)-\cos\left(a\right)\mathrm{Ci}\left(\frac{b}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x,x)
```

```
[Out] sin(a)*Si(b/x) - cos(a)*Ci(b/x)
```

3.37 $\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=13

$$-\frac{\sin\left(a+\frac{b}{x}\right)}{b}$$

```
[Out] -sin(a+b/x)/b
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3380, 2637}

$$-\frac{\sin\left(a+\frac{b}{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b/x]/x^2,x]
```

```
[Out] -(Sin[a + b/x]/b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^2}\,dx &= -\text{Subst}\left(\int \cos(a+bx)\,dx, x, \frac{1}{x}\right)\\ &= -\frac{\sin\left(a+\frac{b}{x}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{\sin\left(a+\frac{b}{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b/x]/x^2,x]
```

```
[Out] -(Sin[a + b/x]/b)
```

fricas [A] time = 0.93, size = 15, normalized size = 1.15

$$-\frac{\sin\left(\frac{ax+b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x^2,x, algorithm="fricas")
```

```
[Out] -sin((a*x + b)/x)/b
```

giac [A] time = 0.36, size = 15, normalized size = 1.15

$$-\frac{\sin\left(\frac{ax+b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x^2,x, algorithm="giac")
```

```
[Out] -sin((a*x + b)/x)/b
```

maple [A] time = 0.02, size = 14, normalized size = 1.08

$$-\frac{\sin\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b/x)/x^2,x)
```

```
[Out] -sin(a+b/x)/b
```

maxima [A] time = 2.00, size = 13, normalized size = 1.00

$$-\frac{\sin\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x^2,x, algorithm="maxima")
```

```
[Out] -sin(a + b/x)/b
```

mupad [B] time = 0.26, size = 13, normalized size = 1.00

$$-\frac{\sin\left(a+\frac{b}{x}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b/x)/x^2,x)
```

```
[Out] -sin(a + b/x)/b
```

sympy [A] time = 0.98, size = 15, normalized size = 1.15

$$\begin{cases} -\frac{\sin\left(a+\frac{b}{x}\right)}{b} & \text{for}\ b \neq 0 \\ -\frac{\cos\left(a\right)}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x**2,x)
```

```
[Out] Piecewise((-sin(a + b/x)/b, Ne(b, 0)), (-cos(a)/x, True))
```

3.38 $\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^3}\,dx$

Optimal. Leaf size=30

$$-\frac{\cos\left(a+\frac{b}{x}\right)}{b^2}-\frac{\sin\left(a+\frac{b}{x}\right)}{bx}$$

```
[Out] -cos(a+b/x)/b^2-sin(a+b/x)/b/x
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3380, 3296, 2638}

$$-\frac{\cos\left(a+\frac{b}{x}\right)}{b^2}-\frac{\sin\left(a+\frac{b}{x}\right)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b/x]/x^3,x]
```

```
[Out] -(Cos[a + b/x]/b^2) - Sin[a + b/x]/(b*x)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^3}\,dx = -\operatorname{Subst}\left(\int x\cos(a+bx)\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\sin\left(a+\frac{b}{x}\right)}{bx} + \frac{\operatorname{Subst}\left(\int \sin(a+bx)\,dx, x, \frac{1}{x}\right)}{b}$$
$$= -\frac{\cos\left(a+\frac{b}{x}\right)}{b^2} - \frac{\sin\left(a+\frac{b}{x}\right)}{bx}$$

Mathematica [A] time = 0.05, size = 29, normalized size = 0.97

$$-\frac{b\sin\left(a+\frac{b}{x}\right)+x\cos\left(a+\frac{b}{x}\right)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b/x]/x^3,x]
[Out] -((x*Cos[a + b/x] + b*Sin[a + b/x])/(b^2*x))
```

fricas [A] time = 0.75, size = 33, normalized size = 1.10

$$-\frac{x\cos\left(\frac{ax+b}{x}\right)+b\sin\left(\frac{ax+b}{x}\right)}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x^3,x, algorithm="fricas")
[Out] -(x*cos((a*x + b)/x) + b*sin((a*x + b)/x))/(b^2*x)
```

giac [A] time = 0.39, size = 49, normalized size = 1.63

$$\frac{a\sin\left(\frac{ax+b}{x}\right)-\frac{(ax+b)\sin\left(\frac{ax+b}{x}\right)}{x}-\cos\left(\frac{ax+b}{x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x^3,x, algorithm="giac")
[Out] (a*sin((a*x + b)/x) - (a*x + b)*sin((a*x + b)/x)/x - cos((a*x + b)/x))/b^2
```

maple [A] time = 0.04, size = 42, normalized size = 1.40

$$-\frac{\cos\left(a+\frac{b}{x}\right)+\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)-a\sin\left(a+\frac{b}{x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b/x)/x^3,x)
```

```
[Out] -1/b^2*(cos(a+b/x)+(a+b/x)*sin(a+b/x)-a*sin(a+b/x))
```

maxima [C] time = 1.69, size = 51, normalized size = 1.70

$$-\frac{\left(\Gamma\left(2,\frac{ib}{x}\right)+\Gamma\left(2,-\frac{ib}{x}\right)\right)\cos(a)-\left(i\,\Gamma\left(2,\frac{ib}{x}\right)-i\,\Gamma\left(2,-\frac{ib}{x}\right)\right)\sin(a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*((gamma(2, I*b/x) + gamma(2, -I*b/x))*cos(a) - (I*gamma(2, I*b/x) - I*
gamma(2, -I*b/x))*sin(a))/b^2
```

mupad [B] time = 0.28, size = 30, normalized size = 1.00

$$-\frac{\cos\left(a+\frac{b}{x}\right)}{b^2}-\frac{\sin\left(a+\frac{b}{x}\right)}{b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b/x)/x^3,x)
```

```
[Out] - cos(a + b/x)/b^2 - sin(a + b/x)/(b*x)
```

sympy [A] time = 1.80, size = 31, normalized size = 1.03

$$\begin{cases} -\frac{\sin\left(a+\frac{b}{x}\right)}{bx}-\frac{\cos\left(a+\frac{b}{x}\right)}{b^2} & \text{for } b\neq 0 \\ -\frac{\cos(a)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x**3,x)
```

```
[Out] Piecewise((-sin(a + b/x)/(b*x) - cos(a + b/x)/b**2, Ne(b, 0)), (-cos(a)/(2*
x**2), True))
```

3.39 $\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^4} dx$

Optimal. Leaf size=46

$$\frac{2\sin\left(a+\frac{b}{x}\right)}{b^3}-\frac{2\cos\left(a+\frac{b}{x}\right)}{b^2x}-\frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}$$

```
[Out] -2*cos(a+b/x)/b^2/x+2*sin(a+b/x)/b^3-sin(a+b/x)/b/x^2
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3380, 3296, 2637}

$$\frac{2\sin\left(a+\frac{b}{x}\right)}{b^3}-\frac{2\cos\left(a+\frac{b}{x}\right)}{b^2x}-\frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b/x]/x^4,x]
```

```
[Out] (-2*Cos[a + b/x])/(b^2*x) + (2*Sin[a + b/x])/b^3 - Sin[a + b/x]/(b*x^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{\cos\left(a+\frac{b}{x}\right)}{x^4}\,dx = -\operatorname{Subst}\left(\int x^2\cos(a+bx)\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\sin\left(a+\frac{b}{x}\right)}{bx^2} + \frac{2\operatorname{Subst}\left(\int x\sin(a+bx)\,dx, x, \frac{1}{x}\right)}{b}$$
$$= -\frac{2\cos\left(a+\frac{b}{x}\right)}{b^2x} - \frac{\sin\left(a+\frac{b}{x}\right)}{bx^2} + \frac{2\operatorname{Subst}\left(\int \cos(a+bx)\,dx, x, \frac{1}{x}\right)}{b^2}$$
$$= -\frac{2\cos\left(a+\frac{b}{x}\right)}{b^2x} + \frac{2\sin\left(a+\frac{b}{x}\right)}{b^3} - \frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}$$

Mathematica [**A**] time = 0.00, size = 46, normalized size = 1.00

$$\frac{2\sin\left(a+\frac{b}{x}\right)}{b^3} - \frac{2\cos\left(a+\frac{b}{x}\right)}{b^2x} - \frac{\sin\left(a+\frac{b}{x}\right)}{bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b/x]/x^4,x]
[Out] (-2*Cos[a + b/x])/(b^2*x) + (2*Sin[a + b/x])/b^3 - Sin[a + b/x]/(b*x^2)
```

fricas [**A**] time = 0.71, size = 43, normalized size = 0.93

$$-\frac{2\,bx\cos\left(\frac{ax+b}{x}\right) + \left(b^2-2\,x^2\right)\sin\left(\frac{ax+b}{x}\right)}{b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x^4,x, algorithm="fricas")
[Out] -(2*b*x*cos((a*x + b)/x) + (b^2 - 2*x^2)*sin((a*x + b)/x))/(b^3*x^2)
```

giac [**B**] time = 0.43, size = 107, normalized size = 2.33

$$-\frac{a^2\sin\left(\frac{ax+b}{x}\right) - 2\,a\cos\left(\frac{ax+b}{x}\right) - \frac{2\,(ax+b)a\sin\left(\frac{ax+b}{x}\right)}{x} + \frac{2\,(ax+b)\cos\left(\frac{ax+b}{x}\right)}{x} + \frac{(ax+b)^2\sin\left(\frac{ax+b}{x}\right)}{x^2} - 2\,\sin\left(\frac{ax+b}{x}\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x^4,x, algorithm="giac")
```

```
[Out] -(a^2*sin((a*x + b)/x) - 2*a*cos((a*x + b)/x) - 2*(a*x + b)*a*sin((a*x + b)
/x)/x + 2*(a*x + b)*cos((a*x + b)/x)/x + (a*x + b)^2*sin((a*x + b)/x)/x^2 -
 2*sin((a*x + b)/x))/b^3
```

maple [A] time = 0.04, size = 92, normalized size = 2.00

$$-\frac{\left(a+\frac{b}{x}\right)^{2}\sin\left(a+\frac{b}{x}\right)-2\sin\left(a+\frac{b}{x}\right)+2\cos\left(a+\frac{b}{x}\right)\left(a+\frac{b}{x}\right)-2a\left(\cos\left(a+\frac{b}{x}\right)+\left(a+\frac{b}{x}\right)\sin\left(a+\frac{b}{x}\right)\right)+a^{2}\sin}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b/x)/x^4,x)
```

```
[Out] -1/b^3*((a+b/x)^2*sin(a+b/x)-2*sin(a+b/x)+2*cos(a+b/x)*(a+b/x)-2*a*(cos(a+b
/x)+(a+b/x)*sin(a+b/x))+a^2*sin(a+b/x))
```

maxima [C] time = 1.23, size = 50, normalized size = 1.09

$$\frac{\left(i\,\Gamma\left(3,\frac{i\,b}{x}\right)-i\,\Gamma\left(3,-\frac{i\,b}{x}\right)\right)\cos\left(a\right)+\left(\Gamma\left(3,\frac{i\,b}{x}\right)+\Gamma\left(3,-\frac{i\,b}{x}\right)\right)\sin\left(a\right)}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*((I*gamma(3, I*b/x) - I*gamma(3, -I*b/x))*cos(a) + (gamma(3, I*b/x) + g
amma(3, -I*b/x))*sin(a))/b^3
```

mupad [B] time = 0.37, size = 47, normalized size = 1.02

$$\frac{2\,\sin\left(a+\frac{b}{x}\right)}{b^{3}}-\frac{b^{2}\,\sin\left(a+\frac{b}{x}\right)+2\,b\,x\,\cos\left(a+\frac{b}{x}\right)}{b^{3}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b/x)/x^4,x)
```

```
[Out] (2*sin(a + b/x))/b^3 - (b^2*sin(a + b/x) + 2*b*x*cos(a + b/x))/(b^3*x^2)
```

sympy [A] time = 3.04, size = 46, normalized size = 1.00

$$\begin{cases} -\frac{\sin\left(a+\frac{b}{x}\right)}{bx^{2}}-\frac{2\cos\left(a+\frac{b}{x}\right)}{b^{2}x}+\frac{2\sin\left(a+\frac{b}{x}\right)}{b^{3}} & \text{for}\ b\neq 0\\ -\frac{\cos\left(a\right)}{3x^{3}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x)/x**4,x)
```

```
[Out] Piecewise((-sin(a + b/x)/(b*x**2) - 2*cos(a + b/x)/(b**2*x) + 2*sin(a + b/x
)/b**3, Ne(b, 0)), (-cos(a)/(3*x**3), True))
```

3.40 $\int \cos\left(a+\frac{b}{x^2}\right)\,dx$

Optimal. Leaf size=79

$$\sqrt{2\pi}\sqrt{b}\sin(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+\sqrt{2\pi}\sqrt{b}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+x\cos\left(a+\frac{b}{x^2}\right)$$

```
[Out] x*cos(a+b/x^2)+cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*b^(1/2)*2^(1/2)*
Pi^(1/2)+FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*sin(a)*b^(1/2)*2^(1/2)*Pi^(1/
2)
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3360, 3388, 3353, 3352, 3351}

$$\sqrt{2\pi}\sqrt{b}\sin(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{x}\right)+\sqrt{2\pi}\sqrt{b}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+x\cos\left(a+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b/x^2],x]
```

```
[Out] x*Cos[a + b/x^2] + Sqrt[b]*Sqrt[2*Pi]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/
x] + Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3360

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> -Dist[f^(-1), Subst[Int[(a + b*Cos[c + d/x^n])^p/x^2, x], x, 1/(e
 + f*x)], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && ILtQ[n, 0] &&
 EqQ[n, -2]
```

Rule 3388

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_), x_Symbol] :> Simp[((e*x
)^(m + 1)*Cos[c + d*x^n])/(e*(m + 1)), x] + Dist[(d*n)/(e^n*(m + 1)), Int[(
e*x)^(m + n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos\left(a+\frac{b}{x^2}\right)dx &= -\operatorname{Subst}\left(\int \frac{\cos\left(a+bx^2\right)}{x^2}dx, x, \frac{1}{x}\right)\\ &= x\cos\left(a+\frac{b}{x^2}\right)+(2b)\operatorname{Subst}\left(\int \sin\left(a+bx^2\right)dx, x, \frac{1}{x}\right)\\ &= x\cos\left(a+\frac{b}{x^2}\right)+(2b\cos(a))\operatorname{Subst}\left(\int \sin\left(bx^2\right)dx, x, \frac{1}{x}\right)+(2b\sin(a))\operatorname{Subst}\left(\int \cos\left(bx^2\right.\right.\\ &= x\cos\left(a+\frac{b}{x^2}\right)+\sqrt{b}\sqrt{2\pi}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+\sqrt{b}\sqrt{2\pi}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)\end{aligned}$$

Mathematica [A] time = 0.13, size = 80, normalized size = 1.01

$$\sqrt{2\pi}\sqrt{b}\left(\sin(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)+\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\right)-x\sin(a)\sin\left(\frac{b}{x^2}\right)+x\cos(a)\cos\left(\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b/x^2],x]
```

```
[Out] x*Cos[a]*Cos[b/x^2] + Sqrt[b]*Sqrt[2*Pi]*(Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/P
i])/x] + FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a]) - x*Sin[a]*Sin[b/x^2]
```

fricas [A] time = 0.80, size = 73, normalized size = 0.92

$$\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)+\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)\sin(a)+x\cos\left(\frac{ax^2+b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x^2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x) + sqrt(2)*pi
*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x)*sin(a) + x*cos((a*x^2 + b)/x^
2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(a + \frac{b}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate(cos(a + b/x^2), x)
```

maple [A] time = 0.02, size = 57, normalized size = 0.72

$$x\cos\left(a + \frac{b}{x^2}\right) + \sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right) + \sin(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b/x^2),x)
```

```
[Out] x*cos(a+b/x^2)+b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi
^(1/2)/x)+sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

maxima [C] time = 2.37, size = 127, normalized size = 1.61

$$\frac{\sqrt{2}\left(2\,\sqrt{2}\,bx^2\sqrt{\frac{1}{x^4}}\,\cos\left(\frac{ax^2+b}{x^2}\right) + \left(\left((i+1)\,\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{\frac{ib}{x^2}}\right)-1\right) - (i-1)\,\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{-\frac{ib}{x^2}}\right)-1\right)\right)\cos(a) + \left(-(i-\right.\right.\right.}{4\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(2*sqrt(2)*b*x^2*sqrt(x^(-4))*cos((a*x^2 + b)/x^2) + (((I + 1)*
sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) - (I - 1)*sqrt(pi)*(erf(sqrt(-I*b/x^2)) -
 1))*cos(a) + (-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) + (I + 1)*sqrt(pi
)*(erf(sqrt(-I*b/x^2)) - 1))*sin(a))*b*(b^2/x^4)^(1/4))*sqrt(x^4)/(b*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+\frac{b}{x^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b/x^2),x)
[Out] int(cos(a + b/x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(a+\frac{b}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x**2),x)
[Out] Integral(cos(a + b/x**2), x)
```

3.41 $\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\sin(a)\mathrm{Si}\left(\frac{b}{x^2}\right)-\frac{1}{2}\cos(a)\mathrm{Ci}\left(\frac{b}{x^2}\right)$$

```
[Out] -1/2*Ci(b/x^2)*cos(a)+1/2*Si(b/x^2)*sin(a)
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3378, 3376, 3375}

$$\frac{1}{2}\sin(a)\mathrm{Si}\left(\frac{b}{x^2}\right)-\frac{1}{2}\cos(a)\mathrm{CosIntegral}\left(\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b/x^2]/x,x]
```

```
[Out] -(Cos[a]*CosIntegral[b/x^2])/2 + (Sin[a]*SinIntegral[b/x^2])/2
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x}\,dx = \cos(a)\int \frac{\cos\left(\frac{b}{x^2}\right)}{x}\,dx - \sin(a)\int \frac{\sin\left(\frac{b}{x^2}\right)}{x}\,dx$$
$$= -\frac{1}{2}\cos(a)\mathrm{Ci}\left(\frac{b}{x^2}\right) + \frac{1}{2}\sin(a)\mathrm{Si}\left(\frac{b}{x^2}\right)$$

Mathematica [**A**] time = 0.04, size = 24, normalized size = 0.96

$$\frac{1}{2}\left(\sin(a)\mathrm{Si}\left(\frac{b}{x^2}\right) - \cos(a)\mathrm{Ci}\left(\frac{b}{x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b/x^2]/x,x]
```

```
[Out] (-(Cos[a]*CosIntegral[b/x^2]) + Sin[a]*SinIntegral[b/x^2])/2
```

fricas [**A**] time = 0.57, size = 29, normalized size = 1.16

$$-\frac{1}{4}\left(\mathrm{Ci}\left(\frac{b}{x^2}\right) + \mathrm{Ci}\left(-\frac{b}{x^2}\right)\right)\cos(a) + \frac{1}{2}\,\sin(a)\,\mathrm{Si}\left(\frac{b}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x^2)/x,x, algorithm="fricas")
```

```
[Out] -1/4*(cos_integral(b/x^2) + cos_integral(-b/x^2))*cos(a) + 1/2*sin(a)*sin_i
ntegral(b/x^2)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x^2)/x,x, algorithm="giac")
```

```
[Out] integrate(cos(a + b/x^2)/x, x)
```

maple [**A**] time = 0.02, size = 22, normalized size = 0.88

$$-\frac{\mathrm{Ci}\left(\frac{b}{x^2}\right)\cos(a)}{2} + \frac{\mathrm{Si}\left(\frac{b}{x^2}\right)\sin(a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b/x^2)/x,x)
```

```
[Out] -1/2*Ci(b/x^2)*cos(a)+1/2*Si(b/x^2)*sin(a)
```

maxima [C] time = 1.34, size = 43, normalized size = 1.72

$$-\frac{1}{4}\left(\mathrm{Ei}\left(\frac{i\,b}{x^2}\right)+\mathrm{Ei}\left(-\frac{i\,b}{x^2}\right)\right)\cos\left(a\right)-\frac{1}{4}\left(i\,\mathrm{Ei}\left(\frac{i\,b}{x^2}\right)-i\,\mathrm{Ei}\left(-\frac{i\,b}{x^2}\right)\right)\sin\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x^2)/x,x, algorithm="maxima")
```

```
[Out] -1/4*(Ei(I*b/x^2) + Ei(-I*b/x^2))*cos(a) - 1/4*(I*Ei(I*b/x^2) - I*Ei(-I*b/x
^2))*sin(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\frac{\sin\left(a\right)\,\mathrm{sinint}\left(\frac{b}{x^2}\right)}{2}-\frac{\cos\left(a\right)\,\mathrm{cosint}\left(\frac{b}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b/x^2)/x,x)
```

```
[Out] (sin(a)*sinint(b/x^2))/2 - (cos(a)*cosint(b/x^2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x**2)/x,x)
```

```
[Out] Integral(cos(a + b/x**2)/x, x)
```

3.42 $\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$

Optimal. Leaf size=74

$$\frac{\sqrt{\frac{\pi}{2}}\sin(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}$$

```
[Out] -1/2*cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*2^(1/2)*Pi^(1/2)/b^(1/2)+1
/2*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*sin(a)*2^(1/2)*Pi^(1/2)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3384, 3354, 3352, 3351}

$$\frac{\sqrt{\frac{\pi}{2}}\sin(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}-\frac{\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b/x^2]/x^2,x]
```

```
[Out] -((Sqrt[Pi/2]*Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x])/Sqrt[b]) + (Sqrt[Pi/
2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/Sqrt[b]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3354

```
Int[Cos[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Cos[c], Int
[Cos[d*(e + f*x)^2], x], x] - Dist[Sin[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3384

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_)]*(x_)^(m_.), x_Symbol] :> Dist[2/n, Subst[I
nt[Cos[a + b*x^2], x], x, x^(n/2)], x] /; FreeQ[{a, b, m, n}, x] && EqQ[m,
n/2 - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^2}\,dx &= -\text{Subst}\left(\int \cos\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= -\left(\cos(a)\,\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \frac{1}{x}\right)\right)+\sin(a)\,\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\sqrt{\frac{\pi}{2}}\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{\sqrt{b}}+\frac{\sqrt{\frac{\pi}{2}}\,S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 62, normalized size = 0.84

$$-\frac{\sqrt{\frac{\pi}{2}}\left(\cos(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)-\sin(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b/x^2]/x^2,x]
```

```
[Out] -((Sqrt[Pi/2]*(Cos[a]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x] - FresnelS[(Sqrt[b]*
Sqrt[2/Pi])/x]*Sin[a]))/Sqrt[b])
```

fricas [A] time = 0.69, size = 65, normalized size = 0.88

$$-\frac{\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\cos(a)\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)-\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{\frac{b}{\pi}}}{x}\right)\sin(a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x^2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x) - sqrt
(2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x)*sin(a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a + \frac{b}{x^2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(a + b/x^2)/x^2, x)
```

maple [A] time = 0.02, size = 48, normalized size = 0.65

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right) - \sin(a)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b/x^2)/x^2,x)
```

```
[Out] -1/2*2^(1/2)*Pi^(1/2)/b^(1/2)*(cos(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)-
sin(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

maxima [C] time = 1.24, size = 98, normalized size = 1.32

$$-\frac{\sqrt{2}\,\sqrt{x^4}\left(\left(-(i-1)\,\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{\frac{ib}{x^2}}\right)-1\right)+(i+1)\,\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{-\frac{ib}{x^2}}\right)-1\right)\right)\cos(a)+\left(-(i+1)\,\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{\frac{ib}{x^2}}\right)-1\right.\right.\right.}{8\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x^2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*sqrt(x^4)*((-(I - 1)*sqrt(pi)*(erf(sqrt(I*b/x^2)) - 1) + (I +
1)*sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*cos(a) + (-(I + 1)*sqrt(pi)*(erf(sqr
t(I*b/x^2)) - 1) + (I - 1)*sqrt(pi)*(erf(sqrt(-I*b/x^2)) - 1))*sin(a))*(b^2
/x^4)^(1/4)/(b*x)
```

mupad [B] time = 0.41, size = 55, normalized size = 0.74

$$\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{b}}{x\,\sqrt{\pi}}\right)\,\sin(a)}{2\,\sqrt{b}} - \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{b}}{x\,\sqrt{\pi}}\right)\,\cos(a)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b/x^2)/x^2,x)
```

```
[Out] (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*b^(1/2))/(x*pi^(1/2)))*sin(a))/(2*b^(1/
2)) - (2^(1/2)*pi^(1/2)*fresnelc((2^(1/2)*b^(1/2))/(x*pi^(1/2)))*cos(a))/(2
*b^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x**2)/x**2,x)
```

```
[Out] Integral(cos(a + b/x**2)/x**2, x)
```

3.43 $\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^3}\,dx$

Optimal. Leaf size=15

$$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b}$$

```
[Out] -1/2*sin(a+b/x^2)/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3380, 2637}

$$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b/x^2]/x^3,x]
```

```
[Out] -Sin[a + b/x^2]/(2*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^3}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \cos(a+bx)\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b/x^2]/x^3,x]
```

```
[Out] -1/2*Sin[a + b/x^2]/b
```

fricas [A] time = 0.99, size = 17, normalized size = 1.13

$$-\frac{\sin\left(\frac{ax^2+b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*sin((a*x^2 + b)/x^2)/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x^2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(cos(a + b/x^2)/x^3, x)
```

maple [A] time = 0.02, size = 14, normalized size = 0.93

$$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b/x^2)/x^3,x)
```

```
[Out] -1/2*sin(a+b/x^2)/b
```

maxima [A]   time = 0.66, size = 13, normalized size = 0.87

$$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x^2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*sin(a + b/x^2)/b
```

mupad [B]   time = 0.27, size = 13, normalized size = 0.87

$$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b/x^2)/x^3,x)
```

```
[Out] -sin(a + b/x^2)/(2*b)
```

sympy [A]   time = 2.82, size = 22, normalized size = 1.47

$$\begin{cases} -\frac{\sin\left(a+\frac{b}{x^2}\right)}{2b} & \text{for } b \neq 0 \\ -\frac{\cos(a)}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x**2)/x**3,x)
```

```
[Out] Piecewise((-sin(a + b/x**2)/(2*b), Ne(b, 0)), (-cos(a)/(2*x**2), True))
```

3.44 $\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^4} dx$

Optimal. Leaf size=97

$$\frac{\sqrt{\frac{\pi}{2}}\sin(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}}-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2bx}$$

```
[Out] -1/2*sin(a+b/x^2)/b/x+1/4*cos(a)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*2^(1/
2)*Pi^(1/2)/b^(3/2)+1/4*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x)*sin(a)*2^(1/2)
*Pi^(1/2)/b^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3410, 3386, 3353, 3352, 3351}

$$\frac{\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}}{x}\right)}{2b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}}-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2bx}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b/x^2]/x^4,x]
```

```
[Out] (Sqrt[Pi/2]*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x])/(2*b^(3/2)) + (Sqrt[Pi
/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a])/(2*b^(3/2)) - Sin[a + b/x^2]/(
2*b*x)
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3353

```
Int[Sin[(c_) + (d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Dist[Sin[c], Int
[Cos[d*(e + f*x)^2], x], x] + Dist[Cos[c], Int[Sin[d*(e + f*x)^2], x], x] /
; FreeQ[{c, d, e, f}, x]
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3410

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> -Subst[Int[(a + b*Cos[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a
, b, c, d}, x] && IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m] && EqQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^4}\,dx &= -\mathrm{Subst}\left(\int x^2\cos\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\sin\left(a+\frac{b}{x^2}\right)}{2bx} + \frac{\mathrm{Subst}\left(\int \sin\left(a+bx^2\right)\,dx, x, \frac{1}{x}\right)}{2b}\\
&= -\frac{\sin\left(a+\frac{b}{x^2}\right)}{2bx} + \frac{\cos(a)\,\mathrm{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \frac{1}{x}\right)}{2b} + \frac{\sin(a)\,\mathrm{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \frac{1}{x}\right)}{2b}\\
&= \frac{\sqrt{\frac{\pi}{2}}\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)}{2b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}\,C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right)\sin(a)}{2b^{3/2}} - \frac{\sin\left(a+\frac{b}{x^2}\right)}{2bx}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 88, normalized size = 0.91

$$\frac{\sqrt{2\pi}\,x\sin(a)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right) + \sqrt{2\pi}\,x\cos(a)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}}{x}\right) - 2\sqrt{b}\,\sin\left(a+\frac{b}{x^2}\right)}{4b^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b/x^2]/x^4,x]
```

```
[Out] (Sqrt[2*Pi]*x*Cos[a]*FresnelS[(Sqrt[b]*Sqrt[2/Pi])/x] + Sqrt[2*Pi]*x*Fresne
lC[(Sqrt[b]*Sqrt[2/Pi])/x]*Sin[a] - 2*Sqrt[b]*Sin[a + b/x^2])/(4*b^(3/2)*x)
```

fricas [A] time = 0.68, size = 84, normalized size = 0.87

$$\frac{\sqrt{2}\,\pi x\sqrt{\frac{b}{\pi}}\,\cos(a)\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{\frac{b}{\pi}}}{x}\right)+\sqrt{2}\,\pi x\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{\frac{b}{\pi}}}{x}\right)\sin(a)-2\,b\sin\left(\frac{ax^2+b}{x^2}\right)}{4\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x^2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*x*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*sqrt(b/pi)/x) + sqr
t(2)*pi*x*sqrt(b/pi)*fresnel_cos(sqrt(2)*sqrt(b/pi)/x)*sin(a) - 2*b*sin((a*
x^2 + b)/x^2))/(b^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(cos(a + b/x^2)/x^4, x)
```

maple [A] time = 0.02, size = 64, normalized size = 0.66

$$-\frac{\sin\left(a+\frac{b}{x^2}\right)}{2bx}+\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\mathrm{S}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)+\sin(a)\,\mathrm{FresnelC}\left(\frac{\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}\,x}\right)\right)}{4b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b/x^2)/x^4,x)
```

```
[Out] -1/2*sin(a+b/x^2)/b/x+1/4/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(b^(1/2)
*2^(1/2)/Pi^(1/2)/x)+sin(a)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)/x))
```

maxima [C] time = 1.29, size = 74, normalized size = 0.76

$$\frac{\sqrt{2}\left(x^4\right)^{\frac{3}{2}}\left(\left(-(i+1)\,\Gamma\left(\frac{3}{2},\frac{ib}{x^2}\right)+(i-1)\,\Gamma\left(\frac{3}{2},-\frac{ib}{x^2}\right)\right)\cos(a)+\left((i-1)\,\Gamma\left(\frac{3}{2},\frac{ib}{x^2}\right)-(i+1)\,\Gamma\left(\frac{3}{2},-\frac{ib}{x^2}\right)\right)\sin(a)\right)\left(\frac{b^2}{x^4}\right)^{\frac{3}{4}}}{8\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x^2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*((-(I + 1)*gamma(3/2, I*b/x^2) + (I - 1)*gamma(3/2, -I*b/x^2))*
cos(a) + ((I - 1)*gamma(3/2, I*b/x^2) - (I + 1)*gamma(3/2, -I*b/x^2))*sin(a
))*(x^4)^(3/2)*(b^2/x^4)^(3/4)/(b^3*x^3)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b/x^2)/x^4,x)
```

```
[Out] int(cos(a + b/x^2)/x^4, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+\frac{b}{x^2}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b/x**2)/x**4,x)
```

```
[Out] Integral(cos(a + b/x**2)/x**4, x)
```

3.45 $\int \frac{\cos^2(\sqrt{x})}{\sqrt{x}} \, dx$

Optimal. Leaf size=19

$$\sqrt{x} + \sin(\sqrt{x})\cos(\sqrt{x})$$

```
[Out] cos(x^(1/2))*sin(x^(1/2))+x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3380, 2635, 8}

$$\sqrt{x} + \sin(\sqrt{x})\cos(\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] Sqrt[x] + Cos[Sqrt[x]]*Sin[Sqrt[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \cos^2(x)\,dx, x, \sqrt{x}\right)$$
$$= \cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right) + \text{Subst}\left(\int 1\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x} + \cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.03, size = 18, normalized size = 0.95

$$\sqrt{x} + \frac{1}{2}\sin\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]]^2/Sqrt[x],x]
[Out] Sqrt[x] + Sin[2*Sqrt[x]]/2
```

fricas [A] time = 0.53, size = 13, normalized size = 0.68

$$\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right) + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="fricas")
[Out] cos(sqrt(x))*sin(sqrt(x)) + sqrt(x)
```

giac [A] time = 0.38, size = 12, normalized size = 0.63

$$\sqrt{x} + \frac{1}{2}\sin\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="giac")
[Out] sqrt(x) + 1/2*sin(2*sqrt(x))
```

maple [A] time = 0.04, size = 14, normalized size = 0.74

$$\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right) + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))^2/x^(1/2),x)
```

```
[Out] cos(x^(1/2))*sin(x^(1/2))+x^(1/2)
```

maxima [A] time = 0.30, size = 12, normalized size = 0.63

$$\sqrt{x} + \frac{1}{2} \sin\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x) + 1/2*sin(2*sqrt(x))
```

mupad [B] time = 0.37, size = 12, normalized size = 0.63

$$\frac{\sin\left(2\sqrt{x}\right)}{2} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))^2/x^(1/2),x)
```

```
[Out] sin(2*x^(1/2))/2 + x^(1/2)
```

sympy [B] time = 0.26, size = 39, normalized size = 2.05

$$\sqrt{x}\sin^2\left(\sqrt{x}\right) + \sqrt{x}\cos^2\left(\sqrt{x}\right) + \sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2))**2/x**(1/2),x)
```

```
[Out] sqrt(x)*sin(sqrt(x))**2 + sqrt(x)*cos(sqrt(x))**2 + sin(sqrt(x))*cos(sqrt(x
))
```

3.46 $\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$2\sin\left(\sqrt{x}\right)$$

```
[Out] 2*sin(x^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3380, 2637}

$$2\sin\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sin[Sqrt[x]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt{x}\right)\\ &= 2\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$2\sin\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sin[Sqrt[x]]
```

fricas [A] time = 0.78, size = 6, normalized size = 0.75

$$2 \sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sin(sqrt(x))
```

giac [A] time = 0.38, size = 6, normalized size = 0.75

$$2 \sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sin(sqrt(x))
```

maple [A] time = 0.02, size = 7, normalized size = 0.88

$$2 \sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))/x^(1/2),x)
```

```
[Out] 2*sin(x^(1/2))
```

maxima [A] time = 0.90, size = 6, normalized size = 0.75

$$2 \sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sin(sqrt(x))
```

mupad [B] time = 0.03, size = 6, normalized size = 0.75

$$2 \sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))/x^(1/2),x)
```

```
[Out] 2*sin(x^(1/2))
```

sympy [**A**] time = 0.25, size = 7, normalized size = 0.88

$$2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*sin(sqrt(x))
```

3.47 $\int \cos\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$2\sqrt{x}\,\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3362, 3296, 2638}

$$2\sqrt{x}\,\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3362

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\int \cos\left(\sqrt{x}\right)\,dx = 2\,\text{Subst}\left(\int x\cos(x)\,dx, x, \sqrt{x}\right)$$
$$= 2\sqrt{x}\,\sin\left(\sqrt{x}\right) - 2\,\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{x}\right)$$
$$= 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\,\sin\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$2\sqrt{x}\,\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]],x]
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

fricas [A] time = 0.62, size = 16, normalized size = 0.73

$$2\,\sqrt{x}\,\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

giac [A] time = 0.49, size = 16, normalized size = 0.73

$$2\,\sqrt{x}\,\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

maple [A] time = 0.03, size = 17, normalized size = 0.77

$$2\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x)
```

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

maxima [A] time = 0.85, size = 16, normalized size = 0.73

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

mupad [B] time = 0.28, size = 16, normalized size = 0.73

$$2\cos\left(\sqrt{x}\right)+2\sqrt{x}\,\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x)
[Out] 2*cos(x^(1/2)) + 2*x^(1/2)*sin(x^(1/2))
```

sympy [A] time = 0.24, size = 20, normalized size = 0.91

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2)),x)
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.48 $\int \cos^2\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=36

$$\frac{x}{2}+\frac{1}{2}\cos^2\left(\sqrt{x}\right)+\sqrt{x}\,\sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$$

```
[Out] 1/2*x+1/2*cos(x^(1/2))^2+cos(x^(1/2))*sin(x^(1/2))*x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3362, 3310, 30}

$$\frac{x}{2}+\frac{1}{2}\cos^2\left(\sqrt{x}\right)+\sqrt{x}\,\sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[x]]^2,x]
```

```
[Out] x/2 + Cos[Sqrt[x]]^2/2 + Sqrt[x]*Cos[Sqrt[x]]*Sin[Sqrt[x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3362

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\begin{aligned}\int \cos^2\left(\sqrt{x}\right)\,dx &= 2\,\text{Subst}\left(\int x\cos^2(x)\,dx, x, \sqrt{x}\right)\\ &= \frac{1}{2}\cos^2\left(\sqrt{x}\right)+\sqrt{x}\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)+\text{Subst}\left(\int x\,dx, x, \sqrt{x}\right)\\ &= \frac{x}{2}+\frac{1}{2}\cos^2\left(\sqrt{x}\right)+\sqrt{x}\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.86

$$\frac{1}{4}\left(2\left(x+\sqrt{x}\sin\left(2\sqrt{x}\right)\right)+\cos\left(2\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]]^2,x]
[Out] (Cos[2*Sqrt[x]] + 2*(x + Sqrt[x]*Sin[2*Sqrt[x]]))/4
```

fricas [A] time = 1.26, size = 24, normalized size = 0.67

$$\sqrt{x}\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)+\frac{1}{2}\cos\left(\sqrt{x}\right)^2+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))^2,x, algorithm="fricas")
[Out] sqrt(x)*cos(sqrt(x))*sin(sqrt(x)) + 1/2*cos(sqrt(x))^2 + 1/2*x
```

giac [A] time = 0.41, size = 23, normalized size = 0.64

$$\frac{1}{2}\sqrt{x}\sin\left(2\sqrt{x}\right)+\frac{1}{2}x+\frac{1}{4}\cos\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))^2,x, algorithm="giac")
[Out] 1/2*sqrt(x)*sin(2*sqrt(x)) + 1/2*x + 1/4*cos(2*sqrt(x))
```

maple [A] time = 0.02, size = 34, normalized size = 0.94

$$2\sqrt{x}\left(\frac{\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)}{2}+\frac{\sqrt{x}}{2}\right)-\frac{x}{2}-\frac{\left(\sin^2\left(\sqrt{x}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))^2,x)
```

```
[Out] 2*x^(1/2)*(1/2*cos(x^(1/2))*sin(x^(1/2))+1/2*x^(1/2))-1/2*x-1/2*sin(x^(1/2)
)^2
```

maxima [A] time = 0.66, size = 23, normalized size = 0.64

$$\frac{1}{2}\sqrt{x}\sin\left(2\sqrt{x}\right)+\frac{1}{2}x+\frac{1}{4}\cos\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x)*sin(2*sqrt(x)) + 1/2*x + 1/4*cos(2*sqrt(x))
```

mupad [B] time = 0.35, size = 23, normalized size = 0.64

$$\frac{x}{2}-\frac{\sin\left(\sqrt{x}\right)^2}{2}+\frac{\sqrt{x}\ \sin\left(2\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))^2,x)
```

```
[Out] x/2 - sin(x^(1/2))^2/2 + (x^(1/2)*sin(2*x^(1/2)))/2
```

sympy [A] time = 0.25, size = 51, normalized size = 1.42

$$\sqrt{x}\sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)+\frac{x\sin^2\left(\sqrt{x}\right)}{2}+\frac{x\cos^2\left(\sqrt{x}\right)}{2}-\frac{\sin^2\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2))**2,x)
```

```
[Out] sqrt(x)*sin(sqrt(x))*cos(sqrt(x)) + x*sin(sqrt(x))**2/2 + x*cos(sqrt(x))**2
/2 - sin(sqrt(x))**2/2
```

3.49 $\int x^{3/2} \cos\left(a + b\sqrt[3]{x}\right) dx$

Optimal. Leaf size=235

$$\frac{405405\sqrt{\frac{\pi}{2}}\sin(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{64b^{15/2}}+\frac{405405\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{64b^{15/2}}-\frac{405405\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)}{64b^7}+\frac{135135\sqrt{x}}{3}$$

```
[Out] -3861/8*x^(7/6)*cos(a+b*x^(1/3))/b^4+39/2*x^(11/6)*cos(a+b*x^(1/3))/b^2-405
405/64*x^(1/6)*sin(a+b*x^(1/3))/b^7+27027/16*x^(5/6)*sin(a+b*x^(1/3))/b^5-4
29/4*x^(3/2)*sin(a+b*x^(1/3))/b^3+3*x^(13/6)*sin(a+b*x^(1/3))/b+405405/128*
cos(a)*FresnelS(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^(15/2)
+405405/128*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1
/2)/b^(15/2)+135135/32*cos(a+b*x^(1/3))*x^(1/2)/b^6
```

Rubi [A] time = 0.35, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3416, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{405405\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[6]{x}\right)}{64b^{15/2}}+\frac{405405\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{64b^{15/2}}-\frac{429x^{3/2}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3}+\frac{27027x}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Cos[a + b*x^(1/3)],x]
```

```
[Out] (135135*Sqrt[x]*Cos[a + b*x^(1/3)])/(32*b^6) - (3861*x^(7/6)*Cos[a + b*x^(1
/3)])/(8*b^4) + (39*x^(11/6)*Cos[a + b*x^(1/3)])/(2*b^2) + (405405*Sqrt[Pi/
2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)])/(64*b^(15/2)) + (405405*Sqr
t[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a])/(64*b^(15/2)) - (40540
5*x^(1/6)*Sin[a + b*x^(1/3)])/(64*b^7) + (27027*x^(5/6)*Sin[a + b*x^(1/3)])
/(16*b^5) - (429*x^(3/2)*Sin[a + b*x^(1/3)])/(4*b^3) + (3*x^(13/6)*Sin[a +
b*x^(1/3)])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3416

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\int x^{3/2}\cos\left(a+b\sqrt[3]{x}\right)\,dx = 3\,\mathrm{Subst}\left(\int x^{13/2}\cos(a+bx)\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{39\,\mathrm{Subst}\left(\int x^{11/2}\sin(a+bx)\,dx, x, \sqrt[3]{x}\right)}{2b}$$
$$= \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} + \frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{429\,\mathrm{Subst}\left(\int x^{9/2}\cos(a+bx)\,dx, x\right.}{4b^2}$$
$$= \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{429x^{3/2}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{3x^{13/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} + \frac{3861\,\mathrm{Su}}{}$$
$$= -\frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{429x^{3/2}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{3x^{1}}{}$$
$$= -\frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} + \frac{27027x^{5/6}\sin\left(a+b\sqrt[3]{x}\right)}{16b^5} - \frac{4}{}$$
$$= \frac{135135\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)}{32b^6} - \frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} + \frac{2}{}$$
$$= \frac{135135\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)}{32b^6} - \frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{4}{}$$
$$= \frac{135135\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)}{32b^6} - \frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{4}{}$$
$$= \frac{135135\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)}{32b^6} - \frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{4}{}$$
$$= \frac{135135\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)}{32b^6} - \frac{3861x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{39x^{11/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} + \frac{4}{}$$

Mathematica [A] time = 0.58, size = 165, normalized size = 0.70

$$\frac{6\sqrt{b}\,\sqrt[6]{x}\left(26\left(16b^5x^{5/3} - 396b^3x + 3465b\sqrt[3]{x}\right)\cos\left(a+b\sqrt[3]{x}\right) + \left(64b^6x^2 - 2288b^4x^{4/3} + 36036b^2x^{2/3} - 135135\right)\mathrm{s}}{128b^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Cos[a + b*x^(1/3)],x]
```

```
[Out] (405405*Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)] + 405405*Sqr
t[2*Pi]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a] + 6*Sqrt[b]*x^(1/6)*(26
*(3465*b*x^(1/3) - 396*b^3*x + 16*b^5*x^(5/3))*Cos[a + b*x^(1/3)] + (-13513
```

```
5 + 36036*b^2*x^(2/3) - 2288*b^4*x^(4/3) + 64*b^6*x^2)*Sin[a + b*x^(1/3)]))
/(128*b^(15/2))
```

fricas [A] time = 0.60, size = 145, normalized size = 0.62

$$\frac{3\left(135135\,\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\cos(a)\,\mathrm{S}\left(\sqrt{2}\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)+135135\,\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin(a)+52\left(16\,b^{6}x^{\frac{11}{6}}-396\,b^{4}x^{\frac{7}{6}}+3\cdots\right.\right.}{128\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(a+b*x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/128*(135135*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x^(1/6)*sqrt
(b/pi)) + 135135*sqrt(2)*pi*sqrt(b/pi)*fresnel_cos(sqrt(2)*x^(1/6)*sqrt(b/p
i))*sin(a) + 52*(16*b^6*x^(11/6) - 396*b^4*x^(7/6) + 3465*b^2*sqrt(x))*cos(
b*x^(1/3) + a) - 2*(2288*b^5*x^(3/2) - 36036*b^3*x^(5/6) - (64*b^7*x^2 - 13
5135*b)*x^(1/6))*sin(b*x^(1/3) + a))/b^8
```

giac [C] time = 2.21, size = 241, normalized size = 1.03

$$-\frac{3\left(64i\,b^{6}x^{\frac{13}{6}}-416\,b^{5}x^{\frac{11}{6}}-2288i\,b^{4}x^{\frac{3}{2}}+10296\,b^{3}x^{\frac{7}{6}}+36036i\,b^{2}x^{\frac{5}{6}}-90090\,b\sqrt{x}-135135i\,x^{\frac{1}{6}}\right)e^{\left(i\,bx^{\frac{1}{3}}+i\,a\right)}}{128\,b^{7}}-\frac{3\left(-6\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(a+b*x^(1/3)),x, algorithm="giac")
```

```
[Out] -3/128*(64*I*b^6*x^(13/6) - 416*b^5*x^(11/6) - 2288*I*b^4*x^(3/2) + 10296*b
^3*x^(7/6) + 36036*I*b^2*x^(5/6) - 90090*b*sqrt(x) - 135135*I*x^(1/6))*e^(I
*b*x^(1/3) + I*a)/b^7 - 3/128*(-64*I*b^6*x^(13/6) - 416*b^5*x^(11/6) + 2288
*I*b^4*x^(3/2) + 10296*b^3*x^(7/6) - 36036*I*b^2*x^(5/6) - 90090*b*sqrt(x)
+ 135135*I*x^(1/6))*e^(-I*b*x^(1/3) - I*a)/b^7 + 405405/256*I*sqrt(2)*sqrt(
pi)*erf(-1/2*sqrt(2)*x^(1/6)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b^7*(
-I*b/abs(b) + 1)*sqrt(abs(b))) - 405405/256*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqr
t(2)*x^(1/6)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b^7*(I*b/abs(b) + 1)*
sqrt(abs(b)))
```

maple [A] time = 0.04, size = 196, normalized size = 0.83

$$\frac{3x^{\frac{13}{6}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{b}-\frac{39\left(-\frac{x^{\frac{11}{6}}\cos\left(a+b\,x^{\frac{1}{3}}\right)}{2b}+\frac{\frac{11x^{\frac{3}{2}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{4b}-\frac{99\left(-\frac{x^{\frac{7}{6}}\cos\left(a+b\,x^{\frac{1}{3}}\right)}{2b}+\frac{\frac{7x^{\frac{5}{6}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{4b}-\frac{35\left(-\frac{\sqrt{x}\,\cos\left(a+b\,x^{\frac{1}{3}}\right)}{2b}+\frac{\frac{3x^{\frac{1}{6}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{4b}\cdots}{\cdots}\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{4b}}{b}\cdots\right.}{b}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*cos(a+b*x^(1/3)),x)
```

```
[Out] 3*x^(13/6)*sin(a+b*x^(1/3))/b-39/b*(-1/2/b*x^(11/6)*cos(a+b*x^(1/3))+11/2/b
*(1/2/b*x^(3/2)*sin(a+b*x^(1/3))-9/2/b*(-1/2/b*x^(7/6)*cos(a+b*x^(1/3))+7/2
/b*(1/2/b*x^(5/6)*sin(a+b*x^(1/3))-5/2/b*(-1/2/b*x^(1/2)*cos(a+b*x^(1/3))+3
/2/b*(1/2*x^(1/6)*sin(a+b*x^(1/3))/b-1/4/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*F
resnelS(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))+sin(a)*FresnelC(x^(1/6)*b^(1/2)*2
^(1/2)/Pi^(1/2))))))))
```

maxima [C] time = 0.81, size = 135, normalized size = 0.57

$$-\frac{3\left(\sqrt{2}\sqrt{\pi}\left(\left(-\left(135135i+135135\right)\cos\left(a\right)+\left(135135i-135135\right)\sin\left(a\right)\right)\operatorname{erf}\left(\sqrt{i\,b}\,x^{\frac{1}{6}}\right)+\left(\left(135135i-135135\right)\cos\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(a+b*x^(1/3)),x, algorithm="maxima")
```

```
[Out] -3/512*(sqrt(2)*sqrt(pi)*((-(135135*I + 135135)*cos(a) + (135135*I - 135135
)*sin(a))*erf(sqrt(I*b)*x^(1/6)) + ((135135*I - 135135)*cos(a) - (135135*I
+ 135135)*sin(a))*erf(sqrt(-I*b)*x^(1/6)))*b^(3/2) - 208*(16*b^7*x^(11/6) -
 396*b^5*x^(7/6) + 3465*b^3*sqrt(x))*cos(b*x^(1/3) + a) - 8*(64*b^8*x^(13/6
) - 2288*b^6*x^(3/2) + 36036*b^4*x^(5/6) - 135135*b^2*x^(1/6))*sin(b*x^(1/3
) + a))/b^9
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\cos\left(a+b\,x^{1/3}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*cos(a + b*x^(1/3)),x)
```

```
[Out] int(x^(3/2)*cos(a + b*x^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\cos\left(a+b\sqrt[3]{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*cos(a+b*x**(1/3)),x)
```

```
[Out] Integral(x**(3/2)*cos(a + b*x**(1/3)), x)
```

3.50 $\int \sqrt{x} \cos\left(a + b\sqrt[3]{x}\right) dx$

Optimal. Leaf size=169

$$\frac{315\sqrt{\frac{\pi}{2}}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{8b^{9/2}}-\frac{315\sqrt{\frac{\pi}{2}}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{8b^{9/2}}-\frac{315\sqrt[6]{x}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4}-\frac{105\sqrt{x}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3}$$

```
[Out] -315/8*x^(1/6)*cos(a+b*x^(1/3))/b^4+21/2*x^(5/6)*cos(a+b*x^(1/3))/b^2+3*x^(
7/6)*sin(a+b*x^(1/3))/b+315/16*cos(a)*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(
1/2))*2^(1/2)*Pi^(1/2)/b^(9/2)-315/16*FresnelS(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(
1/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(9/2)-105/4*sin(a+b*x^(1/3))*x^(1/2)/b^3
```

Rubi [A] time = 0.20, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3416, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{315\sqrt{\frac{\pi}{2}}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[6]{x}\right)}{8b^{9/2}}-\frac{315\sqrt{\frac{\pi}{2}}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{8b^{9/2}}+\frac{21x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2}-\frac{105\sqrt{x}\sin\left(a+\right.}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Cos[a + b*x^(1/3)],x]
```

```
[Out] (-315*x^(1/6)*Cos[a + b*x^(1/3)])/(8*b^4) + (21*x^(5/6)*Cos[a + b*x^(1/3)])
/(2*b^2) + (315*Sqrt[Pi/2]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)])/(8*
b^(9/2)) - (315*Sqrt[Pi/2]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a])/(8*
b^(9/2)) - (105*Sqrt[x]*Sin[a + b*x^(1/3)])/(4*b^3) + (3*x^(7/6)*Sin[a + b*
x^(1/3)])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3416

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\int \sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)dx = 3\,\text{Subst}\left(\int x^{7/2}\cos(a+bx)\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{3x^{7/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{21\,\text{Subst}\left(\int x^{5/2}\sin(a+bx)\,dx, x, \sqrt[3]{x}\right)}{2b}$$
$$= \frac{21x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} + \frac{3x^{7/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{105\,\text{Subst}\left(\int x^{3/2}\cos(a+bx)\,dx, x,\right.}{4b^2}$$
$$= \frac{21x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{105\sqrt{x}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{3x^{7/6}\sin\left(a+b\sqrt[3]{x}\right)}{b} + \frac{315\,\text{Subst}}{}$$
$$= -\frac{315\sqrt[6]{x}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{21x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{105\sqrt{x}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{3x^{7/6}\text{s}}{}$$
$$= -\frac{315\sqrt[6]{x}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{21x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{105\sqrt{x}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{3x^{7/6}\text{s}}{}$$
$$= -\frac{315\sqrt[6]{x}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{21x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} - \frac{105\sqrt{x}\sin\left(a+b\sqrt[3]{x}\right)}{4b^3} + \frac{3x^{7/6}\text{s}}{}$$
$$= -\frac{315\sqrt[6]{x}\cos\left(a+b\sqrt[3]{x}\right)}{8b^4} + \frac{21x^{5/6}\cos\left(a+b\sqrt[3]{x}\right)}{2b^2} + \frac{315\sqrt{\frac{\pi}{2}}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{8b^{9/2}}$$

Mathematica [A] time = 0.35, size = 141, normalized size = 0.83

$$\frac{6\sqrt{b}\sqrt[6]{x}\left(2b\sqrt[3]{x}\left(4b^2x^{2/3}-35\right)\sin\left(a+b\sqrt[3]{x}\right)+7\left(4b^2x^{2/3}-15\right)\cos\left(a+b\sqrt[3]{x}\right)\right)+315\sqrt{2\pi}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{\right.}}{16b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Cos[a + b*x^(1/3)],x]
```

```
[Out] (315*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)] - 315*Sqrt[2*Pi
]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a] + 6*Sqrt[b]*x^(1/6)*(7*(-15 +
 4*b^2*x^(2/3))*Cos[a + b*x^(1/3)] + 2*b*(-35 + 4*b^2*x^(2/3))*x^(1/3)*Sin[
a + b*x^(1/3)]))/(16*b^(9/2))
```

fricas [A] time = 0.77, size = 118, normalized size = 0.70

$$\frac{3\left(105\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\cos(a)\,C\left(\sqrt{2}x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)-105\sqrt{2}\pi\sqrt{\frac{b}{\pi}}\,S\left(\sqrt{2}x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin(a)+14\left(4b^3x^{\frac{5}{6}}-15bx^{\frac{1}{6}}\right)\cos\left(bx^{\frac{1}{3}}+\right.\right.}{16\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(a+b*x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/16*(105*sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x^(1/6)*sqrt(b/p
i)) - 105*sqrt(2)*pi*sqrt(b/pi)*fresnel_sin(sqrt(2)*x^(1/6)*sqrt(b/pi))*sin
(a) + 14*(4*b^3*x^(5/6) - 15*b*x^(1/6))*cos(b*x^(1/3) + a) + 4*(4*b^4*x^(7/
6) - 35*b^2*sqrt(x))*sin(b*x^(1/3) + a))/b^5
```

giac [C] time = 0.50, size = 193, normalized size = 1.14

$$-\frac{3\left(8i\,b^{3}x^{\frac{7}{6}}-28\,b^{2}x^{\frac{5}{6}}-70i\,b\sqrt{x}+105\,x^{\frac{1}{6}}\right)e^{\left(i\,bx^{\frac{1}{3}}+i\,a\right)}}{16\,b^{4}}-\frac{3\left(-8i\,b^{3}x^{\frac{7}{6}}-28\,b^{2}x^{\frac{5}{6}}+70i\,b\sqrt{x}+105\,x^{\frac{1}{6}}\right)e^{\left(-i\,bx^{\frac{1}{3}}-i\,a\right)}}{16\,b^{4}}-315$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(a+b*x^(1/3)),x, algorithm="giac")
```

```
[Out] -3/16*(8*I*b^3*x^(7/6) - 28*b^2*x^(5/6) - 70*I*b*sqrt(x) + 105*x^(1/6))*e^(
I*b*x^(1/3) + I*a)/b^4 - 3/16*(-8*I*b^3*x^(7/6) - 28*b^2*x^(5/6) + 70*I*b*s
qrt(x) + 105*x^(1/6))*e^(-I*b*x^(1/3) - I*a)/b^4 - 315/32*sqrt(2)*sqrt(pi)*
erf(-1/2*sqrt(2)*x^(1/6)*(-I*b/abs(b) + 1)*sqrt(abs(b)))*e^(I*a)/(b^4*(-I*b
/abs(b) + 1)*sqrt(abs(b))) - 315/32*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x^(1/
6)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b^4*(I*b/abs(b) + 1)*sqrt(abs(b
)))
```

maple [A] time = 0.03, size = 131, normalized size = 0.78

$$\frac{3x^{\frac{7}{6}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{b}-\frac{21\left(-\frac{x^{\frac{5}{6}}\cos\left(a+b\,x^{\frac{1}{3}}\right)}{2b}+\frac{\frac{5\sqrt{x}\,\sin\left(a+b\,x^{\frac{1}{3}}\right)}{4b}-\frac{15\left(-\frac{x^{\frac{1}{6}}\cos\left(a+b\,x^{\frac{1}{3}}\right)}{2b}+\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)-\sin(a)\mathrm{S}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{4b^{\frac{3}{2}}}\right)}{4b}}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*cos(a+b*x^(1/3)),x)
```

```
[Out] 3*x^(7/6)*sin(a+b*x^(1/3))/b-21/b*(-1/2/b*x^(5/6)*cos(a+b*x^(1/3))+5/2/b*(1
/2/b*x^(1/2)*sin(a+b*x^(1/3))-3/2/b*(-1/2/b*x^(1/6)*cos(a+b*x^(1/3))+1/4/b^
```

```
(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))-s
in(a)*FresnelS(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))))))
```

maxima [C] time = 1.24, size = 111, normalized size = 0.66

$$-\frac{3\left(\sqrt{2}\sqrt{\pi}\left(((105i-105)\cos(a)+(105i+105)\sin(a))\,\mathrm{erf}\left(\sqrt{i\,b}\,x^{\frac{1}{6}}\right)+(-(105i+105)\cos(a)-(105i-105)\,\mathrm{si}\right.\right.}{64\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(a+b*x^(1/3)),x, algorithm="maxima")
```

```
[Out] -3/64*(sqrt(2)*sqrt(pi)*(((105*I - 105)*cos(a) + (105*I + 105)*sin(a))*erf(
sqrt(I*b)*x^(1/6)) + (-(105*I + 105)*cos(a) - (105*I - 105)*sin(a))*erf(sqr
t(-I*b)*x^(1/6)))*b^(3/2) - 56*(4*b^4*x^(5/6) - 15*b^2*x^(1/6))*cos(b*x^(1/
3) + a) - 16*(4*b^5*x^(7/6) - 35*b^3*sqrt(x))*sin(b*x^(1/3) + a))/b^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\cos\left(a+b\,x^{1/3}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*cos(a + b*x^(1/3)),x)
```

```
[Out] int(x^(1/2)*cos(a + b*x^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*cos(a+b*x**(1/3)),x)
```

```
[Out] Integral(sqrt(x)*cos(a + b*x**(1/3)), x)
```

3.51 $\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=99

$$-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{b^{3/2}}+\frac{3\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)}{b}$$

```
[Out] 3*x^(1/6)*sin(a+b*x^(1/3))/b-3/2*cos(a)*FresnelS(x^(1/6)*b^(1/2)*2^(1/2)/Pi
^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)-3/2*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1
/2))*sin(a)*2^(1/2)*Pi^(1/2)/b^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3416, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[6]{x}\right)}{b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{b^{3/2}}+\frac{3\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^(1/3)]/Sqrt[x],x]
```

```
[Out] (-3*Sqrt[Pi/2]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)])/b^(3/2) - (3*Sq
rt[Pi/2]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a])/b^(3/2) + (3*x^(1/6)*
Sin[a + b*x^(1/3)])/b
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3416

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx &= 3\,\text{Subst}\left(\int \sqrt{x}\,\cos(a+bx)\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{3\sqrt[6]{x}\,\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{3\,\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{2b}\\
&= \frac{3\sqrt[6]{x}\,\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{(3\cos(a))\,\text{Subst}\left(\int \frac{\sin(bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{2b} - \frac{(3\sin(a))\,\text{Subst}\left(\int \frac{\cos(bx}{\sqrt{x}}\right.}{2b}\\
&= \frac{3\sqrt[6]{x}\,\sin\left(a+b\sqrt[3]{x}\right)}{b} - \frac{(3\cos(a))\,\text{Subst}\left(\int \sin\left(bx^2\right)\,dx, x, \sqrt[6]{x}\right)}{b} - \frac{(3\sin(a))\,\text{Subst}\left(\int \text{co}\right.}{b}\\
&= -\frac{3\sqrt{\frac{\pi}{2}}\,\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{b^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a)}{b^{3/2}} + \frac{3\sqrt[6]{x}\,\sin\left(a+b\sqrt[3]{x}\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 94, normalized size = 0.95

$$-\frac{3\left(\sqrt{2\pi}\,\sin(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)+\sqrt{2\pi}\,\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)-2\sqrt{b}\sqrt[6]{x}\sin\left(a+b\sqrt[3]{x}\right)\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^(1/3)]/Sqrt[x],x]
```

```
[Out] (-3*(Sqrt[2*Pi]*Cos[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)] + Sqrt[2*Pi]*Fr
esnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a] - 2*Sqrt[b]*x^(1/6)*Sin[a + b*x^(
1/3)]))/(2*b^(3/2))
```

fricas [A] time = 0.89, size = 78, normalized size = 0.79

$$-\frac{3\left(\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\cos(a)\,\mathrm{S}\left(\sqrt{2}\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)+\sqrt{2}\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin(a)-2\,bx^{\frac{1}{6}}\sin\left(bx^{\frac{1}{3}}+a\right)\right)}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))/x^(1/2),x, algorithm="fricas")
```

```
[Out] -3/2*(sqrt(2)*pi*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x^(1/6)*sqrt(b/pi))
+ sqrt(2)*pi*sqrt(b/pi)*fresnel_cos(sqrt(2)*x^(1/6)*sqrt(b/pi))*sin(a) - 2*
b*x^(1/6)*sin(b*x^(1/3) + a))/b^2
```

giac [C] time = 0.52, size = 143, normalized size = 1.44

$$-\frac{3i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\,x^{\frac{1}{6}}\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(ia)}}{4\,b\left(-\frac{ib}{|b|}+1\right)\sqrt{|b|}}+\frac{3i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{2}\,x^{\frac{1}{6}}\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}\right)e^{(-ia)}}{4\,b\left(\frac{ib}{|b|}+1\right)\sqrt{|b|}}-\frac{3i\,x^{\frac{1}{6}}e^{\left(ibx^{\frac{1}{3}}+ia\right)}}{2\,b}+\frac{3i\,x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))/x^(1/2),x, algorithm="giac")
```

```
[Out] -3/4*I*sqrt(2)*sqrt(pi)*erf(-1/2*sqrt(2)*x^(1/6)*(-I*b/abs(b) + 1)*sqrt(abs
(b)))*e^(I*a)/(b*(-I*b/abs(b) + 1)*sqrt(abs(b))) + 3/4*I*sqrt(2)*sqrt(pi)*e
rf(-1/2*sqrt(2)*x^(1/6)*(I*b/abs(b) + 1)*sqrt(abs(b)))*e^(-I*a)/(b*(I*b/abs
(b) + 1)*sqrt(abs(b))) - 3/2*I*x^(1/6)*e^(I*b*x^(1/3) + I*a)/b + 3/2*I*x^(1
/6)*e^(-I*b*x^(1/3) - I*a)/b
```

maple [A] time = 0.02, size = 64, normalized size = 0.65

$$\frac{3x^{\frac{1}{6}}\sin\left(a+b\,x^{\frac{1}{3}}\right)}{b}-\frac{3\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\mathrm{S}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(a)\,\mathrm{FresnelC}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*x^(1/3))/x^(1/2),x)
```

```
[Out] 3*x^(1/6)*sin(a+b*x^(1/3))/b-3/2/b^(3/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(
x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))+sin(a)*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/P
i^(1/2)))
```

maxima [C] time = 1.03, size = 73, normalized size = 0.74

$$\frac{3\left(\sqrt{2}\sqrt{\pi}\left(\left(-\left(i+1\right)\cos\left(a\right)+\left(i-1\right)\sin\left(a\right)\right)\operatorname{erf}\left(\sqrt{i\,b}\,x^{\frac{1}{6}}\right)+\left(\left(i-1\right)\cos\left(a\right)-\left(i+1\right)\sin\left(a\right)\right)\operatorname{erf}\left(\sqrt{-i\,b}\,x^{\frac{1}{6}}\right)\right)b^{\frac{3}{2}}+}{8\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*(sqrt(2)*sqrt(pi)*((-(I + 1)*cos(a) + (I - 1)*sin(a))*erf(sqrt(I*b)*x^(
1/6)) + ((I - 1)*cos(a) - (I + 1)*sin(a))*erf(sqrt(-I*b)*x^(1/6)))*b^(3/2)
+ 8*b^2*x^(1/6)*sin(b*x^(1/3) + a))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x^{1/3}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^(1/3))/x^(1/2),x)
```

```
[Out] int(cos(a + b*x^(1/3))/x^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x**(1/3))/x**(1/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))/sqrt(x), x)
```

3.52 $\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=110

$$-4\sqrt{2\pi}\,b^{3/2}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)+4\sqrt{2\pi}\,b^{3/2}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)+\frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}}-\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}$$

```
[Out] 4*b*sin(a+b*x^(1/3))/x^(1/6)-4*b^(3/2)*cos(a)*FresnelC(x^(1/6)*b^(1/2)*2^(1
/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)+4*b^(3/2)*FresnelS(x^(1/6)*b^(1/2)*2^(1/2)/P
i^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)-2*cos(a+b*x^(1/3))/x^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3416, 3297, 3306, 3305, 3351, 3304, 3352}

$$-4\sqrt{2\pi}\,b^{3/2}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[6]{x}\right)+4\sqrt{2\pi}\,b^{3/2}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)+\frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}}-\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^(1/3)]/x^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x^(1/3)])/Sqrt[x] - 4*b^(3/2)*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt
[b]*Sqrt[2/Pi]*x^(1/6)] + 4*b^(3/2)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*
x^(1/6)]*Sin[a] + (4*b*Sin[a + b*x^(1/3)])/x^(1/6)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3416

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{\cos(a+bx)}{x^{5/2}}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} - (2b)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}} - \left(4b^2\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}} - \left(4b^2\cos(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right) + (4$$
$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}} - \left(8b^2\cos(a)\right)\text{Subst}\left(\int \cos\left(bx^2\right)\,dx, x, \sqrt[6]{x}\right) + ($$
$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} - 4b^{3/2}\sqrt{2\pi}\,\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right) + 4b^{3/2}\sqrt{2\pi}\,S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)\sin(a$$

Mathematica [A] time = 0.25, size = 110, normalized size = 1.00

$$-4\sqrt{2\pi}\,b^{3/2}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)+4\sqrt{2\pi}\,b^{3/2}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)+\frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}}-\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^(1/3)]/x^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x^(1/3)])/Sqrt[x] - 4*b^(3/2)*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt
[b]*Sqrt[2/Pi]*x^(1/6)] + 4*b^(3/2)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*
x^(1/6)]*Sin[a] + (4*b*Sin[a + b*x^(1/3)])/x^(1/6)
```

fricas [A] time = 1.26, size = 96, normalized size = 0.87

$$-\frac{2\left(2\sqrt{2}\,\pi bx\sqrt{\frac{b}{\pi}}\,\cos(a)\,\mathrm{C}\left(\sqrt{2}\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right) - 2\sqrt{2}\,\pi bx\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(\sqrt{2}\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin(a) - 2\,bx^{\frac{5}{6}}\sin\left(bx^{\frac{1}{3}}+a\right) + \sqrt{x}\,\cos\left(bx\right.\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))/x^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*sqrt(2)*pi*b*x*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x^(1/6)*sqrt(b/p
i)) - 2*sqrt(2)*pi*b*x*sqrt(b/pi)*fresnel_sin(sqrt(2)*x^(1/6)*sqrt(b/pi))*s
in(a) - 2*b*x^(5/6)*sin(b*x^(1/3) + a) + sqrt(x)*cos(b*x^(1/3) + a))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx^{\frac{1}{3}} + a\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^(1/3) + a)/x^(3/2), x)
```

maple [A] time = 0.03, size = 78, normalized size = 0.71

$$-\frac{2\cos\left(a+b\,x^{\frac{1}{3}}\right)}{\sqrt{x}} - 4b\left(-\frac{\sin\left(a+b\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}} + \sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\,\left(\cos(a)\,\mathrm{FresnelC}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right) - \sin(a)\mathrm{S}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*x^(1/3))/x^(3/2),x)
```

```
[Out] -2*cos(a+b*x^(1/3))/x^(1/2)-4*b*(-1/x^(1/6)*sin(a+b*x^(1/3))+b^(1/2)*2^(1/2
)*Pi^(1/2)*(cos(a)*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))-sin(a)*Fresne
lS(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))))
```

maxima [C] time = 1.96, size = 74, normalized size = 0.67

$$-\frac{3\left(\left((i-1)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2}, i\,bx^{\frac{1}{3}}\right) - (i+1)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2}, -i\,bx^{\frac{1}{3}}\right)\right)\cos(a) + \left((i+1)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2}, i\,bx^{\frac{1}{3}}\right) - (i-1)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2},\right.\right.\right.}{4\,x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))/x^(3/2),x, algorithm="maxima")
```

```
[Out] -3/4*(((I - 1)*sqrt(2)*gamma(-3/2, I*b*x^(1/3)) - (I + 1)*sqrt(2)*gamma(-3/
2, -I*b*x^(1/3)))*cos(a) + ((I + 1)*sqrt(2)*gamma(-3/2, I*b*x^(1/3)) - (I -
 1)*sqrt(2)*gamma(-3/2, -I*b*x^(1/3)))*sin(a))*sqrt(b*x^(1/3))*b/x^(1/6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a + b\,x^{1/3}\right)}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^(1/3))/x^(3/2),x)
```

```
[Out] int(cos(a + b*x^(1/3))/x^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a + b\sqrt[3]{x}\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x**(1/3))/x**(3/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))/x**(3/2), x)
```

3.53 $\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=184

$$-\frac{32}{315}\sqrt{2\pi}\,b^{9/2}\sin(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)-\frac{32}{315}\sqrt{2\pi}\,b^{9/2}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)-\frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}}-\frac{16b^3\sin\left(a+\right.}{315\sqrt{x}}$$

```
[Out] -2/3*cos(a+b*x^(1/3))/x^(3/2)+8/105*b^2*cos(a+b*x^(1/3))/x^(5/6)-32/315*b^4
*cos(a+b*x^(1/3))/x^(1/6)+4/21*b*sin(a+b*x^(1/3))/x^(7/6)-32/315*b^(9/2)*co
s(a)*FresnelS(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)-32/315*b^(
9/2)*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)-16/
315*b^3*sin(a+b*x^(1/3))/x^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3416, 3297, 3306, 3305, 3351, 3304, 3352}

$$-\frac{32}{315}\sqrt{2\pi}\,b^{9/2}\sin(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[6]{x}\right)-\frac{32}{315}\sqrt{2\pi}\,b^{9/2}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)+\frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}}-\frac{16b^3\,\mathrm{si}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^(1/3)]/x^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x^(1/3)])/(3*x^(3/2)) + (8*b^2*Cos[a + b*x^(1/3)])/(105*x^(5/
6)) - (32*b^4*Cos[a + b*x^(1/3)])/(315*x^(1/6)) - (32*b^(9/2)*Sqrt[2*Pi]*Co
s[a]*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)])/315 - (32*b^(9/2)*Sqrt[2*Pi]*Fre
snelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a])/315 + (4*b*Sin[a + b*x^(1/3)])/(2
1*x^(7/6)) - (16*b^3*Sin[a + b*x^(1/3)])/(315*Sqrt[x])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3416

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx = 3\,\text{Subst}\left(\int \frac{\cos(a+bx)}{x^{11/2}}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} - \frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^{9/2}}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} - \frac{1}{21}\left(4b^2\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^{7/2}}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} + \frac{1}{105}\left(8b^3\right)\text{Subst}\left(\int \frac{\text{sin}}{}\right.$$
$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} - \frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{315\sqrt{x}} + \frac{}{3}$$
$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} - \frac{1}{}$$
$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} - \frac{1}{}$$
$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} - \frac{1}{}$$
$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}} - \frac{32}{315}b^{9/2}\sqrt{2\pi}\,\cos(a)S$$

Mathematica [A] time = 0.25, size = 180, normalized size = 0.98

$$-\frac{2\left(16\sqrt{2\pi}\,b^{9/2}x^{3/2}\sin(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)+16\sqrt{2\pi}\,b^{9/2}x^{3/2}\cos(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)+16b^4x^{4/3}\cos\left(a+b\sqrt[3]{x}\right)+8\right.}{315x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^(1/3)]/x^(5/2),x]
```

```
[Out] (-2*(105*Cos[a + b*x^(1/3)] - 12*b^2*x^(2/3)*Cos[a + b*x^(1/3)] + 16*b^4*x^
(4/3)*Cos[a + b*x^(1/3)] + 16*b^(9/2)*Sqrt[2*Pi]*x^(3/2)*Cos[a]*FresnelS[Sq
rt[b]*Sqrt[2/Pi]*x^(1/6)] + 16*b^(9/2)*Sqrt[2*Pi]*x^(3/2)*FresnelC[Sqrt[b]*
Sqrt[2/Pi]*x^(1/6)]*Sin[a] - 30*b*x^(1/3)*Sin[a + b*x^(1/3)] + 8*b^3*x*Sin[
a + b*x^(1/3)]))/(315*x^(3/2))
```

fricas [A] time = 0.81, size = 134, normalized size = 0.73

$$-\frac{2\left(16\sqrt{2}\,\pi b^4x^2\sqrt{\frac{b}{\pi}}\,\cos(a)\,\mathrm{S}\left(\sqrt{2}\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)+16\sqrt{2}\,\pi b^4x^2\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(\sqrt{2}\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin(a)+\left(16\,b^4x^{\frac{11}{6}}-12\,b^2x^{\frac{7}{6}}+105\right.\right.}{315\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/315*(16*sqrt(2)*pi*b^4*x^2*sqrt(b/pi)*cos(a)*fresnel_sin(sqrt(2)*x^(1/6)
*sqrt(b/pi)) + 16*sqrt(2)*pi*b^4*x^2*sqrt(b/pi)*fresnel_cos(sqrt(2)*x^(1/6)
*sqrt(b/pi))*sin(a) + (16*b^4*x^(11/6) - 12*b^2*x^(7/6) + 105*sqrt(x))*cos(
b*x^(1/3) + a) + 2*(4*b^3*x^(3/2) - 15*b*x^(5/6))*sin(b*x^(1/3) + a))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx^{\frac{1}{3}}+a\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^(1/3) + a)/x^(5/2), x)
```

maple [A] time = 0.04, size = 129, normalized size = 0.70

$$-\frac{2\cos\left(a+b\,x^{\frac{1}{3}}\right)}{3x^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(a+b\,x^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}}+\frac{2b\left(-\frac{\cos\left(a+b\,x^{\frac{1}{3}}\right)}{5x^{\frac{5}{6}}}-\frac{2b\left(-\frac{\sin\left(a+b\,x^{\frac{1}{3}}\right)}{3\sqrt{x}}+\frac{2b\left(-\frac{\cos\left(a+b\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}}-\sqrt{b}\,\sqrt{2}\,\sqrt{\pi}\left(\cos(a)\mathrm{S}\left(\frac{x^{\frac{1}{6}}\sqrt{b}\,\sqrt{2}}{\sqrt{\pi}}\right)+\sin(a)\,\mathrm{FresnelC}\left(x^{\cdots}\right.\right.\right.}{3}\right.}{5}\right.}{7}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*x^(1/3))/x^(5/2),x)
```

```
[Out] -2/3*cos(a+b*x^(1/3))/x^(3/2)-4/3*b*(-1/7/x^(7/6)*sin(a+b*x^(1/3))+2/7*b*(-
1/5/x^(5/6)*cos(a+b*x^(1/3))-2/5*b*(-1/3/x^(1/2)*sin(a+b*x^(1/3))+2/3*b*(-c
os(a+b*x^(1/3))/x^(1/6)-b^(1/2)*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelS(x^(1/6)*b
^(1/2)*2^(1/2)/Pi^(1/2))+sin(a)*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2)))
))))
```

maxima [C] time = 1.62, size = 76, normalized size = 0.41

$$\frac{3\left(\left(-(i+1)\sqrt{2}\,\Gamma\left(-\frac{9}{2}, i\,bx^{\frac{1}{3}}\right)+(i-1)\sqrt{2}\,\Gamma\left(-\frac{9}{2}, -i\,bx^{\frac{1}{3}}\right)\right)\cos(a)+\left((i-1)\sqrt{2}\,\Gamma\left(-\frac{9}{2}, i\,bx^{\frac{1}{3}}\right)-(i+1)\sqrt{2}\,\Gamma\left(-\frac{9}{2}, -\right.\right.\right.}{4\,x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))/x^(5/2),x, algorithm="maxima")
```

```
[Out] 3/4*((-(I + 1)*sqrt(2)*gamma(-9/2, I*b*x^(1/3)) + (I - 1)*sqrt(2)*gamma(-9/
2, -I*b*x^(1/3)))*cos(a) + ((I - 1)*sqrt(2)*gamma(-9/2, I*b*x^(1/3)) - (I +
 1)*sqrt(2)*gamma(-9/2, -I*b*x^(1/3)))*sin(a))*sqrt(b*x^(1/3))*b^4/x^(1/6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x^{1/3}\right)}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^(1/3))/x^(5/2),x)
```

```
[Out] int(cos(a + b*x^(1/3))/x^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x**(1/3))/x**(5/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))/x**(5/2), x)
```

3.54 $\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx$

Optimal. Leaf size=250

$$\frac{256\sqrt{2\pi}\,b^{15/2}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{675675}-\frac{256\sqrt{2\pi}\,b^{15/2}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{675675}-\frac{256b^7\sin\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}}+\frac{128b^6\cos\left(a\right.}{675675}$$

```
[Out] -2/5*cos(a+b*x^(1/3))/x^(5/2)+8/715*b^2*cos(a+b*x^(1/3))/x^(11/6)-32/45045*
b^4*cos(a+b*x^(1/3))/x^(7/6)+4/65*b*sin(a+b*x^(1/3))/x^(13/6)-16/6435*b^3*s
in(a+b*x^(1/3))/x^(3/2)+64/225225*b^5*sin(a+b*x^(1/3))/x^(5/6)-256/675675*b
^7*sin(a+b*x^(1/3))/x^(1/6)+256/675675*b^(15/2)*cos(a)*FresnelC(x^(1/6)*b^(
1/2)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)-256/675675*b^(15/2)*FresnelS(x^(1/6
)*b^(1/2)*2^(1/2)/Pi^(1/2))*sin(a)*2^(1/2)*Pi^(1/2)+128/675675*b^6*cos(a+b*
x^(1/3))/x^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3416, 3297, 3306, 3305, 3351, 3304, 3352}

$$\frac{256\sqrt{2\pi}\,b^{15/2}\cos(a)\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt[6]{x}\right)}{675675}-\frac{256\sqrt{2\pi}\,b^{15/2}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right)}{675675}+\frac{64b^5\sin\left(a+b\sqrt[3]{x}\right)}{225225x^{5/6}}-\frac{16b^3\,\text{si}}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^(1/3)]/x^(7/2),x]
```

```
[Out] (-2*Cos[a + b*x^(1/3)])/(5*x^(5/2)) + (8*b^2*Cos[a + b*x^(1/3)])/(715*x^(11
/6)) - (32*b^4*Cos[a + b*x^(1/3)])/(45045*x^(7/6)) + (128*b^6*Cos[a + b*x^(
1/3)])/(675675*Sqrt[x]) + (256*b^(15/2)*Sqrt[2*Pi]*Cos[a]*FresnelC[Sqrt[b]*
Sqrt[2/Pi]*x^(1/6)])/675675 - (256*b^(15/2)*Sqrt[2*Pi]*FresnelS[Sqrt[b]*Sqr
t[2/Pi]*x^(1/6)]*Sin[a])/675675 + (4*b*Sin[a + b*x^(1/3)])/(65*x^(13/6)) -
(16*b^3*Sin[a + b*x^(1/3)])/(6435*x^(3/2)) + (64*b^5*Sin[a + b*x^(1/3)])/(2
25225*x^(5/6)) - (256*b^7*Sin[a + b*x^(1/3)])/(675675*x^(1/6))
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3416

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\int \frac{\cos\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx = 3\,\text{Subst}\left(\int \frac{\cos(a+bx)}{x^{17/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} - \frac{1}{5}(2b)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^{15/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{1}{65}\left(4b^2\right)\text{Subst}\left(\int \frac{\cos(a+bx)}{x^{13/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} + \frac{1}{715}\left(8b^3\right)\text{Subst}\left(\int \text{sin}\ldots\right.$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \frac{16b^3\sin\left(a+b\sqrt[3]{x}\right)}{6435x^{3/2}} + \ldots$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \ldots$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{4b\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} - \ldots$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{128b^6\cos\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} \ldots$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{128b^6\cos\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} \ldots$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{128b^6\cos\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} \ldots$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{128b^6\cos\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} \ldots$$

$$= -\frac{2\cos\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b^2\cos\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{32b^4\cos\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}} + \frac{128b^6\cos\left(a+b\sqrt[3]{x}\right)}{675675\sqrt{x}} \ldots$$

Mathematica [A] time = 0.34, size = 238, normalized size = 0.95

$$\frac{2\left(128\sqrt{2\pi}\,b^{15/2}x^{5/2}\cos(a)C\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right) - 128\sqrt{2\pi}\,b^{15/2}x^{5/2}\sin(a)S\left(\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt[6]{x}\right) - 128b^7x^{7/3}\sin\left(a+b\sqrt[3]{x}\right) \ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^(1/3)]/x^(7/2),x]
```

```
[Out] (2*(-135135*Cos[a + b*x^(1/3)] + 3780*b^2*x^(2/3)*Cos[a + b*x^(1/3)] - 240*
b^4*x^(4/3)*Cos[a + b*x^(1/3)] + 64*b^6*x^2*Cos[a + b*x^(1/3)] + 128*b^(15/
2)*Sqrt[2*Pi]*x^(5/2)*Cos[a]*FresnelC[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)] - 128*b^(
15/2)*Sqrt[2*Pi]*x^(5/2)*FresnelS[Sqrt[b]*Sqrt[2/Pi]*x^(1/6)]*Sin[a] + 2079
0*b*x^(1/3)*Sin[a + b*x^(1/3)] - 840*b^3*x*Sin[a + b*x^(1/3)] + 96*b^5*x^(5
/3)*Sin[a + b*x^(1/3)] - 128*b^7*x^(7/3)*Sin[a + b*x^(1/3)]))/(675675*x^(5/
2))
```

fricas [A] time = 0.65, size = 164, normalized size = 0.66

$$\frac{2\left(128\sqrt{2}\,\pi b^7x^3\sqrt{\frac{b}{\pi}}\,\cos(a)\,C\left(\sqrt{2}\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)-128\sqrt{2}\,\pi b^7x^3\sqrt{\frac{b}{\pi}}\,S\left(\sqrt{2}\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin(a)-\left(240\,b^4x^{\frac{11}{6}}-3780\,b^2x^{\frac{7}{6}}-\right.\right.}{675675\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/675675*(128*sqrt(2)*pi*b^7*x^3*sqrt(b/pi)*cos(a)*fresnel_cos(sqrt(2)*x^(1
/6)*sqrt(b/pi)) - 128*sqrt(2)*pi*b^7*x^3*sqrt(b/pi)*fresnel_sin(sqrt(2)*x^(
1/6)*sqrt(b/pi))*sin(a) - (240*b^4*x^(11/6) - 3780*b^2*x^(7/6) - (64*b^6*x^
2 - 135135)*sqrt(x))*cos(b*x^(1/3) + a) + 2*(48*b^5*x^(13/6) - 420*b^3*x^(3
/2) - (64*b^7*x^2 - 10395*b)*x^(5/6))*sin(b*x^(1/3) + a))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx^{\frac{1}{3}} + a\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^(1/3) + a)/x^(7/2), x)
```

maple [A] time = 0.03, size = 180, normalized size = 0.72

$$2b\left(-\frac{\sin\left(a+b\,x\right.}{x^{\frac{1}{6}}}\right.$$

$$2b\left(-\frac{\cos\left(a+b\,x^{\frac{1}{3}}\right)}{3\sqrt{x}}-\right.$$

$$2b\left(-\frac{\sin\left(a+b\,x^{\frac{1}{3}}\right)}{5x^{\frac{5}{6}}}+\right.$$

$$2b\left(-\frac{\cos\left(a+b\,x^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}}-\right.$$

$$2b\left(-\frac{\sin\left(a+b\,x^{\frac{1}{3}}\right)}{9x^{\frac{3}{2}}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*x^(1/3))/x^(7/2),x)
```

```
[Out] -2/5*cos(a+b*x^(1/3))/x^(5/2)-4/5*b*(-1/13/x^(13/6)*sin(a+b*x^(1/3))+2/13*b
*(-1/11/x^(11/6)*cos(a+b*x^(1/3))-2/11*b*(-1/9/x^(3/2)*sin(a+b*x^(1/3))+2/9
*b*(-1/7/x^(7/6)*cos(a+b*x^(1/3))-2/7*b*(-1/5/x^(5/6)*sin(a+b*x^(1/3))+2/5*
b*(-1/3*cos(a+b*x^(1/3))/x^(1/2)-2/3*b*(-1/x^(1/6)*sin(a+b*x^(1/3))+b^(1/2)
*2^(1/2)*Pi^(1/2)*(cos(a)*FresnelC(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2))-sin(a)
*FresnelS(x^(1/6)*b^(1/2)*2^(1/2)/Pi^(1/2)))))))))
```

maxima [C] time = 1.29, size = 76, normalized size = 0.30

$$\frac{3\left(\left((i-1)\,\sqrt{2}\,\Gamma\left(-\frac{15}{2}, i\,bx^{\frac{1}{3}}\right) - (i+1)\,\sqrt{2}\,\Gamma\left(-\frac{15}{2}, -i\,bx^{\frac{1}{3}}\right)\right)\cos(a) + \left((i+1)\,\sqrt{2}\,\Gamma\left(-\frac{15}{2}, i\,bx^{\frac{1}{3}}\right) - (i-1)\,\sqrt{2}\,\Gamma\left(-\frac{15}{2},\right.\right.\right.}{4\,x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))/x^(7/2),x, algorithm="maxima")
```

```
[Out] 3/4*(((I - 1)*sqrt(2)*gamma(-15/2, I*b*x^(1/3)) - (I + 1)*sqrt(2)*gamma(-15
/2, -I*b*x^(1/3)))*cos(a) + ((I + 1)*sqrt(2)*gamma(-15/2, I*b*x^(1/3)) - (I
 - 1)*sqrt(2)*gamma(-15/2, -I*b*x^(1/3)))*sin(a))*sqrt(b*x^(1/3))*b^7/x^(1/
6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a + b\,x^{1/3}\right)}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^(1/3))/x^(7/2),x)
```

```
[Out] int(cos(a + b*x^(1/3))/x^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a + b\sqrt[3]{x}\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x**(1/3))/x**(7/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))/x**(7/2), x)
```

3.55 $\int x^{3/2} \cos^2\left(a+b\sqrt[3]{x}\right) dx$

Optimal. Leaf size=310

$$\frac{405405\sqrt{\pi}\,\sin(2a)C\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)}{32768b^{15/2}}+\frac{405405\sqrt{\pi}\,\cos(2a)S\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)}{32768b^{15/2}}-\frac{405405\sqrt[6]{x}\,\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{16384b^7}+\frac{135135\sqrt{x}\,\cos}{2048}$$

```
[Out] 3861/256*x^(7/6)/b^4-39/16*x^(11/6)/b^2+1/5*x^(5/2)-3861/128*x^(7/6)*cos(a+
b*x^(1/3))^2/b^4+39/8*x^(11/6)*cos(a+b*x^(1/3))^2/b^2+27027/512*x^(5/6)*cos
(a+b*x^(1/3))*sin(a+b*x^(1/3))/b^5-429/32*x^(3/2)*cos(a+b*x^(1/3))*sin(a+b*
x^(1/3))/b^3+3/2*x^(13/6)*cos(a+b*x^(1/3))*sin(a+b*x^(1/3))/b-405405/16384*
x^(1/6)*sin(2*a+2*b*x^(1/3))/b^7+405405/32768*cos(2*a)*FresnelS(2*x^(1/6)*b
^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(15/2)+405405/32768*FresnelC(2*x^(1/6)*b^(1/2)/
Pi^(1/2))*sin(2*a)*Pi^(1/2)/b^(15/2)-135135/4096*x^(1/2)/b^6+135135/2048*co
s(a+b*x^(1/3))^2*x^(1/2)/b^6
```

Rubi [A] time = 0.36, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3416, 3311, 30, 3312, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{405405\sqrt{\pi}\,\sin(2a)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)}{32768b^{15/2}}+\frac{405405\sqrt{\pi}\,\cos(2a)S\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)}{32768b^{15/2}}+\frac{39x^{11/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2}-\frac{3861x^{7/6}\cos^2}{128}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Cos[a + b*x^(1/3)]^2,x]
```

```
[Out] (-135135*Sqrt[x])/(4096*b^6) + (3861*x^(7/6))/(256*b^4) - (39*x^(11/6))/(16
*b^2) + x^(5/2)/5 + (135135*Sqrt[x]*Cos[a + b*x^(1/3)]^2)/(2048*b^6) - (386
1*x^(7/6)*Cos[a + b*x^(1/3)]^2)/(128*b^4) + (39*x^(11/6)*Cos[a + b*x^(1/3)]
^2)/(8*b^2) + (405405*Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[P
i]])/(32768*b^(15/2)) + (405405*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[
Pi]]*Sin[2*a])/(32768*b^(15/2)) + (27027*x^(5/6)*Cos[a + b*x^(1/3)]*Sin[a +
 b*x^(1/3)])/(512*b^5) - (429*x^(3/2)*Cos[a + b*x^(1/3)]*Sin[a + b*x^(1/3)]
)/(32*b^3) + (3*x^(13/6)*Cos[a + b*x^(1/3)]*Sin[a + b*x^(1/3)])/(2*b) - (40
5405*x^(1/6)*Sin[2*(a + b*x^(1/3))])/(16384*b^7)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3416

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\int x^{3/2}\cos^2\left(a+b\sqrt[3]{x}\right)\,dx = 3\,\text{Subst}\left(\int x^{13/2}\cos^2(a+bx)\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{39x^{11/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} + \frac{3x^{13/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b} + \frac{3}{2}\,\text{Subst}\Bigg(\int x^{13/2}$$

$$= \frac{x^{5/2}}{5} - \frac{3861x^{7/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{39x^{11/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} - \frac{429x^{3/2}\cos\left(a+b\sqrt[3]{x}\right.}{32b^3}$$

$$= -\frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6} - \frac{3861x^{7/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{39x^{11}}{}$$

$$= \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6} - \frac{3861x^{7/6}\cos^2\left(a+b\sqrt[3]{x}\right.}{128b^4}$$

$$= -\frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6} - \frac{3861x^{7/}}{}$$

$$= -\frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6} - \frac{3861x^{7/}}{}$$

$$= -\frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6} - \frac{3861x^{7/}}{}$$

$$= -\frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6} - \frac{3861x^{7/}}{}$$

$$= -\frac{135135\sqrt{x}}{4096b^6} + \frac{3861x^{7/6}}{256b^4} - \frac{39x^{11/6}}{16b^2} + \frac{x^{5/2}}{5} + \frac{135135\sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{2048b^6} - \frac{3861x^{7/}}{}$$

Mathematica [A] time = 0.64, size = 174, normalized size = 0.56

$$2\sqrt{b}\,\sqrt[6]{x}\,\left(780\left(256b^5x^{5/3} - 1584b^3x + 3465b\sqrt[3]{x}\right)\cos\left(2\left(a+b\sqrt[3]{x}\right)\right) + 15\left(4096b^6x^2 - 36608b^4x^{4/3} + 144144b^2x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Cos[a + b*x^(1/3)]^2,x]
```

```
[Out] (2027025*Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]] + 2027025
*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a] + 2*Sqrt[b]*x^(1/
6)*(16384*b^7*x^(7/3) + 780*(3465*b*x^(1/3) - 1584*b^3*x + 256*b^5*x^(5/3))
*Cos[2*(a + b*x^(1/3))] + 15*(-135135 + 144144*b^2*x^(2/3) - 36608*b^4*x^(4
/3) + 4096*b^6*x^2)*Sin[2*(a + b*x^(1/3))]))/(163840*b^(15/2))
```

fricas [A] time = 0.98, size = 184, normalized size = 0.59

$$-\frac{399360\, b^6 x^{\frac{11}{6}} - 2471040\, b^4 x^{\frac{7}{6}} - 2027025\,\pi\sqrt{\frac{b}{\pi}}\,\cos(2\,a)\,\mathrm{S}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right) - 2027025\,\pi\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin(2\,a) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(a+b*x^(1/3))^2,x, algorithm="fricas")
```

```
[Out] -1/163840*(399360*b^6*x^(11/6) - 2471040*b^4*x^(7/6) - 2027025*pi*sqrt(b/pi
)*cos(2*a)*fresnel_sin(2*x^(1/6)*sqrt(b/pi)) - 2027025*pi*sqrt(b/pi)*fresne
l_cos(2*x^(1/6)*sqrt(b/pi))*sin(2*a) - 3120*(256*b^6*x^(11/6) - 1584*b^4*x^
(7/6) + 3465*b^2*sqrt(x))*cos(b*x^(1/3) + a)^2 + 60*(36608*b^5*x^(3/2) - 14
4144*b^3*x^(5/6) - (4096*b^7*x^2 - 135135*b)*x^(1/6))*cos(b*x^(1/3) + a)*si
n(b*x^(1/3) + a) - 8*(4096*b^8*x^2 - 675675*b^2)*sqrt(x))/b^8
```

giac [C] time = 1.25, size = 224, normalized size = 0.72

$$\frac{1}{5}\,x^{\frac{5}{2}} - \frac{3\left(4096i\,b^6 x^{\frac{13}{6}} - 13312\,b^5 x^{\frac{11}{6}} - 36608i\,b^4 x^{\frac{3}{2}} + 82368\,b^3 x^{\frac{7}{6}} + 144144i\,b^2 x^{\frac{5}{6}} - 180180\,b\sqrt{x} - 135135i\,x^{\frac{1}{6}}\right)e^{\left(2i\right.}}{32768\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(a+b*x^(1/3))^2,x, algorithm="giac")
```

```
[Out] 1/5*x^(5/2) - 3/32768*(4096*I*b^6*x^(13/6) - 13312*b^5*x^(11/6) - 36608*I*b
^4*x^(3/2) + 82368*b^3*x^(7/6) + 144144*I*b^2*x^(5/6) - 180180*b*sqrt(x) -
135135*I*x^(1/6))*e^(2*I*b*x^(1/3) + 2*I*a)/b^7 - 3/32768*(-4096*I*b^6*x^(1
3/6) - 13312*b^5*x^(11/6) + 36608*I*b^4*x^(3/2) + 82368*b^3*x^(7/6) - 14414
4*I*b^2*x^(5/6) - 180180*b*sqrt(x) + 135135*I*x^(1/6))*e^(-2*I*b*x^(1/3) -
2*I*a)/b^7 + 405405/65536*I*sqrt(pi)*erf(-sqrt(b)*x^(1/6)*(-I*b/abs(b) + 1)
)*e^(2*I*a)/(b^(15/2)*(-I*b/abs(b) + 1)) - 405405/65536*I*sqrt(pi)*erf(-sqr
t(b)*x^(1/6)*(I*b/abs(b) + 1))*e^(-2*I*a)/(b^(15/2)*(I*b/abs(b) + 1))
```

maple [A] time = 0.06, size = 219, normalized size = 0.71

$$\frac{x^{\frac{5}{2}}}{5}+\frac{3x^{\frac{13}{6}}\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}-\frac{39\left(-\frac{x^{\frac{11}{6}}\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}+\frac{11x^{\frac{3}{2}}\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{16b}-\frac{99\left(-\frac{x^{\frac{7}{6}}\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b}+\frac{7x^{\frac{5}{6}}\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{16b}-\frac{35\left(-\frac{\sqrt{x}\,\cos\left(2a+2b\,x \cdots\right.}{4b}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots\right.}{4b}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*cos(a+b*x^(1/3))^2,x)
```

```
[Out] 1/5*x^(5/2)+3/4/b*x^(13/6)*sin(2*a+2*b*x^(1/3))-39/4/b*(-1/4/b*x^(11/6)*cos
(2*a+2*b*x^(1/3))+11/4/b*(1/4/b*x^(3/2)*sin(2*a+2*b*x^(1/3))-9/4/b*(-1/4/b*
x^(7/6)*cos(2*a+2*b*x^(1/3))+7/4/b*(1/4/b*x^(5/6)*sin(2*a+2*b*x^(1/3))-5/4/
b*(-1/4/b*x^(1/2)*cos(2*a+2*b*x^(1/3))+3/4/b*(1/4*x^(1/6)*sin(2*a+2*b*x^(1/
3))/b-1/8/b^(3/2)*Pi^(1/2)*(cos(2*a)*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))+s
in(2*a)*FresnelC(2*x^(1/6)*b^(1/2)/Pi^(1/2))))))))
```

maxima [C] time = 1.24, size = 161, normalized size = 0.52

$$\frac{262144\,b^9x^{\frac{5}{2}} - 4^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\left((-(2027025i + 2027025)\,\cos(2\,a) + (2027025i - 2027025)\,\sin(2\,a))\,\mathrm{erf}\left(\sqrt{2i\,b}\,x^{\frac{1}{6}}\right) + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*cos(a+b*x^(1/3))^2,x, algorithm="maxima")
```

```
[Out] 1/1310720*(262144*b^9*x^(5/2) - 4^(1/4)*sqrt(2)*sqrt(pi)*((-(2027025*I + 20
27025)*cos(2*a) + (2027025*I - 2027025)*sin(2*a))*erf(sqrt(2*I*b)*x^(1/6))
+ ((2027025*I - 2027025)*cos(2*a) - (2027025*I + 2027025)*sin(2*a))*erf(sqr
t(-2*I*b)*x^(1/6)))*b^(3/2) + 12480*(256*b^7*x^(11/6) - 1584*b^5*x^(7/6) +
3465*b^3*sqrt(x))*cos(2*b*x^(1/3) + 2*a) + 240*(4096*b^8*x^(13/6) - 36608*b
^6*x^(3/2) + 144144*b^4*x^(5/6) - 135135*b^2*x^(1/6))*sin(2*b*x^(1/3) + 2*a
))/b^9
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\cos\left(a + b\,x^{1/3}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*cos(a + b*x^(1/3))^2,x)
```

```
[Out] int(x^(3/2)*cos(a + b*x^(1/3))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\cos^2\left(a + b\sqrt[3]{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*cos(a+b*x**(1/3))**2,x)
```

```
[Out] Integral(x**(3/2)*cos(a + b*x**(1/3))**2, x)
```

3.56 $\int \sqrt{x} \cos^2\left(a+b\sqrt[3]{x}\right) dx$

Optimal. Leaf size=218

$$\frac{315\sqrt{\pi}\cos(2a)C\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{512b^{9/2}}-\frac{315\sqrt{\pi}\sin(2a)S\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{512b^{9/2}}-\frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4}-\frac{105\sqrt{x}\sin\left(a+b\sqrt[3]{x}\right)\cos\left(a\right.}{32b^3}$$

```
[Out] 315/256*x^(1/6)/b^4-21/16*x^(5/6)/b^2+1/3*x^(3/2)-315/128*x^(1/6)*cos(a+b*x
^(1/3))^2/b^4+21/8*x^(5/6)*cos(a+b*x^(1/3))^2/b^2+3/2*x^(7/6)*cos(a+b*x^(1/
3))*sin(a+b*x^(1/3))/b+315/512*cos(2*a)*FresnelC(2*x^(1/6)*b^(1/2)/Pi^(1/2)
)*Pi^(1/2)/b^(9/2)-315/512*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))*sin(2*a)*Pi
^(1/2)/b^(9/2)-105/32*cos(a+b*x^(1/3))*sin(a+b*x^(1/3))*x^(1/2)/b^3
```

Rubi [A] time = 0.25, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3416, 3311, 30, 3312, 3306, 3305, 3351, 3304, 3352}

$$\frac{315\sqrt{\pi}\cos(2a)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{512b^{9/2}}-\frac{315\sqrt{\pi}\sin(2a)S\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{512b^{9/2}}+\frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2}-\frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right.}{128b^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Cos[a + b*x^(1/3)]^2,x]
```

```
[Out] (315*x^(1/6))/(256*b^4) - (21*x^(5/6))/(16*b^2) + x^(3/2)/3 - (315*x^(1/6)*
Cos[a + b*x^(1/3)]^2)/(128*b^4) + (21*x^(5/6)*Cos[a + b*x^(1/3)]^2)/(8*b^2)
 + (315*Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]])/(512*b^(9
/2)) - (315*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a])/(512*
b^(9/2)) - (105*Sqrt[x]*Cos[a + b*x^(1/3)]*Sin[a + b*x^(1/3)])/(32*b^3) + (
3*x^(7/6)*Cos[a + b*x^(1/3)]*Sin[a + b*x^(1/3)])/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3416

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\int \sqrt{x}\cos^2\left(a+b\sqrt[3]{x}\right)dx = 3\,\text{Subst}\left(\int x^{7/2}\cos^2(a+bx)\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} + \frac{3x^{7/6}\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{2b} + \frac{3}{2}\,\text{Subst}\left(\int x^{7/2}\,dx\right.$$

$$= \frac{x^{3/2}}{3} - \frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} - \frac{105\sqrt{x}\cos\left(a+b\sqrt[3]{x}\right)\text{s}}{32b^3}$$

$$= -\frac{21x^{5/6}}{16b^2} + \frac{x^{3/2}}{3} - \frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} - \frac{105\sqrt{x}\cos\left(\right.}{}$$

$$= \frac{315\sqrt[6]{x}}{256b^4} - \frac{21x^{5/6}}{16b^2} + \frac{x^{3/2}}{3} - \frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} - \frac{105}{}$$

$$= \frac{315\sqrt[6]{x}}{256b^4} - \frac{21x^{5/6}}{16b^2} + \frac{x^{3/2}}{3} - \frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} - \frac{105}{}$$

$$= \frac{315\sqrt[6]{x}}{256b^4} - \frac{21x^{5/6}}{16b^2} + \frac{x^{3/2}}{3} - \frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} - \frac{105}{}$$

$$= \frac{315\sqrt[6]{x}}{256b^4} - \frac{21x^{5/6}}{16b^2} + \frac{x^{3/2}}{3} - \frac{315\sqrt[6]{x}\cos^2\left(a+b\sqrt[3]{x}\right)}{128b^4} + \frac{21x^{5/6}\cos^2\left(a+b\sqrt[3]{x}\right)}{8b^2} + \frac{315}{}$$

Mathematica [A] time = 0.44, size = 148, normalized size = 0.68

$$\frac{2\sqrt{b}\sqrt[6]{x}\left(63\left(16b^2x^{2/3}-15\right)\cos\left(2\left(a+b\sqrt[3]{x}\right)\right)+4b\sqrt[3]{x}\left(9\left(16b^2x^{2/3}-35\right)\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)+64b^3x\right)\right)+945\sqrt{}}{1536b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Cos[a + b*x^(1/3)]^2,x]
```

```
[Out] (945*Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]] - 945*Sqrt[Pi
]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a] + 2*Sqrt[b]*x^(1/6)*(63*(
-15 + 16*b^2*x^(2/3))*Cos[2*(a + b*x^(1/3))] + 4*b*x^(1/3)*(64*b^3*x + 9*(-
35 + 16*b^2*x^(2/3))*Sin[2*(a + b*x^(1/3))])))/(1536*b^(9/2))
```

fricas [A] time = 0.73, size = 144, normalized size = 0.66

$$\frac{512\,b^5x^{\frac{3}{2}} - 2016\,b^3x^{\frac{5}{6}} + 945\,\pi\sqrt{\frac{b}{\pi}}\cos(2\,a)\,\text{C}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right) - 945\,\pi\sqrt{\frac{b}{\pi}}\,\text{S}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin(2\,a) + 252\left(16\,b^3x^{\frac{5}{6}} - 15\right.}{1536\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(a+b*x^(1/3))^2,x, algorithm="fricas")
```

```
[Out] 1/1536*(512*b^5*x^(3/2) - 2016*b^3*x^(5/6) + 945*pi*sqrt(b/pi)*cos(2*a)*fre
snel_cos(2*x^(1/6)*sqrt(b/pi)) - 945*pi*sqrt(b/pi)*fresnel_sin(2*x^(1/6)*sq
rt(b/pi))*sin(2*a) + 252*(16*b^3*x^(5/6) - 15*b*x^(1/6))*cos(b*x^(1/3) + a)
^2 + 144*(16*b^4*x^(7/6) - 35*b^2*sqrt(x))*cos(b*x^(1/3) + a)*sin(b*x^(1/3)
 + a) + 1890*b*x^(1/6))/b^5
```

giac [C] time = 0.45, size = 176, normalized size = 0.81

$$\frac{1}{3}x^{\frac{3}{2}} - \frac{3\left(64i\,b^3x^{\frac{7}{6}} - 112\,b^2x^{\frac{5}{6}} - 140i\,b\sqrt{x} + 105\,x^{\frac{1}{6}}\right)e^{\left(2i\,bx^{\frac{1}{3}}+2i\,a\right)}}{512\,b^4} - \frac{3\left(-64i\,b^3x^{\frac{7}{6}} - 112\,b^2x^{\frac{5}{6}} + 140i\,b\sqrt{x} + 105\,x^{\frac{1}{6}}\right)e^{\left(\right.}}{512\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(a+b*x^(1/3))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^(3/2) - 3/512*(64*I*b^3*x^(7/6) - 112*b^2*x^(5/6) - 140*I*b*sqrt(x) +
 105*x^(1/6))*e^(2*I*b*x^(1/3) + 2*I*a)/b^4 - 3/512*(-64*I*b^3*x^(7/6) - 11
2*b^2*x^(5/6) + 140*I*b*sqrt(x) + 105*x^(1/6))*e^(-2*I*b*x^(1/3) - 2*I*a)/b
^4 - 315/1024*sqrt(pi)*erf(-sqrt(b)*x^(1/6)*(-I*b/abs(b) + 1))*e^(2*I*a)/(b
^(9/2)*(-I*b/abs(b) + 1)) - 315/1024*sqrt(pi)*erf(-sqrt(b)*x^(1/6)*(I*b/abs
(b) + 1))*e^(-2*I*a)/(b^(9/2)*(I*b/abs(b) + 1))
```

maple [A] time = 0.06, size = 145, normalized size = 0.67

$$\frac{x^{\frac{3}{2}}}{3} + \frac{3x^{\frac{7}{6}}\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b} - \frac{21\left(-\frac{x^{\frac{5}{6}}\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b} + \frac{\frac{5\sqrt{x}\,\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{16b} - \frac{15\left(-\frac{x^{\frac{1}{6}}\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b} + \frac{\sqrt{\pi}\left(\cos(2a)\,\mathrm{FresnelC}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right) - \sin(2a)\mathrm{S}\left(\frac{2x^{\frac{1}{6}}}{\sqrt{\ }}\right.\right.}{8b^{\frac{3}{2}}}\right.}{16b}}{b}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*cos(a+b*x^(1/3))^2,x)
```

```
[Out] 1/3*x^(3/2)+3/4/b*x^(7/6)*sin(2*a+2*b*x^(1/3))-21/4/b*(-1/4/b*x^(5/6)*cos(2
*a+2*b*x^(1/3))+5/4/b*(1/4/b*x^(1/2)*sin(2*a+2*b*x^(1/3))-3/4/b*(-1/4/b*x^(
```

```
1/6)*cos(2*a+2*b*x^(1/3))+1/8/b^(3/2)*Pi^(1/2)*(cos(2*a)*FresnelC(2*x^(1/6)
*b^(1/2)/Pi^(1/2))-sin(2*a)*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))))))
```

maxima [C] time = 1.24, size = 137, normalized size = 0.63

$$\frac{4096\,b^{6}x^{\frac{3}{2}} - 4^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\left(\left((945i - 945)\,\cos\left(2\,a\right) + (945i + 945)\,\sin\left(2\,a\right)\right)\operatorname{erf}\left(\sqrt{2i\,b}\,x^{\frac{1}{6}}\right) + \left(-\,(945i + 945)\,\cos\left(2\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*cos(a+b*x^(1/3))^2,x, algorithm="maxima")
```

```
[Out] 1/12288*(4096*b^6*x^(3/2) - 4^(1/4)*sqrt(2)*sqrt(pi)*(((945*I - 945)*cos(2*
a) + (945*I + 945)*sin(2*a))*erf(sqrt(2*I*b)*x^(1/6)) + (-(945*I + 945)*cos
(2*a) - (945*I - 945)*sin(2*a))*erf(sqrt(-2*I*b)*x^(1/6)))*b^(3/2) + 1008*(
16*b^4*x^(5/6) - 15*b^2*x^(1/6))*cos(2*b*x^(1/3) + 2*a) + 576*(16*b^5*x^(7/
6) - 35*b^3*sqrt(x))*sin(2*b*x^(1/3) + 2*a))/b^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{x}\,\cos\left(a + b\,x^{1/3}\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*cos(a + b*x^(1/3))^2,x)
```

```
[Out] int(x^(1/2)*cos(a + b*x^(1/3))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\cos^{2}\left(a + b\sqrt[3]{x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*cos(a+b*x**(1/3))**2,x)
```

```
[Out] Integral(sqrt(x)*cos(a + b*x**(1/3))**2, x)
```

3.57 $\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=102

$$-\frac{3\sqrt{\pi}\,\sin(2a)C\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{3\sqrt{\pi}\,\cos(2a)S\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)}{8b^{3/2}}+\frac{3\sqrt[6]{x}\,\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{4b}+\sqrt{x}$$

```
[Out] 3/4*x^(1/6)*sin(2*a+2*b*x^(1/3))/b-3/8*cos(2*a)*FresnelS(2*x^(1/6)*b^(1/2)/
Pi^(1/2))*Pi^(1/2)/b^(3/2)-3/8*FresnelC(2*x^(1/6)*b^(1/2)/Pi^(1/2))*sin(2*a
)*Pi^(1/2)/b^(3/2)+x^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3416, 3312, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\pi}\,\sin(2a)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{3\sqrt{\pi}\,\cos(2a)S\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)}{8b^{3/2}}+\frac{3\sqrt[6]{x}\,\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{4b}+\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^(1/3)]^2/Sqrt[x],x]
```

```
[Out] Sqrt[x] - (3*Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]])/(8*b
^(3/2)) - (3*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a])/(8*b
^(3/2)) + (3*x^(1/6)*Sin[2*(a + b*x^(1/3))])/(4*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3416

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx = 3\,\text{Subst}\left(\int \sqrt{x}\,\cos^2(a+bx)\,dx, x, \sqrt[3]{x}\right)$$
$$= 3\,\text{Subst}\left(\int \left(\frac{\sqrt{x}}{2}+\frac{1}{2}\sqrt{x}\,\cos(2a+2bx)\right)dx, x, \sqrt[3]{x}\right)$$
$$= \sqrt{x} + \frac{3}{2}\text{Subst}\left(\int \sqrt{x}\,\cos(2a+2bx)\,dx, x, \sqrt[3]{x}\right)$$
$$= \sqrt{x} + \frac{3\sqrt[6]{x}\,\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{4b} - \frac{3\,\text{Subst}\left(\int \frac{\sin(2a+2bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{8b}$$
$$= \sqrt{x} + \frac{3\sqrt[6]{x}\,\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{4b} - \frac{(3\cos(2a))\,\text{Subst}\left(\int \frac{\sin(2bx)}{\sqrt{x}}\,dx, x, \sqrt[3]{x}\right)}{8b} - \frac{(3\sin(2a))\,\text{Su}}{}$$
$$= \sqrt{x} + \frac{3\sqrt[6]{x}\,\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{4b} - \frac{(3\cos(2a))\,\text{Subst}\left(\int \sin\left(2bx^2\right)dx, x, \sqrt[6]{x}\right)}{4b} - \frac{(3\sin(2a}{}$$
$$= \sqrt{x} - \frac{3\sqrt{\pi}\,\cos(2a)S\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{3\sqrt{\pi}\,C\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a)}{8b^{3/2}} + \frac{3\sqrt[6]{x}\,\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)}{4b}$$

Mathematica [A] time = 0.18, size = 103, normalized size = 1.01

$$\frac{-3\sqrt{\pi}\,\sin(2a)C\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right) - 3\sqrt{\pi}\,\cos(2a)S\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right) + 2\sqrt{b}\,\sqrt[6]{x}\,\left(3\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)+4b\sqrt[3]{x}\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^(1/3)]^2/Sqrt[x],x]
```

```
[Out] (-3*Sqrt[Pi]*Cos[2*a]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]] - 3*Sqrt[Pi]*F
resnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a] + 2*Sqrt[b]*x^(1/6)*(4*b*x^(
1/3) + 3*Sin[2*(a + b*x^(1/3))]))/(8*b^(3/2))
```

fricas [A] time = 1.47, size = 90, normalized size = 0.88

$$-\frac{3\,\pi\sqrt{\frac{b}{\pi}}\,\cos\left(2\,a\right)\text{S}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right) + 3\,\pi\sqrt{\frac{b}{\pi}}\,\text{C}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right) - 12\,bx^{\frac{1}{6}}\cos\left(bx^{\frac{1}{3}}+a\right)\sin\left(bx^{\frac{1}{3}}+a\right) - 8\,b^2\sqrt{x}}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(3*pi*sqrt(b/pi)*cos(2*a)*fresnel_sin(2*x^(1/6)*sqrt(b/pi)) + 3*pi*sqr
t(b/pi)*fresnel_cos(2*x^(1/6)*sqrt(b/pi))*sin(2*a) - 12*b*x^(1/6)*cos(b*x^(
1/3) + a)*sin(b*x^(1/3) + a) - 8*b^2*sqrt(x))/b^2
```

giac [C] time = 0.53, size = 124, normalized size = 1.22

$$\sqrt{x} - \frac{3i\,x^{\frac{1}{6}}e^{\left(2i\,bx^{\frac{1}{3}}+2i\,a\right)}}{8\,b} + \frac{3i\,x^{\frac{1}{6}}e^{\left(-2i\,bx^{\frac{1}{3}}-2i\,a\right)}}{8\,b} - \frac{3i\,\sqrt{\pi}\ \mathrm{erf}\left(-\sqrt{b}\,x^{\frac{1}{6}}\left(-\frac{i\,b}{|b|}+1\right)\right)e^{(2i\,a)}}{16\,b^{\frac{3}{2}}\left(-\frac{i\,b}{|b|}+1\right)} + \frac{3i\,\sqrt{\pi}\ \mathrm{erf}\left(-\sqrt{b}\,x^{\frac{1}{6}}\left(\frac{i\,b}{|b|}+1\right)\right)e^{(-2}}{16\,b^{\frac{3}{2}}\left(\frac{i\,b}{|b|}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x) - 3/8*I*x^(1/6)*e^(2*I*b*x^(1/3) + 2*I*a)/b + 3/8*I*x^(1/6)*e^(-2*I
*b*x^(1/3) - 2*I*a)/b - 3/16*I*sqrt(pi)*erf(-sqrt(b)*x^(1/6)*(-I*b/abs(b) +
 1))*e^(2*I*a)/(b^(3/2)*(-I*b/abs(b) + 1)) + 3/16*I*sqrt(pi)*erf(-sqrt(b)*x
^(1/6)*(I*b/abs(b) + 1))*e^(-2*I*a)/(b^(3/2)*(I*b/abs(b) + 1))
```

maple [A] time = 0.06, size = 67, normalized size = 0.66

$$\sqrt{x} + \frac{3x^{\frac{1}{6}}\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{4b} - \frac{3\sqrt{\pi}\ \left(\cos\left(2a\right)\mathrm{S}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right)+\sin\left(2a\right)\mathrm{FresnelC}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right)\right)}{8b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*x^(1/3))^2/x^(1/2),x)
```

```
[Out] x^(1/2)+3/4*x^(1/6)*sin(2*a+2*b*x^(1/3))/b-3/8/b^(3/2)*Pi^(1/2)*(cos(2*a)*F
resnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))+sin(2*a)*FresnelC(2*x^(1/6)*b^(1/2)/Pi^
(1/2)))
```

maxima [C] time = 1.27, size = 95, normalized size = 0.93

$$\frac{4^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\left(\left(-\left(3i+3\right)\,\cos\left(2\,a\right)+\left(3i-3\right)\,\sin\left(2\,a\right)\right)\mathrm{erf}\left(\sqrt{2i\,b}\,x^{\frac{1}{6}}\right)+\left(\left(3i-3\right)\,\cos\left(2\,a\right)-\left(3i+3\right)\,\sin\left(2\,a\right)\right)\mathrm{erf}\left(\right.}{64\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/64*(4^(1/4)*sqrt(2)*sqrt(pi)*((-(3*I + 3)*cos(2*a) + (3*I - 3)*sin(2*a))*
erf(sqrt(2*I*b)*x^(1/6)) + ((3*I - 3)*cos(2*a) - (3*I + 3)*sin(2*a))*erf(sq
```

```
rt(-2*I*b)*x^(1/6)))*b^(3/2) + 64*b^3*sqrt(x) + 48*b^2*x^(1/6)*sin(2*b*x^(1
/3) + 2*a))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a + b\,x^{1/3}\right)^2}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^(1/3))^2/x^(1/2),x)
```

```
[Out] int(cos(a + b*x^(1/3))^2/x^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(a + b\sqrt[3]{x}\right)}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x**(1/3))**2/x**(1/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))**2/sqrt(x), x)
```

3.58 $\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=116

$$-8\sqrt{\pi}\,b^{3/2}\cos(2a)C\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)+8\sqrt{\pi}\,b^{3/2}\sin(2a)S\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)-\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}+\frac{8b\sin\left(a+b\sqrt[3]{x}\right)\cos\left(a+b\right.}{\sqrt[6]{x}}$$

```
[Out] 8*b*cos(a+b*x^(1/3))*sin(a+b*x^(1/3))/x^(1/6)-8*b^(3/2)*cos(2*a)*FresnelC(2
*x^(1/6)*b^(1/2)/Pi^(1/2))*Pi^(1/2)+8*b^(3/2)*FresnelS(2*x^(1/6)*b^(1/2)/Pi
^(1/2))*sin(2*a)*Pi^(1/2)-2*cos(a+b*x^(1/3))^2/x^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3416, 3314, 30, 3312, 3306, 3305, 3351, 3304, 3352}

$$-8\sqrt{\pi}\,b^{3/2}\cos(2a)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)+8\sqrt{\pi}\,b^{3/2}\sin(2a)S\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)-\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}}+\frac{8b\sin\left(a+b\sqrt[3]{x}\right)\text{cc}}{\sqrt[6]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^(1/3)]^2/x^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x^(1/3)]^2)/Sqrt[x] - 8*b^(3/2)*Sqrt[Pi]*Cos[2*a]*FresnelC[(2
*Sqrt[b]*x^(1/6))/Sqrt[Pi]] + 8*b^(3/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*x^(1/6
))/Sqrt[Pi]]*Sin[2*a] + (8*b*Cos[a + b*x^(1/3)]*Sin[a + b*x^(1/3)])/x^(1/6)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3416

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{\cos^2(a+bx)}{x^{5/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}} + \left(8b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \sqrt[3]{\right.$$

$$= 16b^2\sqrt[6]{x} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}} - \left(16b^2\right)\text{Subst}\left(\int \left(\frac{}{2}\right.\right.$$

$$= -\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}} - \left(8b^2\right)\text{Subst}\left(\int \frac{\cos(2a+2b}{\sqrt{x}}\right.$$

$$= -\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}} - \left(8b^2\cos(2a)\right)\text{Subst}\left(\int \frac{\cos(}{\sqrt{}}\right.$$

$$= -\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{\sqrt[6]{x}} - \left(16b^2\cos(2a)\right)\text{Subst}\left(\int \cos\right.$$

$$= -\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{\sqrt{x}} - 8b^{3/2}\sqrt{\pi}\cos(2a)C\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) + 8b^{3/2}\sqrt{\pi}\,S\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)\sin(2a) +$$

Mathematica [A] time = 0.25, size = 116, normalized size = 1.00

$$\frac{8\sqrt{\pi}\,b^{3/2}\sqrt{x}\,\sin(2a)S\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right) + 4b\sqrt[3]{x}\,\sin\left(2\left(a+b\sqrt[3]{x}\right)\right) - \cos\left(2\left(a+b\sqrt[3]{x}\right)\right) - 1}{\sqrt{x}} - 8\sqrt{\pi}\,b^{3/2}\cos(2a)C\left(\frac{2\sqrt{b}\sqrt[6]{}}{\sqrt{\pi}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^(1/3)]^2/x^(3/2),x]
```

```
[Out] -8*b^(3/2)*Sqrt[Pi]*Cos[2*a]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]] + (-1 -
 Cos[2*(a + b*x^(1/3))] + 8*b^(3/2)*Sqrt[Pi]*Sqrt[x]*FresnelS[(2*Sqrt[b]*x^
(1/6))/Sqrt[Pi]]*Sin[2*a] + 4*b*x^(1/3)*Sin[2*(a + b*x^(1/3))])/Sqrt[x]
```

fricas [A] time = 0.87, size = 100, normalized size = 0.86

$$-\frac{2\left(4\,\pi bx\sqrt{\frac{b}{\pi}}\,\cos\left(2\,a\right)\mathrm{C}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right) - 4\,\pi bx\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right) - 4\,bx^{\frac{5}{6}}\cos\left(bx^{\frac{1}{3}}+a\right)\sin\left(bx^{\frac{1}{3}}+a\right) + \sqrt{}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(4*pi*b*x*sqrt(b/pi)*cos(2*a)*fresnel_cos(2*x^(1/6)*sqrt(b/pi)) - 4*pi*b
*x*sqrt(b/pi)*fresnel_sin(2*x^(1/6)*sqrt(b/pi))*sin(2*a) - 4*b*x^(5/6)*cos(
b*x^(1/3) + a)*sin(b*x^(1/3) + a) + sqrt(x)*cos(b*x^(1/3) + a)^2)/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx^{\frac{1}{3}} + a\right)^2}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^(1/3) + a)^2/x^(3/2), x)
```

maple [A] time = 0.06, size = 87, normalized size = 0.75

$$-\frac{1}{\sqrt{x}}-\frac{\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{\sqrt{x}}-4b\left(-\frac{\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}}+2\sqrt{b}\,\sqrt{\pi}\,\left(\cos\left(2a\right)\mathrm{FresnelC}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right)-\sin\left(2a\right)\mathrm{S}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*x^(1/3))^2/x^(3/2),x)
```

```
[Out] -1/x^(1/2)-1/x^(1/2)*cos(2*a+2*b*x^(1/3))-4*b*(-1/x^(1/6)*sin(2*a+2*b*x^(1/
3))+2*b^(1/2)*Pi^(1/2)*(cos(2*a)*FresnelC(2*x^(1/6)*b^(1/2)/Pi^(1/2))-sin(2
*a)*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))))
```

maxima [C] time = 1.30, size = 87, normalized size = 0.75

$$-\frac{\sqrt{2}\left(\left(\left(3i-3\right)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2},2i\,bx^{\frac{1}{3}}\right)-\left(3i+3\right)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2},-2i\,bx^{\frac{1}{3}}\right)\right)\cos\left(2\,a\right)+\left(\left(3i+3\right)\,\sqrt{2}\,\Gamma\left(-\frac{3}{2},2i\,bx^{\frac{1}{3}}\right)-\left(3i-3\right)\right.\right.}{4\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(sqrt(2)*(((3*I - 3)*sqrt(2)*gamma(-3/2, 2*I*b*x^(1/3)) - (3*I + 3)*sq
rt(2)*gamma(-3/2, -2*I*b*x^(1/3)))*cos(2*a) + ((3*I + 3)*sqrt(2)*gamma(-3/2
, 2*I*b*x^(1/3)) - (3*I - 3)*sqrt(2)*gamma(-3/2, -2*I*b*x^(1/3)))*sin(2*a))
*sqrt(b*x^(1/3))*b*x^(1/3) + 4)/sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x^{1/3}\right)^2}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^(1/3))^2/x^(3/2),x)
```

```
[Out] int(cos(a + b*x^(1/3))^2/x^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x**(1/3))**2/x**(3/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))**2/x**(3/2), x)
```

3.59 $\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=228

$$-\frac{512}{315}\sqrt{\pi}\,b^{9/2}\sin(2a)C\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)-\frac{512}{315}\sqrt{\pi}\,b^{9/2}\cos(2a)S\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)-\frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{315\sqrt[6]{x}}-\frac{128b^3\sin\left(a+b\sqrt[3]{x}\right.}{315\sqrt{}}$$

```
[Out] -16/105*b^2/x^(5/6)+256/315*b^4/x^(1/6)-2/3*cos(a+b*x^(1/3))^2/x^(3/2)+32/1
05*b^2*cos(a+b*x^(1/3))^2/x^(5/6)-512/315*b^4*cos(a+b*x^(1/3))^2/x^(1/6)+8/
21*b*cos(a+b*x^(1/3))*sin(a+b*x^(1/3))/x^(7/6)-512/315*b^(9/2)*cos(2*a)*Fre
snelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))*Pi^(1/2)-512/315*b^(9/2)*FresnelC(2*x^(1/
6)*b^(1/2)/Pi^(1/2))*sin(2*a)*Pi^(1/2)-128/315*b^3*cos(a+b*x^(1/3))*sin(a+b
*x^(1/3))/x^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3416, 3314, 30, 3313, 12, 3306, 3305, 3351, 3304, 3352}

$$-\frac{512}{315}\sqrt{\pi}\,b^{9/2}\sin(2a)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)-\frac{512}{315}\sqrt{\pi}\,b^{9/2}\cos(2a)S\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)+\frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}}-\frac{512b^4\cos^2\left(\right.}{315\sqrt[6]{}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^(1/3)]^2/x^(5/2),x]
```

```
[Out] (-16*b^2)/(105*x^(5/6)) + (256*b^4)/(315*x^(1/6)) - (2*Cos[a + b*x^(1/3)]^2
)/(3*x^(3/2)) + (32*b^2*Cos[a + b*x^(1/3)]^2)/(105*x^(5/6)) - (512*b^4*Cos[
a + b*x^(1/3)]^2)/(315*x^(1/6)) - (512*b^(9/2)*Sqrt[Pi]*Cos[2*a]*FresnelS[(
2*Sqrt[b]*x^(1/6))/Sqrt[Pi]])/315 - (512*b^(9/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[
b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a])/315 + (8*b*Cos[a + b*x^(1/3)]*Sin[a + b*x^(
1/3)])/(21*x^(7/6)) - (128*b^3*Cos[a + b*x^(1/3)]*Sin[a + b*x^(1/3)])/(315*
Sqrt[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3416

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{5/2}}\,dx = 3\,\text{Subst}\left(\int \frac{\cos^2(a+bx)}{x^{11/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{21x^{7/6}} + \frac{1}{21}\left(8b^2\right)\text{Subst}\left(\int \frac{1}{x^{7/2}}\,dx, x,\right.$$

$$= -\frac{16b^2}{105x^{5/6}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right.}{21x^{7/6}}$$

$$= -\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right.}{315\sqrt[6]{x}}$$

$$= -\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right.}{315\sqrt[6]{x}}$$

$$= -\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right.}{315\sqrt[6]{x}}$$

$$= -\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right.}{315\sqrt[6]{x}}$$

$$= -\frac{16b^2}{105x^{5/6}} + \frac{256b^4}{315\sqrt[6]{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{3x^{3/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{105x^{5/6}} - \frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right.}{315\sqrt[6]{x}}$$

Mathematica [A] time = 0.27, size = 185, normalized size = 0.81

$$\frac{-512\sqrt{\pi}\,b^{9/2}x^{3/2}\sin(2a)C\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right) - 512\sqrt{\pi}\,b^{9/2}x^{3/2}\cos(2a)S\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right) - 256b^4x^{4/3}\cos\left(2\left(a+b\sqrt[3]{x}\right)\right) - 64b^3x\,\text{s}}{315x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^(1/3)]^2/x^(5/2),x]
```

```
[Out] (-105 - 105*Cos[2*(a + b*x^(1/3))] + 48*b^2*x^(2/3)*Cos[2*(a + b*x^(1/3))]
- 256*b^4*x^(4/3)*Cos[2*(a + b*x^(1/3))] - 512*b^(9/2)*Sqrt[Pi]*x^(3/2)*Cos
```

```
[2*a]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]] - 512*b^(9/2)*Sqrt[Pi]*x^(3/2)
*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a] + 60*b*x^(1/3)*Sin[2*(a +
b*x^(1/3))] - 64*b^3*x*Sin[2*(a + b*x^(1/3))])/(315*x^(3/2))
```

fricas [A] time = 0.86, size = 154, normalized size = 0.68

$$-\frac{2\left(256\,\pi b^4x^2\sqrt{\frac{b}{\pi}}\,\cos\left(2\,a\right)\mathrm{S}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)+256\,\pi b^4x^2\sqrt{\frac{b}{\pi}}\,\mathrm{C}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin\left(2\,a\right)-128\,b^4x^{\frac{11}{6}}+24\,b^2x^{\frac{7}{6}}+\left(256\,b\right.\right.}{315\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/315*(256*pi*b^4*x^2*sqrt(b/pi)*cos(2*a)*fresnel_sin(2*x^(1/6)*sqrt(b/pi)
) + 256*pi*b^4*x^2*sqrt(b/pi)*fresnel_cos(2*x^(1/6)*sqrt(b/pi))*sin(2*a) -
128*b^4*x^(11/6) + 24*b^2*x^(7/6) + (256*b^4*x^(11/6) - 48*b^2*x^(7/6) + 10
5*sqrt(x))*cos(b*x^(1/3) + a)^2 + 4*(16*b^3*x^(3/2) - 15*b*x^(5/6))*cos(b*x
^(1/3) + a)*sin(b*x^(1/3) + a))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx^{\frac{1}{3}}+a\right)^2}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^(1/3) + a)^2/x^(5/2), x)
```

maple [A] time = 0.06, size = 146, normalized size = 0.64

$$-\frac{1}{3x^{\frac{3}{2}}}-\frac{\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{3x^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}}+\frac{4b\left(-\frac{\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{5x^{\frac{5}{6}}}-\frac{4b\left(-\frac{\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{3\sqrt{x}}+\frac{4b\left(-\frac{\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{6}}}-2\sqrt{b}\,\sqrt{\pi}\left(\cos(2a)\mathrm{S}\left(\frac{2x^{\frac{1}{6}}\sqrt{b}}{\sqrt{\pi}}\right)+\sin(2\right.\right.}{3}\right.}{5}\right.}{7}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*x^(1/3))^2/x^(5/2),x)
```

```
[Out] -1/3/x^(3/2)-1/3/x^(3/2)*cos(2*a+2*b*x^(1/3))-4/3*b*(-1/7/x^(7/6)*sin(2*a+2
*b*x^(1/3))+4/7*b*(-1/5/x^(5/6)*cos(2*a+2*b*x^(1/3))-4/5*b*(-1/3/x^(1/2)*si
n(2*a+2*b*x^(1/3))+4/3*b*(-1/x^(1/6)*cos(2*a+2*b*x^(1/3))-2*b^(1/2)*Pi^(1/2
)*(cos(2*a)*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))+sin(2*a)*FresnelC(2*x^(1/6
)*b^(1/2)/Pi^(1/2))))))
```

maxima [C] time = 1.71, size = 89, normalized size = 0.39

$$\frac{\sqrt{2}\left(\left(-(18i+18)\sqrt{2}\,\Gamma\left(-\frac{9}{2},2i\,bx^{\frac{1}{3}}\right)+(18i-18)\sqrt{2}\,\Gamma\left(-\frac{9}{2},-2i\,bx^{\frac{1}{3}}\right)\right)\cos(2a)+\left((18i-18)\sqrt{2}\,\Gamma\left(-\frac{9}{2},2i\,bx^{\frac{1}{3}}\right)\right.\right.}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(2)*((-(18*I + 18)*sqrt(2)*gamma(-9/2, 2*I*b*x^(1/3)) + (18*I - 18
)*sqrt(2)*gamma(-9/2, -2*I*b*x^(1/3)))*cos(2*a) + ((18*I - 18)*sqrt(2)*gamm
a(-9/2, 2*I*b*x^(1/3)) - (18*I + 18)*sqrt(2)*gamma(-9/2, -2*I*b*x^(1/3)))*s
in(2*a))*sqrt(b*x^(1/3))*b^4*x^(4/3) - 1)/x^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x^{1/3}\right)^2}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^(1/3))^2/x^(5/2),x)
```

```
[Out] int(cos(a + b*x^(1/3))^2/x^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x**(1/3))**2/x**(5/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))**2/x**(5/2), x)
```

3.60 $\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx$

Optimal. Leaf size=328

$$\frac{32768\sqrt{\pi}\,b^{15/2}\cos(2a)C\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{675675}-\frac{32768\sqrt{\pi}\,b^{15/2}\sin(2a)S\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{675675}-\frac{32768b^7\sin\left(a+b\sqrt[3]{x}\right)\cos\left(a+b\sqrt[3]{x}\right)}{675675\sqrt[6]{x}}+$$

```
[Out] -16/715*b^2/x^(11/6)+256/45045*b^4/x^(7/6)-2/5*cos(a+b*x^(1/3))^2/x^(5/2)+3
2/715*b^2*cos(a+b*x^(1/3))^2/x^(11/6)-512/45045*b^4*cos(a+b*x^(1/3))^2/x^(7
/6)+8/65*b*cos(a+b*x^(1/3))*sin(a+b*x^(1/3))/x^(13/6)-128/6435*b^3*cos(a+b*
x^(1/3))*sin(a+b*x^(1/3))/x^(3/2)+2048/225225*b^5*cos(a+b*x^(1/3))*sin(a+b*
x^(1/3))/x^(5/6)-32768/675675*b^7*cos(a+b*x^(1/3))*sin(a+b*x^(1/3))/x^(1/6)
+32768/675675*b^(15/2)*cos(2*a)*FresnelC(2*x^(1/6)*b^(1/2)/Pi^(1/2))*Pi^(1/
2)-32768/675675*b^(15/2)*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))*sin(2*a)*Pi^(
1/2)-4096/675675*b^6/x^(1/2)+8192/675675*b^6*cos(a+b*x^(1/3))^2/x^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3416, 3314, 30, 3312, 3306, 3305, 3351, 3304, 3352}

$$\frac{32768\sqrt{\pi}\,b^{15/2}\cos(2a)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{675675}-\frac{32768\sqrt{\pi}\,b^{15/2}\sin(2a)S\left(\frac{2\sqrt{b}\sqrt[6]{x}}{\sqrt{\pi}}\right)}{675675}-\frac{512b^4\cos^2\left(a+b\sqrt[3]{x}\right)}{45045x^{7/6}}+\frac{32b^2\cos^2}{715}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^(1/3)]^2/x^(7/2),x]
```

```
[Out] (-16*b^2)/(715*x^(11/6)) + (256*b^4)/(45045*x^(7/6)) - (4096*b^6)/(675675*S
qrt[x]) - (2*Cos[a + b*x^(1/3)]^2)/(5*x^(5/2)) + (32*b^2*Cos[a + b*x^(1/3)]
^2)/(715*x^(11/6)) - (512*b^4*Cos[a + b*x^(1/3)]^2)/(45045*x^(7/6)) + (8192
*b^6*Cos[a + b*x^(1/3)]^2)/(675675*Sqrt[x]) + (32768*b^(15/2)*Sqrt[Pi]*Cos[
2*a]*FresnelC[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]])/675675 - (32768*b^(15/2)*Sqrt[
Pi]*FresnelS[(2*Sqrt[b]*x^(1/6))/Sqrt[Pi]]*Sin[2*a])/675675 + (8*b*Cos[a +
b*x^(1/3)]*Sin[a + b*x^(1/3)])/(65*x^(13/6)) - (128*b^3*Cos[a + b*x^(1/3)]*
Sin[a + b*x^(1/3)])/(6435*x^(3/2)) + (2048*b^5*Cos[a + b*x^(1/3)]*Sin[a + b
*x^(1/3)])/(225225*x^(5/6)) - (32768*b^7*Cos[a + b*x^(1/3)]*Sin[a + b*x^(1/
3)])/(675675*x^(1/6))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3416

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a +
b*Cos[c + d*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d, m}, x]
&& IntegerQ[p] && FractionQ[n]
```

Rubi steps

$$\int \frac{\cos^2\left(a+b\sqrt[3]{x}\right)}{x^{7/2}}\,dx = 3\,\text{Subst}\left(\int \frac{\cos^2(a+bx)}{x^{17/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x}\right)}{65x^{13/6}} + \frac{1}{65}\left(8b^2\right)\text{Subst}\left(\int \frac{1}{x^{13/2}}\,dx, x \cdots\right.$$

$$= -\frac{16b^2}{715x^{11/6}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} + \frac{8b\cos\left(a+b\sqrt[3]{x}\right)\sin\left(a+b\sqrt[3]{x} \cdots\right.}{65x^{13/6}}$$

$$= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \frac{512b^4\cos^2\left(a+b \cdots\right.}{45045x^{7/6}}$$

$$= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \cdots$$

$$= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{65536b^8\sqrt[6]{x}}{675675} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \cdots$$

$$= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \cdots$$

$$= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \cdots$$

$$= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \cdots$$

$$= -\frac{16b^2}{715x^{11/6}} + \frac{256b^4}{45045x^{7/6}} - \frac{4096b^6}{675675\sqrt{x}} - \frac{2\cos^2\left(a+b\sqrt[3]{x}\right)}{5x^{5/2}} + \frac{32b^2\cos^2\left(a+b\sqrt[3]{x}\right)}{715x^{11/6}} - \cdots$$

Mathematica [A] time = 0.38, size = 249, normalized size = 0.76

$$\frac{32768\sqrt{\pi}\, b^{15/2}x^{5/2}\cos(2a)C\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)-32768\sqrt{\pi}\, b^{15/2}x^{5/2}\sin(2a)S\left(\frac{2\sqrt{b}\,\sqrt[6]{x}}{\sqrt{\pi}}\right)-16384b^7x^{7/3}\sin\left(2\left(a+b\sqrt[3]{x}\right)\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^(1/3)]^2/x^(7/2),x]
```

```
[Out] (-135135 - 135135*Cos[2*(a + b*x^(1/3))] + 15120*b^2*x^(2/3)*Cos[2*(a + b*x
^(1/3))] - 3840*b^4*x^(4/3)*Cos[2*(a + b*x^(1/3))] + 4096*b^6*x^2*Cos[2*(a
+ b*x^(1/3))] + 32768*b^(15/2)*Sqrt[Pi]*x^(5/2)*Cos[2*a]*FresnelC[(2*Sqrt[b
]*x^(1/6))/Sqrt[Pi]] - 32768*b^(15/2)*Sqrt[Pi]*x^(5/2)*FresnelS[(2*Sqrt[b]*
x^(1/6))/Sqrt[Pi]]*Sin[2*a] + 41580*b*x^(1/3)*Sin[2*(a + b*x^(1/3))] - 6720
*b^3*x*Sin[2*(a + b*x^(1/3))] + 3072*b^5*x^(5/3)*Sin[2*(a + b*x^(1/3))] - 1
6384*b^7*x^(7/3)*Sin[2*(a + b*x^(1/3))])/(675675*x^(5/2))
```

fricas [A] time = 1.24, size = 192, normalized size = 0.59

$$\frac{2\left(16384\,\pi b^7x^3\sqrt{\frac{b}{\pi}}\,\cos(2\,a)\,\mathrm{C}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)-16384\,\pi b^7x^3\sqrt{\frac{b}{\pi}}\,\mathrm{S}\left(2\,x^{\frac{1}{6}}\sqrt{\frac{b}{\pi}}\right)\sin(2\,a)-2048\,b^6x^{\frac{5}{2}}+1920\,b^4x^{\frac{11}{6}}-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/675675*(16384*pi*b^7*x^3*sqrt(b/pi)*cos(2*a)*fresnel_cos(2*x^(1/6)*sqrt(b
/pi)) - 16384*pi*b^7*x^3*sqrt(b/pi)*fresnel_sin(2*x^(1/6)*sqrt(b/pi))*sin(2
*a) - 2048*b^6*x^(5/2) + 1920*b^4*x^(11/6) - 7560*b^2*x^(7/6) - (3840*b^4*x
^(11/6) - 15120*b^2*x^(7/6) - (4096*b^6*x^2 - 135135)*sqrt(x))*cos(b*x^(1/3
) + a)^2 + 4*(768*b^5*x^(13/6) - 1680*b^3*x^(3/2) - (4096*b^7*x^2 - 10395*b
)*x^(5/6))*cos(b*x^(1/3) + a)*sin(b*x^(1/3) + a))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx^{\frac{1}{3}}+a\right)^2}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^(1/3) + a)^2/x^(7/2), x)
```

maple [A] time = 0.06, size = 207, normalized size = 0.63

$$4b\left(-\frac{\cos\left(2a+2\cdots\right.}{3\sqrt{\cdots}}\right.$$

$$4b\left(-\frac{\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{5x^{\frac{5}{6}}}+\cdots\right.$$

$$4b\left(-\frac{\cos\left(2a+2b\,x^{\frac{1}{3}}\right)}{7x^{\frac{7}{6}}}-\cdots\right.$$

$$4b\left(-\frac{\sin\left(2a+2b\,x^{\frac{1}{3}}\right)}{9x^{\frac{3}{2}}}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*x^(1/3))^2/x^(7/2),x)
```

```
[Out] -1/5/x^(5/2)-1/5/x^(5/2)*cos(2*a+2*b*x^(1/3))-4/5*b*(-1/13/x^(13/6)*sin(2*a
+2*b*x^(1/3))+4/13*b*(-1/11/x^(11/6)*cos(2*a+2*b*x^(1/3))-4/11*b*(-1/9/x^(3
/2)*sin(2*a+2*b*x^(1/3))+4/9*b*(-1/7/x^(7/6)*cos(2*a+2*b*x^(1/3))-4/7*b*(-1
/5/x^(5/6)*sin(2*a+2*b*x^(1/3))+4/5*b*(-1/3/x^(1/2)*cos(2*a+2*b*x^(1/3))-4/
3*b*(-1/x^(1/6)*sin(2*a+2*b*x^(1/3))+2*b^(1/2)*Pi^(1/2)*(cos(2*a)*FresnelC(
2*x^(1/6)*b^(1/2)/Pi^(1/2))-sin(2*a)*FresnelS(2*x^(1/6)*b^(1/2)/Pi^(1/2))))
))))))
```

maxima [C] time = 1.83, size = 89, normalized size = 0.27

$$\frac{\sqrt{2}\left(\left((240i-240)\ \sqrt{2}\,\Gamma\left(-\frac{15}{2},2i\,bx^{\frac{1}{3}}\right)-(240i+240)\ \sqrt{2}\,\Gamma\left(-\frac{15}{2},-2i\,bx^{\frac{1}{3}}\right)\right)\cos\left(2\,a\right)+\left((240i+240)\ \sqrt{2}\,\Gamma\left(-\frac{15}{2}\right.\right.\right.}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^(1/3))^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*(sqrt(2)*(((240*I - 240)*sqrt(2)*gamma(-15/2, 2*I*b*x^(1/3)) - (240*I +
 240)*sqrt(2)*gamma(-15/2, -2*I*b*x^(1/3)))*cos(2*a) + ((240*I + 240)*sqrt(
2)*gamma(-15/2, 2*I*b*x^(1/3)) - (240*I - 240)*sqrt(2)*gamma(-15/2, -2*I*b*
x^(1/3)))*sin(2*a))*sqrt(b*x^(1/3))*b^7*x^(7/3) - 1)/x^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x^{1/3}\right)^2}{x^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^(1/3))^2/x^(7/2),x)
```

```
[Out] int(cos(a + b*x^(1/3))^2/x^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(a+b\sqrt[3]{x}\right)}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x**(1/3))**2/x**(7/2),x)
```

```
[Out] Integral(cos(a + b*x**(1/3))**2/x**(7/2), x)
```

3.61 $\int \cos^3\left(\sqrt[3]{x}\right)\,dx$

Optimal. Leaf size=86

$$2x^{2/3}\sin\left(\sqrt[3]{x}\right)+x^{2/3}\sin\left(\sqrt[3]{x}\right)\cos^2\left(\sqrt[3]{x}\right)+\frac{2}{9}\sin^3\left(\sqrt[3]{x}\right)-\frac{14}{3}\sin\left(\sqrt[3]{x}\right)+\frac{2}{3}\sqrt[3]{x}\cos^3\left(\sqrt[3]{x}\right)+4\sqrt[3]{x}\cos\left(\sqrt[3]{x}\right)$$

```
[Out] 4*x^(1/3)*cos(x^(1/3))+2/3*x^(1/3)*cos(x^(1/3))^3-14/3*sin(x^(1/3))+2*x^(2/
3)*sin(x^(1/3))+x^(2/3)*cos(x^(1/3))^2*sin(x^(1/3))+2/9*sin(x^(1/3))^3
```

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3362, 3311, 3296, 2637, 2633}

$$2x^{2/3}\sin\left(\sqrt[3]{x}\right)+x^{2/3}\sin\left(\sqrt[3]{x}\right)\cos^2\left(\sqrt[3]{x}\right)+\frac{2}{9}\sin^3\left(\sqrt[3]{x}\right)-\frac{14}{3}\sin\left(\sqrt[3]{x}\right)+\frac{2}{3}\sqrt[3]{x}\cos^3\left(\sqrt[3]{x}\right)+4\sqrt[3]{x}\cos\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x^(1/3)]^3,x]
```

```
[Out] 4*x^(1/3)*Cos[x^(1/3)] + (2*x^(1/3)*Cos[x^(1/3)]^3)/3 - (14*Sin[x^(1/3)])/3
 + 2*x^(2/3)*Sin[x^(1/3)] + x^(2/3)*Cos[x^(1/3)]^2*Sin[x^(1/3)] + (2*Sin[x^
(1/3)]^3)/9
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
```

```
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3362

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\int \cos^3\left(\sqrt[3]{x}\right) dx = 3\,\text{Subst}\left(\int x^2\cos^3(x)\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{2}{3}\sqrt[3]{x}\cos^3\left(\sqrt[3]{x}\right) + x^{2/3}\cos^2\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right) - \frac{2}{3}\,\text{Subst}\left(\int \cos^3(x)\,dx, x, \sqrt[3]{x}\right) + 2\,\text{Subst}\left(\int\right.$$
$$= \frac{2}{3}\sqrt[3]{x}\cos^3\left(\sqrt[3]{x}\right) + 2x^{2/3}\sin\left(\sqrt[3]{x}\right) + x^{2/3}\cos^2\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right) + \frac{2}{3}\,\text{Subst}\left(\int \left(1-x^2\right) dx, x, -\right.$$
$$= 4\sqrt[3]{x}\cos\left(\sqrt[3]{x}\right) + \frac{2}{3}\sqrt[3]{x}\cos^3\left(\sqrt[3]{x}\right) - \frac{2}{3}\sin\left(\sqrt[3]{x}\right) + 2x^{2/3}\sin\left(\sqrt[3]{x}\right) + x^{2/3}\cos^2\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{x}\right.$$
$$= 4\sqrt[3]{x}\cos\left(\sqrt[3]{x}\right) + \frac{2}{3}\sqrt[3]{x}\cos^3\left(\sqrt[3]{x}\right) - \frac{14}{3}\sin\left(\sqrt[3]{x}\right) + 2x^{2/3}\sin\left(\sqrt[3]{x}\right) + x^{2/3}\cos^2\left(\sqrt[3]{x}\right)\sin\left(\sqrt[3]{\right.$$

Mathematica [A] time = 0.10, size = 66, normalized size = 0.77

$$\frac{1}{36}\left(81\left(x^{2/3}-2\right)\sin\left(\sqrt[3]{x}\right) + \left(9x^{2/3}-2\right)\sin\left(3\sqrt[3]{x}\right) + 162\sqrt[3]{x}\cos\left(\sqrt[3]{x}\right) + 6\sqrt[3]{x}\cos\left(3\sqrt[3]{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x^(1/3)]^3,x]
```

```
[Out] (162*x^(1/3)*Cos[x^(1/3)] + 6*x^(1/3)*Cos[3*x^(1/3)] + 81*(-2 + x^(2/3))*Si
n[x^(1/3)] + (-2 + 9*x^(2/3))*Sin[3*x^(1/3)])/36
```

fricas [A] time = 1.04, size = 48, normalized size = 0.56

$$\frac{2}{3}\,x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)^3 + \frac{1}{9}\left(\left(9\,x^{\frac{2}{3}}-2\right)\cos\left(x^{\frac{1}{3}}\right)^2 + 18\,x^{\frac{2}{3}} - 40\right)\sin\left(x^{\frac{1}{3}}\right) + 4\,x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/3))^3,x, algorithm="fricas")
```

```
[Out] 2/3*x^(1/3)*cos(x^(1/3))^3 + 1/9*((9*x^(2/3) - 2)*cos(x^(1/3))^2 + 18*x^(2/
3) - 40)*sin(x^(1/3)) + 4*x^(1/3)*cos(x^(1/3))
```

giac [A] time = 0.39, size = 47, normalized size = 0.55

$$\frac{1}{36}\left(9\,x^{\frac{2}{3}}-2\right)\sin\left(3\,x^{\frac{1}{3}}\right)+\frac{9}{4}\left(x^{\frac{2}{3}}-2\right)\sin\left(x^{\frac{1}{3}}\right)+\frac{1}{6}\,x^{\frac{1}{3}}\cos\left(3\,x^{\frac{1}{3}}\right)+\frac{9}{2}\,x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/3))^3,x, algorithm="giac")
```

```
[Out] 1/36*(9*x^(2/3) - 2)*sin(3*x^(1/3)) + 9/4*(x^(2/3) - 2)*sin(x^(1/3)) + 1/6*
x^(1/3)*cos(3*x^(1/3)) + 9/2*x^(1/3)*cos(x^(1/3))
```

maple [A] time = 0.04, size = 58, normalized size = 0.67

$$x^{\frac{2}{3}}\left(2+\cos^{2}\left(x^{\frac{1}{3}}\right)\right)\sin\left(x^{\frac{1}{3}}\right)-4\sin\left(x^{\frac{1}{3}}\right)+4x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)+\frac{2x^{\frac{1}{3}}\left(\cos^{3}\left(x^{\frac{1}{3}}\right)\right)}{3}-\frac{2\left(2+\cos^{2}\left(x^{\frac{1}{3}}\right)\right)\sin\left(x^{\frac{1}{3}}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/3))^3,x)
```

```
[Out] x^(2/3)*(2+cos(x^(1/3))^2)*sin(x^(1/3))-4*sin(x^(1/3))+4*x^(1/3)*cos(x^(1/3
))+2/3*x^(1/3)*cos(x^(1/3))^3-2/9*(2+cos(x^(1/3))^2)*sin(x^(1/3))
```

maxima [A] time = 0.85, size = 47, normalized size = 0.55

$$\frac{1}{36}\left(9\,x^{\frac{2}{3}}-2\right)\sin\left(3\,x^{\frac{1}{3}}\right)+\frac{9}{4}\left(x^{\frac{2}{3}}-2\right)\sin\left(x^{\frac{1}{3}}\right)+\frac{1}{6}\,x^{\frac{1}{3}}\cos\left(3\,x^{\frac{1}{3}}\right)+\frac{9}{2}\,x^{\frac{1}{3}}\cos\left(x^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/3))^3,x, algorithm="maxima")
```

```
[Out] 1/36*(9*x^(2/3) - 2)*sin(3*x^(1/3)) + 9/4*(x^(2/3) - 2)*sin(x^(1/3)) + 1/6*
x^(1/3)*cos(3*x^(1/3)) + 9/2*x^(1/3)*cos(x^(1/3))
```

mupad [B] time = 0.50, size = 62, normalized size = 0.72

$$4\,x^{1/3}\cos\left(x^{1/3}\right)-\frac{2\cos\left(x^{1/3}\right)^{2}\sin\left(x^{1/3}\right)}{9}-\frac{40\sin\left(x^{1/3}\right)}{9}+2\,x^{2/3}\sin\left(x^{1/3}\right)+\frac{2\,x^{1/3}\cos\left(x^{1/3}\right)^{3}}{3}+x^{2/3}\cos\left(x^{1/3}\right)^{2}\sin\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/3))^3,x)
```

```
[Out] 4*x^(1/3)*cos(x^(1/3)) - (2*cos(x^(1/3))^2*sin(x^(1/3)))/9 - (40*sin(x^(1/3
)))/9 + 2*x^(2/3)*sin(x^(1/3)) + (2*x^(1/3)*cos(x^(1/3))^3)/3 + x^(2/3)*cos
(x^(1/3))^2*sin(x^(1/3))
```

sympy [B] time = 2.61, size = 513, normalized size = 5.97

$$\frac{54x^{\frac{2}{3}}\tan^5\left(\frac{\sqrt[3]{x}}{2}\right)}{9\tan^6\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+9}+\frac{36x^{\frac{2}{3}}\tan^3\left(\frac{\sqrt[3]{x}}{2}\right)}{9\tan^6\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^4\left(\frac{\sqrt[3]{x}}{2}\right)+27\tan^2\left(\frac{\sqrt[3]{x}}{2}\right)+9}+\frac{}{9\tan^6\left(\frac{\sqrt[3]{x}}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/3))**3,x)
```

```
[Out] 54*x**(2/3)*tan(x**(1/3)/2)**5/(9*tan(x**(1/3)/2)**6 + 27*tan(x**(1/3)/2)**
4 + 27*tan(x**(1/3)/2)**2 + 9) + 36*x**(2/3)*tan(x**(1/3)/2)**3/(9*tan(x**(
1/3)/2)**6 + 27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2 + 9) + 54*x**(2/
3)*tan(x**(1/3)/2)/(9*tan(x**(1/3)/2)**6 + 27*tan(x**(1/3)/2)**4 + 27*tan(x
**(1/3)/2)**2 + 9) - 42*x**(1/3)*tan(x**(1/3)/2)**6/(9*tan(x**(1/3)/2)**6 +
 27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2 + 9) - 18*x**(1/3)*tan(x**(1
/3)/2)**4/(9*tan(x**(1/3)/2)**6 + 27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2
)**2 + 9) + 18*x**(1/3)*tan(x**(1/3)/2)**2/(9*tan(x**(1/3)/2)**6 + 27*tan(x
**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2 + 9) + 42*x**(1/3)/(9*tan(x**(1/3)/2)
**6 + 27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2 + 9) - 84*tan(x**(1/3)/
2)**5/(9*tan(x**(1/3)/2)**6 + 27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2
 + 9) - 152*tan(x**(1/3)/2)**3/(9*tan(x**(1/3)/2)**6 + 27*tan(x**(1/3)/2)**
4 + 27*tan(x**(1/3)/2)**2 + 9) - 84*tan(x**(1/3)/2)/(9*tan(x**(1/3)/2)**6 +
 27*tan(x**(1/3)/2)**4 + 27*tan(x**(1/3)/2)**2 + 9)
```

3.62 $\int \frac{\cos\left(\sqrt[6]{x}\right)}{x^{5/6}} \, dx$

Optimal. Leaf size=8

$$6 \sin\left(\sqrt[6]{x}\right)$$

```
[Out] 6*sin(x^(1/6))
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3380, 2637}

$$6 \sin\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x^(1/6)]/x^(5/6),x]
```

```
[Out] 6*Sin[x^(1/6)]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{\cos\left(\sqrt[6]{x}\right)}{x^{5/6}} \, dx &= 6 \operatorname{Subst}\left(\int \cos(x) \, dx, x, \sqrt[6]{x}\right) \\ &= 6 \sin\left(\sqrt[6]{x}\right) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$6 \sin\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x^(1/6)]/x^(5/6),x]
[Out] 6*Sin[x^(1/6)]
```

fricas [A] time = 0.98, size = 6, normalized size = 0.75

$$6 \sin\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/6))/x^(5/6),x, algorithm="fricas")
[Out] 6*sin(x^(1/6))
```

giac [A] time = 0.36, size = 6, normalized size = 0.75

$$6 \sin\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/6))/x^(5/6),x, algorithm="giac")
[Out] 6*sin(x^(1/6))
```

maple [A] time = 0.02, size = 7, normalized size = 0.88

$$6 \sin\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/6))/x^(5/6),x)
[Out] 6*sin(x^(1/6))
```

maxima [A] time = 0.98, size = 6, normalized size = 0.75

$$6 \sin\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/6))/x^(5/6),x, algorithm="maxima")
[Out] 6*sin(x^(1/6))
```

mupad [B] time = 0.43, size = 6, normalized size = 0.75

$$6 \sin\left(x^{1/6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/6))/x^(5/6),x)
```

```
[Out] 6*sin(x^(1/6))
```

sympy [A] time = 75.63, size = 7, normalized size = 0.88

$$6 \sin\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/6))/x**(5/6),x)
```

```
[Out] 6*sin(x**(1/6))
```

3.63 $\int (ex)^m (b\cos (c+dx^n))^p \, dx$

Optimal. Leaf size=21

$$\text{Int}\left((ex)^m (b\cos (c+dx^n))^p ,x\right)$$

```
[Out] Unintegrable((e*x)^m*(b*cos(c+d*x^n))^p,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^m (b\cos (c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*(b*Cos[c + d*x^n])^p,x]
[Out] Defer[Int][(e*x)^m*(b*Cos[c + d*x^n])^p, x]
```

Rubi steps

$$\int (ex)^m (b\cos (c+dx^n))^p \, dx=\int (ex)^m (b\cos (c+dx^n))^p \, dx$$

Mathematica [A] time = 1.01, size = 0, normalized size = 0.00

$$\int (ex)^m (b\cos (c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*(b*Cos[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^m*(b*Cos[c + d*x^n])^p, x]
```

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m (b\cos (dx^n+c))^p ,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*cos(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*(b*cos(d*x^n + c))^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (b\cos (dx^n + c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*cos(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*cos(d*x^n + c))^p, x)
```

maple [A] time = 0.98, size = 0, normalized size = 0.00

$$\int (ex)^m (b\cos (c + d\,x^n))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*cos(c+d*x^n))^p,x)
[Out] int((e*x)^m*(b*cos(c+d*x^n))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (b\cos (dx^n + c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*cos(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*cos(d*x^n + c))^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (e\,x)^m \,(b\ \cos (c + d\,x^n))^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*cos(c + d*x^n))^p,x)
[Out] int((e*x)^m*(b*cos(c + d*x^n))^p, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos (c + dx^n))^p (ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*cos(c+d*x**n))**p,x)
[Out] Integral((b*cos(c + d*x**n))**p*(e*x)**m, x)
```

3.64 $\int (ex)^m (a + b\cos(c + dx^n))^p\, dx$

Optimal. Leaf size=23

$$\text{Int}\left((ex)^m (a + b\cos(c + dx^n))^p\,, x\right)$$

```
[Out] Unintegrable((e*x)^m*(a+b*cos(c+d*x^n))^p,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int (ex)^m (a + b\cos(c + dx^n))^p\, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^m*(a + b*Cos[c + d*x^n])^p,x]
```

```
[Out] Defer[Int][(e*x)^m*(a + b*Cos[c + d*x^n])^p, x]
```

Rubi steps

$$\int (ex)^m (a + b\cos(c + dx^n))^p\, dx = \int (ex)^m (a + b\cos(c + dx^n))^p\, dx$$

Mathematica [A] time = 1.29, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b\cos(c + dx^n))^p\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^m*(a + b*Cos[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^m*(a + b*Cos[c + d*x^n])^p, x]
```

fricas [A] time = 1.54, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m (b\cos(dx^n + c) + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*cos(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*(b*cos(d*x^n + c) + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (b\cos(dx^n + c) + a)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*cos(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*cos(d*x^n + c) + a)^p, x)
```

maple [A] time = 0.90, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b\cos(c + d\,x^n))^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a+b*cos(c+d*x^n))^p,x)
[Out] int((e*x)^m*(a+b*cos(c+d*x^n))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (b\cos(dx^n + c) + a)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*cos(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*cos(d*x^n + c) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (e\,x)^m\, (a + b\,\cos(c + d\,x^n))^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b*cos(c + d*x^n))^p,x)
[Out] int((e*x)^m*(a + b*cos(c + d*x^n))^p, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m (a + b\cos(c + dx^n))^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*cos(c+d*x**n))**p,x)
[Out] Integral((e*x)**m*(a + b*cos(c + d*x**n))**p, x)
```

3.65 $\int (ex)^{-1+n} (b\cos(c+dx^n))^p \, dx$

Optimal. Leaf size=93

$$-\frac{x^{-n}(ex)^n \sin(c+dx^n)(b\cos(c+dx^n))^{p+1} {}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(dx^n+c)\right)}{bden(p+1)\sqrt{\sin^2(c+dx^n)}}$$

```
[Out] -(e*x)^n*(b*cos(c+d*x^n))^(1+p)*hypergeom([1/2, 1/2+1/2*p],[3/2+1/2*p],cos(
c+d*x^n)^2)*sin(c+d*x^n)/b/d/e/n/(1+p)/(x^n)/(sin(c+d*x^n)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3382, 3380, 2643}

$$-\frac{x^{-n}(ex)^n \sin(c+dx^n)(b\cos(c+dx^n))^{p+1} {}_2F_1\left(\frac{1}{2},\frac{p+1}{2};\frac{p+3}{2};\cos^2(dx^n+c)\right)}{bden(p+1)\sqrt{\sin^2(c+dx^n)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(-1 + n)*(b*Cos[c + d*x^n])^p,x]
```

```
[Out] -(((e*x)^n*(b*Cos[c + d*x^n])^(1 + p)*Hypergeometric2F1[1/2, (1 + p)/2, (3
+ p)/2, Cos[c + d*x^n]^2]*Sin[c + d*x^n])/(b*d*e*n*(1 + p)*x^n*Sqrt[Sin[c +
 d*x^n]^2]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3382

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
```

```
+ b*Cos[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int (ex)^{-1+n} (b\cos (c+dx^n))^p \, dx = \frac{(x^{-n}(ex)^n) \int x^{-1+n} (b\cos (c+dx^n))^p \, dx}{e}$$
$$= \frac{(x^{-n}(ex)^n)\,\mathrm{Subst}\left(\int (b\cos(c+dx))^p \, dx, x, x^n\right)}{en}$$
$$= -\frac{x^{-n}(ex)^n (b\cos (c+dx^n))^{1+p} \, {}_2F_1\left(\frac{1}{2}, \frac{1+p}{2}; \frac{3+p}{2}; \cos^2 (c+dx^n)\right) \sin (c+dx^n)}{bden(1+p)\sqrt{\sin^2 (c+dx^n)}}$$

Mathematica [A] time = 0.17, size = 89, normalized size = 0.96

$$-\frac{x^{1-n}(ex)^{n-1}\sqrt{\sin^2 (c+dx^n)} \cot (c+dx^n) (b\cos (c+dx^n))^p \, {}_2F_1\left(\frac{1}{2}, \frac{p+1}{2}; \frac{p+3}{2}; \cos^2 (dx^n+c)\right)}{dn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(-1 + n)*(b*Cos[c + d*x^n])^p,x]
```

```
[Out] -((x^(1 - n)*(e*x)^(-1 + n)*(b*Cos[c + d*x^n])^p*Cot[c + d*x^n]*Hypergeomet
ric2F1[1/2, (1 + p)/2, (3 + p)/2, Cos[c + d*x^n]^2]*Sqrt[Sin[c + d*x^n]^2])
/(d*n*(1 + p)))
```

fricas [F] time = 1.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((ex)^{n-1} (b\cos (dx^n+c))^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*cos(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^(n - 1)*(b*cos(d*x^n + c))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{n-1} (b\cos (dx^n+c))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*cos(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(n - 1)*(b*cos(d*x^n + c))^p, x)
```

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int (ex)^{-1+n} (b\cos(c+d\,x^n))^p\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+n)*(b*cos(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+n)*(b*cos(c+d*x^n))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{n-1} (b\cos(dx^n+c))^p\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(b*cos(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(n - 1)*(b*cos(d*x^n + c))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^{n-1}\,(b\,\cos(c+d\,x^n))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(n - 1)*(b*cos(c + d*x^n))^p,x)
[Out] int((e*x)^(n - 1)*(b*cos(c + d*x^n))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx^n))^p\,(ex)^{n-1}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+n)*(b*cos(c+d*x**n))**p,x)
[Out] Integral((b*cos(c + d*x**n))**p*(e*x)**(n - 1), x)
```

3.66 $\int (ex)^{-1+2n} (b\cos(c+dx^n))^p \, dx$

Optimal. Leaf size=39

$$\frac{x^{-2n}(ex)^{2n}\text{Int}\left(x^{2n-1}\left(b\cos\left(c+dx^n\right)\right)^p,x\right)}{e}$$

```
[Out] (e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(b*cos(c+d*x^n))^p,x)/e/(x^(2*n))
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^{-1+2n} (b\cos(c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^(-1 + 2*n)*(b*Cos[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(b*Cos[c + d*x^n])^p, x])/(e*x^(2*n))
```

Rubi steps

$$\int (ex)^{-1+2n} (b\cos(c+dx^n))^p \, dx = \frac{\left(x^{-2n}(ex)^{2n}\right)\int x^{-1+2n} (b\cos(c+dx^n))^p \, dx}{e}$$

Mathematica [A] time = 0.93, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (b\cos(c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^(-1 + 2*n)*(b*Cos[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^(-1 + 2*n)*(b*Cos[c + d*x^n])^p, x]
```

fricas [A] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex\right)^{2n-1}\left(b\cos\left(dx^n+c\right)\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*cos(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^(2*n - 1)*(b*cos(d*x^n + c))^p, x)
```

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{2\,n-1} \left(b\cos\left(dx^n + c\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*cos(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)*(b*cos(d*x^n + c))^p, x)
```

maple [**A**] time = 0.91, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} \left(b\cos\left(c + d\,x^n\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+2*n)*(b*cos(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+2*n)*(b*cos(c+d*x^n))^p,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{2\,n-1} \left(b\cos\left(dx^n + c\right)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(b*cos(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(2*n - 1)*(b*cos(d*x^n + c))^p, x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.03

$$\int (e\,x)^{2\,n-1}\,{\left(b\,\cos\left(c+d\,x^n\right)\right)}^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(2*n - 1)*(b*cos(c + d*x^n))^p,x)
[Out] int((e*x)^(2*n - 1)*(b*cos(c + d*x^n))^p, x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(c + dx^{n}\right)\right)^{p} \left(ex\right)^{2n - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+2*n)*(b*cos(c+d*x**n))**p,x)
```

```
[Out] Integral((b*cos(c + d*x**n))**p*(e*x)**(2*n - 1), x)
```

3.67 $\int (ex)^{-1+n} (a + b\cos(c + dx^n))^p \, dx$

Optimal. Leaf size=131

$$\frac{\sqrt{2}\, x^{-n}(ex)^n \sin(c+dx^n)\,(a+b\cos(c+dx^n))^p \left(\frac{a+b\cos(c+dx^n)}{a+b}\right)^{-p} F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}(1-\cos(dx^n+c)),\frac{b(1-\cos(dx^n+}{a+b}\right.}{den\sqrt{\cos(c+dx^n)+1}}$$

```
[Out] (e*x)^n*AppellF1(1/2,-p,1/2,3/2,b*(1-cos(c+d*x^n))/(a+b),1/2-1/2*cos(c+d*x^
n))*(a+b*cos(c+d*x^n))^p*sin(c+d*x^n)*2^(1/2)/d/e/n/(x^n)/(((a+b*cos(c+d*x^
n))/(a+b))^p)/(1+cos(c+d*x^n))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3382, 3380, 2665, 139, 138}

$$\frac{\sqrt{2}\, x^{-n}(ex)^n \sin(c+dx^n)\,(a+b\cos(c+dx^n))^p \left(\frac{a+b\cos(c+dx^n)}{a+b}\right)^{-p} F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}(1-\cos(dx^n+c)),\frac{b(1-\cos(dx^n+}{a+b}\right.}{den\sqrt{\cos(c+dx^n)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(-1 + n)*(a + b*Cos[c + d*x^n])^p,x]
```

```
[Out] (Sqrt[2]*(e*x)^n*AppellF1[1/2, 1/2, -p, 3/2, (1 - Cos[c + d*x^n])/2, (b*(1
- Cos[c + d*x^n]))/(a + b)]*(a + b*Cos[c + d*x^n])^p*Sin[c + d*x^n])/(d*e*n
*x^n*Sqrt[1 + Cos[c + d*x^n]]*((a + b*Cos[c + d*x^n])/(a + b))^p)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3382

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_), x_
Symbol] :> Dist[(e^IntPart[m]*(e*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a
+ b*Cos[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && Inte
gerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int (ex)^{-1+n}\,(a+b\cos(c+dx^n))^p\,dx = \frac{(x^{-n}(ex)^n)\int x^{-1+n}\,(a+b\cos(c+dx^n))^p\,dx}{e}$$

$$= \frac{(x^{-n}(ex)^n)\,\mathrm{Subst}\left(\int (a+b\cos(c+dx))^p\,dx, x, x^n\right)}{en}$$

$$= -\frac{(x^{-n}(ex)^n\sin(c+dx^n))\,\mathrm{Subst}\left(\int \frac{(a+bx)^p}{\sqrt{1-x}\,\sqrt{1+x}}\,dx, x, \cos(c+dx^n)\right)}{den\sqrt{1-\cos(c+dx^n)}\,\sqrt{1+\cos(c+dx^n)}}$$

$$= -\frac{\left(x^{-n}(ex)^n\,(a+b\cos(c+dx^n))^p\left(-\frac{a+b\cos(c+dx^n)}{-a-b}\right)^{-p}\sin(c+dx^n)\right)\mathrm{Subst}\left(\int \cdots\right.}{den\sqrt{1-\cos(c+dx^n)}\,\sqrt{1+\cos(c+dx^n)}}$$

$$= \frac{\sqrt{2}\,x^{-n}(ex)^n F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\frac{1}{2}(1-\cos(c+dx^n)),\frac{b(1-\cos(c+dx^n))}{a+b}\right)(a+b\cos \cdots}{den\sqrt{1+\cos(c+dx^n)}}$$

Mathematica [A] time = 0.43, size = 149, normalized size = 1.14

$$-\frac{x^{-n}(ex)^n \csc\left(c+dx^n\right) \sqrt{-\frac{b(\cos(c+dx^n)-1)}{a+b}} \sqrt{\frac{b(\cos(c+dx^n)+1)}{b-a}} \left(a+b\cos\left(c+dx^n\right)\right)^{p+1} F_1\left(p+1;\frac{1}{2},\frac{1}{2};p+2;\frac{a+b\cos(dx^n}{a-b}\right.}{bden(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Cos[c + d*x^n])^p,x]
```

```
[Out] -(((e*x)^n*AppellF1[1 + p, 1/2, 1/2, 2 + p, (a + b*Cos[c + d*x^n])/(a - b),
 (a + b*Cos[c + d*x^n])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x^n]))/(a + b))]
*Sqrt[(b*(1 + Cos[c + d*x^n]))/(-a + b)]*(a + b*Cos[c + d*x^n])^(1 + p)*Csc
[c + d*x^n])/(b*d*e*n*(1 + p)*x^n))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ex\right)^{n-1}\left(b\cos\left(dx^n+c\right)+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*cos(c+d*x^n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^(n - 1)*(b*cos(d*x^n + c) + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex\right)^{n-1}\left(b\cos\left(dx^n+c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*cos(c+d*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(n - 1)*(b*cos(d*x^n + c) + a)^p, x)
```

maple [F] time = 1.00, size = 0, normalized size = 0.00

$$\int \left(ex\right)^{-1+n}\left(a+b\cos\left(c+d\,x^{n}\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+n)*(a+b*cos(c+d*x^n))^p,x)
```

```
[Out] int((e*x)^(-1+n)*(a+b*cos(c+d*x^n))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex\right)^{n-1}\left(b\cos\left(dx^n+c\right)+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+n)*(a+b*cos(c+d*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(n - 1)*(b*cos(d*x^n + c) + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^{n-1}\,(a+b\,\cos\left(c+d\,x^n\right))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(n - 1)*(a + b*cos(c + d*x^n))^p,x)
```

```
[Out] int((e*x)^(n - 1)*(a + b*cos(c + d*x^n))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+n)*(a+b*cos(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

3.68 $\int (ex)^{-1+2n} (a+b\cos(c+dx^n))^p \, dx$

Optimal. Leaf size=41

$$\frac{x^{-2n}(ex)^{2n}\text{Int}\left(x^{2n-1}\left(a+b\cos\left(c+dx^n\right)\right)^p,x\right)}{e}$$

```
[Out] (e*x)^(2*n)*Unintegrable(x^(-1+2*n)*(a+b*cos(c+d*x^n))^p,x)/e/(x^(2*n))
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ex)^{-1+2n} (a+b\cos(c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Cos[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^(2*n)*Defer[Int][x^(-1 + 2*n)*(a + b*Cos[c + d*x^n])^p, x])/(e*x^(2*
n))
```

Rubi steps

$$\int (ex)^{-1+2n} (a+b\cos(c+dx^n))^p \, dx = \frac{\left(x^{-2n}(ex)^{2n}\right)\int x^{-1+2n}(a+b\cos(c+dx^n))^p \, dx}{e}$$

Mathematica [A] time = 1.14, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n} (a+b\cos(c+dx^n))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Cos[c + d*x^n])^p,x]
```

```
[Out] Integrate[(e*x)^(-1 + 2*n)*(a + b*Cos[c + d*x^n])^p, x]
```

fricas [A] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^{2n-1}(b\cos(dx^n+c)+a)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*cos(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^(2*n - 1)*(b*cos(d*x^n + c) + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{2n-1}(b\cos(dx^n+c)+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*cos(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)*(b*cos(d*x^n + c) + a)^p, x)
```

maple [A] time = 0.94, size = 0, normalized size = 0.00

$$\int (ex)^{-1+2n}(a+b\cos(c+d\,x^n))^p\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(-1+2*n)*(a+b*cos(c+d*x^n))^p,x)
[Out] int((e*x)^(-1+2*n)*(a+b*cos(c+d*x^n))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{2n-1}(b\cos(dx^n+c)+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(-1+2*n)*(a+b*cos(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^(2*n - 1)*(b*cos(d*x^n + c) + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int (e\,x)^{2n-1}\,(a+b\,\cos(c+d\,x^n))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(2*n - 1)*(a + b*cos(c + d*x^n))^p,x)
[Out] int((e*x)^(2*n - 1)*(a + b*cos(c + d*x^n))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(-1+2*n)*(a+b*cos(c+d*x**n))**p,x)
```

```
[Out] Timed out
```

3.69 $\int \frac{\cos(a+bx^n)}{x} dx$

Optimal. Leaf size=26

$$\frac{\cos(a)\text{Ci}\left(bx^n\right)}{n}-\frac{\sin(a)\text{Si}\left(bx^n\right)}{n}$$

```
[Out] Ci(b*x^n)*cos(a)/n-Si(b*x^n)*sin(a)/n
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3378, 3376, 3375}

$$\frac{\cos(a)\text{CosIntegral}\left(bx^n\right)}{n}-\frac{\sin(a)\text{Si}\left(bx^n\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^n]/x,x]
```

```
[Out] (Cos[a]*CosIntegral[b*x^n])/n - (Sin[a]*SinIntegral[b*x^n])/n
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(a+bx^n\right)}{x}\,dx &= \cos(a)\int \frac{\cos\left(bx^n\right)}{x}\,dx-\sin(a)\int \frac{\sin\left(bx^n\right)}{x}\,dx\\ &= \frac{\cos(a)\text{Ci}\left(bx^n\right)}{n}-\frac{\sin(a)\text{Si}\left(bx^n\right)}{n}\end{aligned}$$

Mathematica [A]   time = 0.07, size = 24, normalized size = 0.92

$$\frac{\cos(a)\text{Ci}\left(bx^n\right)-\sin(a)\text{Si}\left(bx^n\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^n]/x,x]
```

```
[Out] (Cos[a]*CosIntegral[b*x^n] - Sin[a]*SinIntegral[b*x^n])/n
```

fricas [A]   time = 1.31, size = 35, normalized size = 1.35

$$\frac{\cos(a)\,\text{Ci}\left(bx^n\right)+\cos(a)\,\text{Ci}\left(-bx^n\right)-2\,\sin(a)\,\text{Si}\left(bx^n\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(a)*cos_integral(b*x^n) + cos(a)*cos_integral(-b*x^n) - 2*sin(a)*si
n_integral(b*x^n))/n
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx^n+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^n + a)/x, x)
```

maple [A]   time = 0.03, size = 25, normalized size = 0.96

$$\frac{-\text{Si}\left(b\,x^n\right)\sin(a)+\text{Ci}\left(b\,x^n\right)\cos(a)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*x^n)/x,x)
```

```
[Out] 1/n*(-Si(b*x^n)*sin(a)+Ci(b*x^n)*cos(a))
```

maxima [C]   time = 1.74, size = 90, normalized size = 3.46

$$\frac{\left(\text{Ei}\left(i\,bx^n\right)+\text{Ei}\left(-i\,bx^n\right)+\text{Ei}\left(i\,be^{\left(n\overline{\log(x)}\right)}\right)+\text{Ei}\left(-i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos(a)+\left(i\,\text{Ei}\left(i\,bx^n\right)-i\,\text{Ei}\left(-i\,bx^n\right)+i\,\text{Ei}\left(i\,be^{\left(n\overline{\log}\right.}\right.\right.}{4\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)/x,x, algorithm="maxima")
```

```
[Out] 1/4*((Ei(I*b*x^n) + Ei(-I*b*x^n) + Ei(I*b*e^(n*conjugate(log(x)))) + Ei(-I*
b*e^(n*conjugate(log(x)))))*cos(a) + (I*Ei(I*b*x^n) - I*Ei(-I*b*x^n) + I*Ei
(I*b*e^(n*conjugate(log(x)))) - I*Ei(-I*b*e^(n*conjugate(log(x)))))*sin(a))
/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos\left(a+b\,x^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n)/x,x)
```

```
[Out] int(cos(a + b*x^n)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+bx^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x**n)/x,x)
```

```
[Out] Integral(cos(a + b*x**n)/x, x)
```

3.70 $\int \frac{\cos^2(a+bx^n)}{x}\,dx$

Optimal. Leaf size=43

$$\frac{\cos(2a)\text{Ci}\left(2bx^n\right)}{2n}-\frac{\sin(2a)\text{Si}\left(2bx^n\right)}{2n}+\frac{\log(x)}{2}$$

```
[Out] 1/2*Ci(2*b*x^n)*cos(2*a)/n+1/2*ln(x)-1/2*Si(2*b*x^n)*sin(2*a)/n
```

Rubi [A] time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3426, 3378, 3376, 3375}

$$\frac{\cos(2a)\text{CosIntegral}\left(2bx^n\right)}{2n}-\frac{\sin(2a)\text{Si}\left(2bx^n\right)}{2n}+\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^n]^2/x,x]
```

```
[Out] (Cos[2*a]*CosIntegral[2*b*x^n])/(2*n) + Log[x]/2 - (Sin[2*a]*SinIntegral[2*
b*x^n])/(2*n)
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3426

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(a+bx^n)}{x}\,dx &= \int \left(\frac{1}{2x}+\frac{\cos(2a+2bx^n)}{2x}\right)dx\\
&= \frac{\log(x)}{2}+\frac{1}{2}\int \frac{\cos(2a+2bx^n)}{x}\,dx\\
&= \frac{\log(x)}{2}+\frac{1}{2}\cos(2a)\int \frac{\cos(2bx^n)}{x}\,dx-\frac{1}{2}\sin(2a)\int \frac{\sin(2bx^n)}{x}\,dx\\
&= \frac{\cos(2a)\mathrm{Ci}(2bx^n)}{2n}+\frac{\log(x)}{2}-\frac{\sin(2a)\mathrm{Si}(2bx^n)}{2n}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 37, normalized size = 0.86

$$\frac{\cos(2a)\mathrm{Ci}(2bx^n)-\sin(2a)\mathrm{Si}(2bx^n)+n\log(x)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^n]^2/x,x]
```

```
[Out] (Cos[2*a]*CosIntegral[2*b*x^n] + n*Log[x] - Sin[2*a]*SinIntegral[2*b*x^n])/
(2*n)
```

fricas [A] time = 0.65, size = 48, normalized size = 1.12

$$\frac{\cos(2a)\,\mathrm{Ci}(2bx^n)+\cos(2a)\,\mathrm{Ci}(-2bx^n)+2n\log(x)-2\sin(2a)\,\mathrm{Si}(2bx^n)}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)^2/x,x, algorithm="fricas")
```

```
[Out] 1/4*(cos(2*a)*cos_integral(2*b*x^n) + cos(2*a)*cos_integral(-2*b*x^n) + 2*n
*log(x) - 2*sin(2*a)*sin_integral(2*b*x^n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx^n+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)^2/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^n + a)^2/x, x)
```

maple [A] time = 0.05, size = 45, normalized size = 1.05

$$-\frac{\mathrm{Si}\left(2b\,x^{n}\right)\sin\left(2a\right)}{2n}+\frac{\mathrm{Ci}\left(2b\,x^{n}\right)\cos\left(2a\right)}{2n}+\frac{\ln\left(b\,x^{n}\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*x^n)^2/x,x)
```

```
[Out] -1/2*Si(2*b*x^n)*sin(2*a)/n+1/2*Ci(2*b*x^n)*cos(2*a)/n+1/2/n*ln(b*x^n)
```

maxima [C] time = 2.36, size = 99, normalized size = 2.30

$$\frac{\left(\mathrm{Ei}\left(2i\,bx^{n}\right)+\mathrm{Ei}\left(-2i\,bx^{n}\right)+\mathrm{Ei}\left(2i\,be^{\left(n\overline{\log\left(x\right)}\right)}\right)+\mathrm{Ei}\left(-2i\,be^{\left(n\overline{\log\left(x\right)}\right)}\right)\right)\cos\left(2\,a\right)+4\,n\log(x)+\left(i\,\mathrm{Ei}\left(2i\,bx^{n}\right)-i\,\mathrm{Ei}\left(\cdot\right.\right.}{8\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*((Ei(2*I*b*x^n) + Ei(-2*I*b*x^n) + Ei(2*I*b*e^(n*conjugate(log(x)))) +
Ei(-2*I*b*e^(n*conjugate(log(x)))))*cos(2*a) + 4*n*log(x) + (I*Ei(2*I*b*x^n
) - I*Ei(-2*I*b*x^n) + I*Ei(2*I*b*e^(n*conjugate(log(x)))) - I*Ei(-2*I*b*e^
(n*conjugate(log(x)))))*sin(2*a))/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(a+b\,x^{n}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n)^2/x,x)
```

```
[Out] int(cos(a + b*x^n)^2/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(a+bx^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x**n)**2/x,x)
```

```
[Out] Integral(cos(a + b*x**n)**2/x, x)
```

3.71 $\int \frac{\cos^3(a+bx^n)}{x} \, dx$

Optimal. Leaf size=67

$$\frac{3\cos(a)\text{Ci}(bx^n)}{4n}+\frac{\cos(3a)\text{Ci}(3bx^n)}{4n}-\frac{3\sin(a)\text{Si}(bx^n)}{4n}-\frac{\sin(3a)\text{Si}(3bx^n)}{4n}$$

```
[Out] 3/4*Ci(b*x^n)*cos(a)/n+1/4*Ci(3*b*x^n)*cos(3*a)/n-3/4*Si(b*x^n)*sin(a)/n-1/
4*Si(3*b*x^n)*sin(3*a)/n
```

Rubi [A] time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3426, 3378, 3376, 3375}

$$\frac{3\cos(a)\text{CosIntegral}(bx^n)}{4n}+\frac{\cos(3a)\text{CosIntegral}(3bx^n)}{4n}-\frac{3\sin(a)\text{Si}(bx^n)}{4n}-\frac{\sin(3a)\text{Si}(3bx^n)}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^n]^3/x,x]
```

```
[Out] (3*Cos[a]*CosIntegral[b*x^n])/(4*n) + (Cos[3*a]*CosIntegral[3*b*x^n])/(4*n)
 - (3*Sin[a]*SinIntegral[b*x^n])/(4*n) - (Sin[3*a]*SinIntegral[3*b*x^n])/(4
*n)
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3426

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx^n)}{x}\,dx &= \int \left(\frac{3\cos(a+bx^n)}{4x}+\frac{\cos(3a+3bx^n)}{4x}\right)dx\\ &= \frac{1}{4}\int \frac{\cos(3a+3bx^n)}{x}\,dx+\frac{3}{4}\int \frac{\cos(a+bx^n)}{x}\,dx\\ &= \frac{1}{4}(3\cos(a))\int \frac{\cos(bx^n)}{x}\,dx+\frac{1}{4}\cos(3a)\int \frac{\cos(3bx^n)}{x}\,dx-\frac{1}{4}(3\sin(a))\int \frac{\sin(bx^n)}{x}\,dx-\\ &= \frac{3\cos(a)\text{Ci}(bx^n)}{4n}+\frac{\cos(3a)\text{Ci}(3bx^n)}{4n}-\frac{3\sin(a)\text{Si}(bx^n)}{4n}-\frac{\sin(3a)\text{Si}(3bx^n)}{4n}\end{aligned}$$

Mathematica [A] time = 0.13, size = 53, normalized size = 0.79

$$\frac{3\cos(a)\text{Ci}(bx^n)+\cos(3a)\text{Ci}(3bx^n)-3\sin(a)\text{Si}(bx^n)-\sin(3a)\text{Si}(3bx^n)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^n]^3/x,x]
```

```
[Out] (3*Cos[a]*CosIntegral[b*x^n] + Cos[3*a]*CosIntegral[3*b*x^n] - 3*Sin[a]*Sin
Integral[b*x^n] - Sin[3*a]*SinIntegral[3*b*x^n])/(4*n)
```

fricas [A] time = 0.82, size = 74, normalized size = 1.10

$$\frac{\cos(3\,a)\,\text{Ci}(3\,bx^n)+3\,\cos(a)\,\text{Ci}(bx^n)+3\,\cos(a)\,\text{Ci}(-bx^n)+\cos(3\,a)\,\text{Ci}(-3\,bx^n)-2\,\sin(3\,a)\,\text{Si}(3\,bx^n)-6\,\text{s}}{8\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)^3/x,x, algorithm="fricas")
```

```
[Out] 1/8*(cos(3*a)*cos_integral(3*b*x^n) + 3*cos(a)*cos_integral(b*x^n) + 3*cos(
a)*cos_integral(-b*x^n) + cos(3*a)*cos_integral(-3*b*x^n) - 2*sin(3*a)*sin_
integral(3*b*x^n) - 6*sin(a)*sin_integral(b*x^n))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx^n+a)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)^3/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^n + a)^3/x, x)
```

maple [A] time = 0.05, size = 52, normalized size = 0.78

$$\frac{-\frac{\operatorname{Si}(3b\,x^{n})\sin(3a)}{4}+\frac{\operatorname{Ci}(3b\,x^{n})\cos(3a)}{4}-\frac{3\operatorname{Si}(b\,x^{n})\sin(a)}{4}+\frac{3\operatorname{Ci}(b\,x^{n})\cos(a)}{4}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*x^n)^3/x,x)
```

```
[Out] 1/n*(-1/4*Si(3*b*x^n)*sin(3*a)+1/4*Ci(3*b*x^n)*cos(3*a)-3/4*Si(b*x^n)*sin(a
)+3/4*Ci(b*x^n)*cos(a))
```

maxima [C] time = 1.86, size = 179, normalized size = 2.67

$$\frac{\left(\operatorname{Ei}\left(3i\,bx^{n}\right)+\operatorname{Ei}\left(-3i\,bx^{n}\right)+\operatorname{Ei}\left(3i\,be^{\left(n\overline{\log(x)}\right)}\right)+\operatorname{Ei}\left(-3i\,be^{\left(n\overline{\log(x)}\right)}\right)\right)\cos\left(3\,a\right)+3\left(\operatorname{Ei}\left(i\,bx^{n}\right)+\operatorname{Ei}\left(-i\,bx^{n}\right)+\operatorname{Ei}\left(i\,be\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)^3/x,x, algorithm="maxima")
```

```
[Out] 1/16*((Ei(3*I*b*x^n) + Ei(-3*I*b*x^n) + Ei(3*I*b*e^(n*conjugate(log(x)))) +
 Ei(-3*I*b*e^(n*conjugate(log(x)))))*cos(3*a) + 3*(Ei(I*b*x^n) + Ei(-I*b*x^
n) + Ei(I*b*e^(n*conjugate(log(x)))) + Ei(-I*b*e^(n*conjugate(log(x)))))*co
s(a) + (I*Ei(3*I*b*x^n) - I*Ei(-3*I*b*x^n) + I*Ei(3*I*b*e^(n*conjugate(log(
x)))) - I*Ei(-3*I*b*e^(n*conjugate(log(x)))))*sin(3*a) + (3*I*Ei(I*b*x^n) -
 3*I*Ei(-I*b*x^n) + 3*I*Ei(I*b*e^(n*conjugate(log(x)))) - 3*I*Ei(-I*b*e^(n*
conjugate(log(x)))))*sin(a))/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x^n\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n)^3/x,x)
```

```
[Out] int(cos(a + b*x^n)^3/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{3}\left(a+bx^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x**n)**3/x,x)
```

```
[Out] Integral(cos(a + b*x**n)**3/x, x)
```

3.72 $\int \frac{\cos^4(a+bx^n)}{x}\,dx$

Optimal. Leaf size=79

$$\frac{\cos(2a)\text{Ci}\left(2bx^n\right)}{2n}+\frac{\cos(4a)\text{Ci}\left(4bx^n\right)}{8n}-\frac{\sin(2a)\text{Si}\left(2bx^n\right)}{2n}-\frac{\sin(4a)\text{Si}\left(4bx^n\right)}{8n}+\frac{3\log(x)}{8}$$

```
[Out] 1/2*Ci(2*b*x^n)*cos(2*a)/n+1/8*Ci(4*b*x^n)*cos(4*a)/n+3/8*ln(x)-1/2*Si(2*b*
x^n)*sin(2*a)/n-1/8*Si(4*b*x^n)*sin(4*a)/n
```

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3426, 3378, 3376, 3375}

$$\frac{\cos(2a)\text{CosIntegral}\left(2bx^n\right)}{2n}+\frac{\cos(4a)\text{CosIntegral}\left(4bx^n\right)}{8n}-\frac{\sin(2a)\text{Si}\left(2bx^n\right)}{2n}-\frac{\sin(4a)\text{Si}\left(4bx^n\right)}{8n}+\frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^n]^4/x,x]
```

```
[Out] (Cos[2*a]*CosIntegral[2*b*x^n])/(2*n) + (Cos[4*a]*CosIntegral[4*b*x^n])/(8*
n) + (3*Log[x])/8 - (Sin[2*a]*SinIntegral[2*b*x^n])/(2*n) - (Sin[4*a]*SinIn
tegral[4*b*x^n])/(8*n)
```

Rule 3375

```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3376

```
Int[Cos[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[CosIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

Rule 3378

```
Int[Cos[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Dist[Cos[c], Int[Cos[d*x
^n]/x, x], x] - Dist[Sin[c], Int[Sin[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x
]
```

Rule 3426

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(a+bx^n)}{x}\,dx &= \int \left(\frac{3}{8x}+\frac{\cos(2a+2bx^n)}{2x}+\frac{\cos(4a+4bx^n)}{8x}\right)dx\\ &= \frac{3\log(x)}{8}+\frac{1}{8}\int \frac{\cos(4a+4bx^n)}{x}\,dx+\frac{1}{2}\int \frac{\cos(2a+2bx^n)}{x}\,dx\\ &= \frac{3\log(x)}{8}+\frac{1}{2}\cos(2a)\int \frac{\cos(2bx^n)}{x}\,dx+\frac{1}{8}\cos(4a)\int \frac{\cos(4bx^n)}{x}\,dx-\frac{1}{2}\sin(2a)\int \frac{\sin(\\ &= \frac{\cos(2a)\mathrm{Ci}(2bx^n)}{2n}+\frac{\cos(4a)\mathrm{Ci}(4bx^n)}{8n}+\frac{3\log(x)}{8}-\frac{\sin(2a)\mathrm{Si}(2bx^n)}{2n}-\frac{\sin(4a)\mathrm{Si}(4bx^n)}{8n}\end{aligned}$$

Mathematica [A] time = 0.14, size = 66, normalized size = 0.84

$$\frac{4\cos(2a)\mathrm{Ci}(2bx^n)+\cos(4a)\mathrm{Ci}(4bx^n)-4\sin(2a)\mathrm{Si}(2bx^n)-\sin(4a)\mathrm{Si}(4bx^n)}{8n}+\frac{3\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^n]^4/x,x]
```

```
[Out] (3*Log[x])/8 + (4*Cos[2*a]*CosIntegral[2*b*x^n] + Cos[4*a]*CosIntegral[4*b*
x^n] - 4*Sin[2*a]*SinIntegral[2*b*x^n] - Sin[4*a]*SinIntegral[4*b*x^n])/(8*
n)
```

fricas [A] time = 0.90, size = 87, normalized size = 1.10

$$\frac{\cos(4\,a)\,\mathrm{Ci}(4\,bx^n)+4\,\cos(2\,a)\,\mathrm{Ci}(2\,bx^n)+4\,\cos(2\,a)\,\mathrm{Ci}(-2\,bx^n)+\cos(4\,a)\,\mathrm{Ci}(-4\,bx^n)+6\,n\log(x)-2\,\sin}{16\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)^4/x,x, algorithm="fricas")
```

```
[Out] 1/16*(cos(4*a)*cos_integral(4*b*x^n) + 4*cos(2*a)*cos_integral(2*b*x^n) + 4
*cos(2*a)*cos_integral(-2*b*x^n) + cos(4*a)*cos_integral(-4*b*x^n) + 6*n*lo
g(x) - 2*sin(4*a)*sin_integral(4*b*x^n) - 8*sin(2*a)*sin_integral(2*b*x^n))
/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx^n+a)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)^4/x,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^n + a)^4/x, x)
```

maple [A] time = 0.05, size = 77, normalized size = 0.97

$$-\frac{\mathrm{Si}\left(4b\,x^{n}\right)\sin\left(4a\right)}{8n}+\frac{\mathrm{Ci}\left(4b\,x^{n}\right)\cos\left(4a\right)}{8n}-\frac{\mathrm{Si}\left(2b\,x^{n}\right)\sin\left(2a\right)}{2n}+\frac{\mathrm{Ci}\left(2b\,x^{n}\right)\cos\left(2a\right)}{2n}+\frac{3\ln\left(b\,x^{n}\right)}{8n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*x^n)^4/x,x)
```

```
[Out] -1/8*Si(4*b*x^n)*sin(4*a)/n+1/8*Ci(4*b*x^n)*cos(4*a)/n-1/2*Si(2*b*x^n)*sin(
2*a)/n+1/2*Ci(2*b*x^n)*cos(2*a)/n+3/8/n*ln(b*x^n)
```

maxima [C] time = 2.62, size = 188, normalized size = 2.38

$$\frac{\left(\mathrm{Ei}\left(4i\,bx^{n}\right)+\mathrm{Ei}\left(-4i\,bx^{n}\right)+\mathrm{Ei}\left(4i\,be^{\left(n\overline{\log\left(x\right)}\right)}\right)+\mathrm{Ei}\left(-4i\,be^{\left(n\overline{\log\left(x\right)}\right)}\right)\right)\cos\left(4\,a\right)+4\left(\mathrm{Ei}\left(2i\,bx^{n}\right)+\mathrm{Ei}\left(-2i\,bx^{n}\right)+\mathrm{Ei}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)^4/x,x, algorithm="maxima")
```

```
[Out] 1/32*((Ei(4*I*b*x^n) + Ei(-4*I*b*x^n) + Ei(4*I*b*e^(n*conjugate(log(x)))) +
 Ei(-4*I*b*e^(n*conjugate(log(x)))))*cos(4*a) + 4*(Ei(2*I*b*x^n) + Ei(-2*I*
b*x^n) + Ei(2*I*b*e^(n*conjugate(log(x)))) + Ei(-2*I*b*e^(n*conjugate(log(x
)))))*cos(2*a) + 12*n*log(x) + (I*Ei(4*I*b*x^n) - I*Ei(-4*I*b*x^n) + I*Ei(4
*I*b*e^(n*conjugate(log(x)))) - I*Ei(-4*I*b*e^(n*conjugate(log(x)))))*sin(4
*a) + (4*I*Ei(2*I*b*x^n) - 4*I*Ei(-2*I*b*x^n) + 4*I*Ei(2*I*b*e^(n*conjugate
(log(x)))) - 4*I*Ei(-2*I*b*e^(n*conjugate(log(x)))))*sin(2*a))/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x^n\right)^4}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n)^4/x,x)
```

```
[Out] int(cos(a + b*x^n)^4/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{4}\left(a+bx^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x**n)**4/x,x)
```

```
[Out] Integral(cos(a + b*x**n)**4/x, x)
```

3.73 $\int \cos(a + bx^n)\, dx$

Optimal. Leaf size=83

$$-\frac{e^{ia}x\left(-ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ibx^n\right)}{2n}-\frac{e^{-ia}x\left(ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ibx^n\right)}{2n}$$

```
[Out] -1/2*exp(I*a)*x*GAMMA(1/n,-I*b*x^n)/n/((-I*b*x^n)^(1/n))-1/2*x*GAMMA(1/n,I*
b*x^n)/exp(I*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3366, 2208}

$$-\frac{e^{ia}x\left(-ibx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},-ibx^n\right)}{2n}-\frac{e^{-ia}x\left(ibx^n\right)^{-1/n}\text{Gamma}\left(\frac{1}{n},ibx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^n],x]
```

```
[Out] -(E^(I*a)*x*Gamma[n^(-1), (-I)*b*x^n])/(2*n*((-I)*b*x^n)^n^(-1)) - (x*Gamma
[n^(-1), I*b*x^n])/(2*E^(I*a)*n*(I*b*x^n)^n^(-1))
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3366

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx^n)\,dx&=\frac{1}{2}\int e^{-ia-ibx^n}\,dx+\frac{1}{2}\int e^{ia+ibx^n}\,dx\\&=-\frac{e^{ia}x\left(-ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-ibx^n\right)}{2n}-\frac{e^{-ia}x\left(ibx^n\right)^{-1/n}\Gamma\left(\frac{1}{n},ibx^n\right)}{2n}\end{aligned}$$

Mathematica [A] time = 0.08, size = 92, normalized size = 1.11

$$-\frac{x\left(b^2x^{2n}\right)^{-1/n}\left((\cos(a)-i\sin(a))\left(-ibx^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},ibx^n\right)+(\cos(a)+i\sin(a))\left(ibx^n\right)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-ibx^n\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^n],x]
```

```
[Out] -1/2*(x*((((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n]*(Cos[a] - I*Sin[a]) + (
I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*x^n]*(Cos[a] + I*Sin[a])))/(n*(b^2*x^(
2*n))^n^(-1))
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\cos\left(bx^n+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\cos\left(bx^n+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^n + a), x)
```

maple [C] time = 0.12, size = 75, normalized size = 0.90

$$x\,\text{hypergeom}\left(\left[\frac{1}{2n}\right],\left[\frac{1}{2},1+\frac{1}{2n}\right],-\frac{x^{2n}b^2}{4}\right)\cos(a)-\frac{b\,x^{1+n}\,\text{hypergeom}\left(\left[\frac{1}{2}+\frac{1}{2n}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{1}{2n}\right],-\frac{x^{2n}b^2}{4}\right)\sin(a)}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*x^n),x)
```

```
[Out] x*hypergeom([1/2/n],[1/2,1+1/2/n],-1/4*x^(2*n)*b^2)*cos(a)-b/(1+n)*x^(1+n)*
hypergeom([1/2+1/2/n],[3/2,3/2+1/2/n],-1/4*x^(2*n)*b^2)*sin(a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(bx^n + a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n),x, algorithm="maxima")
[Out] integrate(cos(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a + b\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n),x)
[Out] int(cos(a + b*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(a + bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x**n),x)
[Out] Integral(cos(a + b*x**n), x)
```

3.74 $\int \cos^2(a+bx^n)\,dx$

Optimal. Leaf size=102

$$-\frac{e^{2ia}2^{-\frac{1}{n}-2}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2ibx^n\right)}{n}-\frac{e^{-2ia}2^{-\frac{1}{n}-2}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2ibx^n\right)}{n}+\frac{x}{2}$$

```
[Out] 1/2*x-2^(-2-1/n)*exp(2*I*a)*x*GAMMA(1/n,-2*I*b*x^n)/n/((-I*b*x^n)^(1/n))-2^
(-2-1/n)*x*GAMMA(1/n,2*I*b*x^n)/exp(2*I*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A] time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3368, 3366, 2208}

$$-\frac{e^{2ia}2^{-\frac{1}{n}-2}x(-ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},-2ibx^n\right)}{n}-\frac{e^{-2ia}2^{-\frac{1}{n}-2}x(ibx^n)^{-1/n}\,\text{Gamma}\left(\frac{1}{n},2ibx^n\right)}{n}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^n]^2,x]
```

```
[Out] x/2 - (2^(-2 - n^(-1))*E^((2*I)*a)*x*Gamma[n^(-1), (-2*I)*b*x^n])/(n*((-I)*
b*x^n)^n^(-1)) - (2^(-2 - n^(-1))*x*Gamma[n^(-1), (2*I)*b*x^n])/(E^((2*I)*a
)*n*(I*b*x^n)^n^(-1))
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Rule 3366

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3368

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx^n)\,dx &= \int \left(\frac{1}{2}+\frac{1}{2}\cos(2a+2bx^n)\right)dx\\ &= \frac{x}{2}+\frac{1}{2}\int \cos(2a+2bx^n)\,dx\\ &= \frac{x}{2}+\frac{1}{4}\int e^{-2ia-2ibx^n}\,dx+\frac{1}{4}\int e^{2ia+2ibx^n}\,dx\\ &= \frac{x}{2}-\frac{2^{-2-\frac{1}{n}}e^{2ia}x(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2ibx^n\right)}{n}-\frac{2^{-2-\frac{1}{n}}e^{-2ia}x(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2ibx^n\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.25, size = 94, normalized size = 0.92

$$-\frac{x\left(e^{2ia}2^{-1/n}(-ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-2ibx^n\right)+e^{-2ia}2^{-1/n}(ibx^n)^{-1/n}\,\Gamma\left(\frac{1}{n},2ibx^n\right)-2n\right)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^n]^2,x]
```

```
[Out] -1/4*(x*(-2*n + (E^((2*I)*a)*Gamma[n^(-1), (-2*I)*b*x^n])/(2^n^(-1)*((-I)*b
*x^n)^n^(-1)) + Gamma[n^(-1), (2*I)*b*x^n]/(2^n^(-1)*E^((2*I)*a)*(I*b*x^n)^
n^(-1))))/n
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\cos(bx^n+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x^n + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(bx^n+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^n + a)^2, x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \cos^2(a + b\,x^n)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*x^n)^2,x)
[Out] int(cos(a+b*x^n)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,x + \frac{1}{2}\,\int \cos\left(2\,bx^n + 2\,a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)^2,x, algorithm="maxima")
[Out] 1/2*x + 1/2*integrate(cos(2*b*x^n + 2*a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a + b\,x^n\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n)^2,x)
[Out] int(cos(a + b*x^n)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^2(a + bx^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x**n)**2,x)
[Out] Integral(cos(a + b*x**n)**2, x)
```

3.75 $\int \cos^3(a+bx^n)\,dx$

Optimal. Leaf size=179

$$-\frac{3e^{ia}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-ibx^n\right)}{8n}-\frac{e^{3ia}3^{-1/n}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-3ibx^n\right)}{8n}-\frac{3e^{-ia}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},ibx^n\right)}{8n}-\frac{e^{-3ia}3^{-1/n}x(ib\cdots}{}$$

```
[Out] -3/8*exp(I*a)*x*GAMMA(1/n,-I*b*x^n)/n/((-I*b*x^n)^(1/n))-3/8*x*GAMMA(1/n,I*
b*x^n)/exp(I*a)/n/((I*b*x^n)^(1/n))-1/8*exp(3*I*a)*x*GAMMA(1/n,-3*I*b*x^n)/
(3^(1/n))/n/((-I*b*x^n)^(1/n))-1/8*x*GAMMA(1/n,3*I*b*x^n)/(3^(1/n))/exp(3*I
*a)/n/((I*b*x^n)^(1/n))
```

Rubi [A] time = 0.08, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3368, 3366, 2208}

$$-\frac{3e^{ia}x(-ibx^n)^{-1/n}\text{Gamma}\left(\frac{1}{n},-ibx^n\right)}{8n}-\frac{e^{3ia}3^{-1/n}x(-ibx^n)^{-1/n}\text{Gamma}\left(\frac{1}{n},-3ibx^n\right)}{8n}-\frac{3e^{-ia}x(ibx^n)^{-1/n}\text{Gamma}\left(\frac{1}{n},\cdots\right)}{8n}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x^n]^3,x]
```

```
[Out] (-3*E^(I*a)*x*Gamma[n^(-1), (-I)*b*x^n])/(8*n*((-I)*b*x^n)^n^(-1)) - (3*x*G
amma[n^(-1), I*b*x^n])/(8*E^(I*a)*n*(I*b*x^n)^n^(-1)) - (E^((3*I)*a)*x*Gamm
a[n^(-1), (-3*I)*b*x^n])/(8*3^n^(-1)*n*((-I)*b*x^n)^n^(-1)) - (x*Gamma[n^(-
1), (3*I)*b*x^n])/(8*3^n^(-1)*E^((3*I)*a)*n*(I*b*x^n)^n^(-1))
```

Rule 2208

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> -Simp[(F^a
*(c + d*x)*Gamma[1/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x)^n*Log[F
]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 3366

```
Int[Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)], x_Symbol] :> Dist[1/2, In
t[E^(-(c*I) - d*I*(e + f*x)^n), x], x] + Dist[1/2, Int[E^(c*I + d*I*(e + f*
x)^n), x], x] /; FreeQ[{c, d, e, f, n}, x]
```

Rule 3368

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_), x_Sy
mbol] :> Int[ExpandTrigReduce[(a + b*Cos[c + d*(e + f*x)^n])^p, x], x] /; F
```

```
reeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx^n)\,dx &= \int \left(\frac{3}{4}\cos(a+bx^n)+\frac{1}{4}\cos(3a+3bx^n)\right)dx\\ &= \frac{1}{4}\int \cos(3a+3bx^n)\,dx+\frac{3}{4}\int \cos(a+bx^n)\,dx\\ &= \frac{1}{8}\int e^{-3ia-3ibx^n}\,dx+\frac{1}{8}\int e^{3ia+3ibx^n}\,dx+\frac{3}{8}\int e^{-ia-ibx^n}\,dx+\frac{3}{8}\int e^{ia+ibx^n}\,dx\\ &= -\frac{3e^{ia}x(-ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},-ibx^n\right)}{8n}-\frac{3e^{-ia}x(ibx^n)^{-1/n}\Gamma\left(\frac{1}{n},ibx^n\right)}{8n}-\frac{3^{-1/n}e^{3ia}x(-ibx^n)^{-1/n}\Gamma(}{8n}\end{aligned}$$

Mathematica [A] time = 0.24, size = 173, normalized size = 0.97

$$-\frac{e^{-3ia}3^{-1/n}x\left(b^2x^{2n}\right)^{-1/n}\left(e^{2ia}3^{\frac{1}{n}+1}(-ibx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},ibx^n\right)+e^{4ia}3^{\frac{1}{n}+1}(ibx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-ibx^n\right)+e^{6ia}(ibx^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{n},-3ibx^n\right.}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x^n]^3,x]
```

```
[Out] -1/8*(x*(3^(1 + n^(-1))*E^((4*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-I)*b*x
^n] + 3^(1 + n^(-1))*E^((2*I)*a)*((-I)*b*x^n)^n^(-1)*Gamma[n^(-1), I*b*x^n]
 + E^((6*I)*a)*(I*b*x^n)^n^(-1)*Gamma[n^(-1), (-3*I)*b*x^n] + ((-I)*b*x^n)^
n^(-1)*Gamma[n^(-1), (3*I)*b*x^n]))/(3^n^(-1)*E^((3*I)*a)*n*(b^2*x^(2*n))^n
^(-1))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\cos(bx^n+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x^n + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(bx^n+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x^n + a)^3, x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \cos^3\left(a + b\,x^n\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*x^n)^3,x)
```

```
[Out] int(cos(a+b*x^n)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(bx^n + a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x^n + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a + b\,x^n\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n)^3,x)
```

```
[Out] int(cos(a + b*x^n)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^3\left(a + bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*x**n)**3,x)
```

```
[Out] Integral(cos(a + b*x**n)**3, x)
```

3.76 $\int x^m \cos(a + bx^n)\, dx$

Optimal. Leaf size=105

$$-\frac{e^{ia}x^{m+1}\left(-ibx^n\right)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-ibx^n\right)}{2n}-\frac{e^{-ia}x^{m+1}\left(ibx^n\right)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},ibx^n\right)}{2n}$$

```
[Out] -1/2*exp(I*a)*x^(1+m)*GAMMA((1+m)/n,-I*b*x^n)/n/((-I*b*x^n)^((1+m)/n))-1/2*
x^(1+m)*GAMMA((1+m)/n,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^((1+m)/n))
```

Rubi [A] time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3424, 2218}

$$-\frac{e^{ia}x^{m+1}\left(-ibx^n\right)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-ibx^n\right)}{2n}-\frac{e^{-ia}x^{m+1}\left(ibx^n\right)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},ibx^n\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cos[a + b*x^n],x]
```

```
[Out] -(E^(I*a)*x^(1 + m)*Gamma[(1 + m)/n, (-I)*b*x^n])/(2*n*((-I)*b*x^n)^((1 + m
)/n)) - (x^(1 + m)*Gamma[(1 + m)/n, I*b*x^n])/(2*E^(I*a)*n*(I*b*x^n)^((1 +
m)/n))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3424

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rubi steps

$$\int x^m \cos(a+bx^n)\,dx = \frac{1}{2}\int e^{-ia-ibx^n}x^m\,dx + \frac{1}{2}\int e^{ia+ibx^n}x^m\,dx$$
$$= -\frac{e^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{2n} - \frac{e^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right)}{2n}$$

Mathematica [A] time = 0.20, size = 115, normalized size = 1.10

$$-\frac{x^{m+1}\left(b^2x^{2n}\right)^{-\frac{m+1}{n}}\left((\cos(a)-i\sin(a))(-ibx^n)^{\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},ibx^n\right)+(\cos(a)+i\sin(a))(ibx^n)^{\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-ibx^n\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cos[a + b*x^n],x]
```

```
[Out] -1/2*(x^(1 + m)*(((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, I*b*x^n]*(Cos[a]
 - I*Sin[a]) + (I*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (-I)*b*x^n]*(Cos[a] +
 I*Sin[a])))/(n*(b^2*x^(2*n))^((1 + m)/n))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\cos\left(bx^n+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(x^m*cos(b*x^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\cos\left(bx^n+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^m*cos(b*x^n + a), x)
```

maple [C] time = 0.16, size = 111, normalized size = 1.06

$$\frac{x^{1+m}\,\text{hypergeom}\left(\left[\frac{m}{2n}+\frac{1}{2n}\right],\left[\frac{1}{2},1+\frac{m}{2n}+\frac{1}{2n}\right],-\frac{x^{2n}b^2}{4}\right)\cos(a)}{1+m}-\frac{b\,x^{1+m+n}\,\text{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2n}+\frac{1}{2n}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{m}{2n}\right.\right.}{1+m+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a+b*x^n),x)
```

```
[Out] 1/(1+m)*x^(1+m)*hypergeom([1/2/n*m+1/2/n],[1/2,1+1/2/n*m+1/2/n],-1/4*x^(2*n
)*b^2)*cos(a)-b/(1+m+n)*x^(1+m+n)*hypergeom([1/2+1/2/n*m+1/2/n],[3/2,3/2+1/
2/n*m+1/2/n],-1/4*x^(2*n)*b^2)*sin(a)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cos(bx^n + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x^m*cos(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \cos(a + b\,x^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a + b*x^n),x)
```

```
[Out] int(x^m*cos(a + b*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cos(a + bx^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(a+b*x**n),x)
```

```
[Out] Integral(x**m*cos(a + b*x**n), x)
```

3.77 $\int x^m \cos^2(a + bx^n)\, dx$

Optimal. Leaf size=141

$$-\frac{e^{2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-2ibx^n\right)}{n}-\frac{e^{-2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},2ibx^n\right)}{n}+\frac{x^{m+1}}{2(m+1)}$$

```
[Out] 1/2*x^(1+m)/(1+m)-exp(2*I*a)*x^(1+m)*GAMMA((1+m)/n,-2*I*b*x^n)/(2^((1+m+2*n
)/n))/n/((-I*b*x^n)^((1+m)/n))-x^(1+m)*GAMMA((1+m)/n,2*I*b*x^n)/(2^((1+m+2*
n)/n))/exp(2*I*a)/n/((I*b*x^n)^((1+m)/n))
```

Rubi [A] time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3426, 3424, 2218}

$$-\frac{e^{2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-2ibx^n\right)}{n}-\frac{e^{-2ia}2^{-\frac{m+2n+1}{n}}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},2ibx^n\right)}{n}+\frac{x}{2(n}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cos[a + b*x^n]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) - (E^((2*I)*a)*x^(1 + m)*Gamma[(1 + m)/n, (-2*I)*b*x^
n])/(2^((1 + m + 2*n)/n)*n*((-I)*b*x^n)^((1 + m)/n)) - (x^(1 + m)*Gamma[(1
+ m)/n, (2*I)*b*x^n])/(2^((1 + m + 2*n)/n)*E^((2*I)*a)*n*(I*b*x^n)^((1 + m)
/n))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3424

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3426

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
```

```
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\cos^2(a+bx^n)\,dx&=\int\left(\frac{x^m}{2}+\frac{1}{2}x^m\cos(2a+2bx^n)\right)dx\\&=\frac{x^{1+m}}{2(1+m)}+\frac{1}{2}\int x^m\cos(2a+2bx^n)\,dx\\&=\frac{x^{1+m}}{2(1+m)}+\frac{1}{4}\int e^{-2ia-2ibx^n}x^m\,dx+\frac{1}{4}\int e^{2ia+2ibx^n}x^m\,dx\\&=\frac{x^{1+m}}{2(1+m)}-\frac{2^{-\frac{1+m+2n}{n}}e^{2ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-2ibx^n\right)}{n}-\frac{2^{-\frac{1+m+2n}{n}}e^{-2ia}x^{1+m}(ibx^n)}{n}\end{aligned}$$

Mathematica [A] time = 0.52, size = 129, normalized size = 0.91

$$-\frac{x^{m+1}\left(e^{2ia}(m+1)2^{-\frac{m+1}{n}}(-ibx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-2ibx^n\right)+e^{-2ia}(m+1)2^{-\frac{m+1}{n}}(ibx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},2ibx^n\right)-2n\right)}{4(m+1)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cos[a + b*x^n]^2,x]
```

```
[Out] -1/4*(x^(1 + m)*(-2*n + (E^((2*I)*a)*(1 + m)*Gamma[(1 + m)/n, (-2*I)*b*x^n]
)/(2^((1 + m)/n)*((-I)*b*x^n)^((1 + m)/n)) + ((1 + m)*Gamma[(1 + m)/n, (2*I
)*b*x^n])/(2^((1 + m)/n)*E^((2*I)*a)*(I*b*x^n)^((1 + m)/n))))/((1 + m)*n)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\cos(bx^n+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*cos(b*x^n + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m\cos(bx^n+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cos(b*x^n + a)^2, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int x^m \left(\cos^2\left(a + b\,x^n\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a+b*x^n)^2,x)
```

```
[Out] int(x^m*cos(a+b*x^n)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{xx^m + (m+1)\int x^m \cos\left(2\,bx^n + 2\,a\right)\,dx}{2\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x*x^m + (m + 1)*integrate(x^m*cos(2*b*x^n + 2*a), x))/(m + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m \cos\left(a + b\,x^n\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a + b*x^n)^2,x)
```

```
[Out] int(x^m*cos(a + b*x^n)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cos^2\left(a + bx^n\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(a+b*x**n)**2,x)
```

```
[Out] Integral(x**m*cos(a + b*x**n)**2, x)
```

3.78 $\int x^m \cos^3(a + bx^n)\, dx$

Optimal. Leaf size=229

$$-\frac{3e^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-ibx^n\right)}{8n}-\frac{3e^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},ibx^n\right)}{8n}-\frac{e^{3ia}3^{-\frac{m+1}{n}}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},\cdot\right.}{8n}$$

```
[Out] -3/8*exp(I*a)*x^(1+m)*GAMMA((1+m)/n,-I*b*x^n)/n/((-I*b*x^n)^((1+m)/n))-3/8*
x^(1+m)*GAMMA((1+m)/n,I*b*x^n)/exp(I*a)/n/((I*b*x^n)^((1+m)/n))-1/8*exp(3*I
*a)*x^(1+m)*GAMMA((1+m)/n,-3*I*b*x^n)/(3^((1+m)/n))/n/((-I*b*x^n)^((1+m)/n)
)-1/8*x^(1+m)*GAMMA((1+m)/n,3*I*b*x^n)/(3^((1+m)/n))/exp(3*I*a)/n/((I*b*x^n
)^((1+m)/n))
```

Rubi [A] time = 0.21, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3426, 3424, 2218}

$$-\frac{3e^{ia}x^{m+1}(-ibx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},-ibx^n\right)}{8n}-\frac{3e^{-ia}x^{m+1}(ibx^n)^{-\frac{m+1}{n}}\text{Gamma}\left(\frac{m+1}{n},ibx^n\right)}{8n}-\frac{e^{3ia}3^{-\frac{m+1}{n}}x^{m+1}(-ibx^n}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cos[a + b*x^n]^3,x]
```

```
[Out] (-3*E^(I*a)*x^(1 + m)*Gamma[(1 + m)/n, (-I)*b*x^n])/(8*n*((-I)*b*x^n)^((1 +
 m)/n)) - (3*x^(1 + m)*Gamma[(1 + m)/n, I*b*x^n])/(8*E^(I*a)*n*(I*b*x^n)^((
1 + m)/n)) - (E^((3*I)*a)*x^(1 + m)*Gamma[(1 + m)/n, (-3*I)*b*x^n])/(8*3^((
1 + m)/n)*n*((-I)*b*x^n)^((1 + m)/n)) - (x^(1 + m)*Gamma[(1 + m)/n, (3*I)*b
*x^n])/(8*3^((1 + m)/n)*E^((3*I)*a)*n*(I*b*x^n)^((1 + m)/n))
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 3424

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Dist[1/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 3426

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^m \cos^3(a+bx^n)\,dx = \int\left(\frac{3}{4}x^m\cos(a+bx^n)+\frac{1}{4}x^m\cos(3a+3bx^n)\right)dx$$
$$=\frac{1}{4}\int x^m\cos(3a+3bx^n)\,dx+\frac{3}{4}\int x^m\cos(a+bx^n)\,dx$$
$$=\frac{1}{8}\int e^{-3ia-3ibx^n}x^m\,dx+\frac{1}{8}\int e^{3ia+3ibx^n}x^m\,dx+\frac{3}{8}\int e^{-ia-ibx^n}x^m\,dx+\frac{3}{8}\int e^{ia+ibx^n}x^m\,dx$$
$$=-\frac{3e^{ia}x^{1+m}(-ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-ibx^n\right)}{8n}-\frac{3e^{-ia}x^{1+m}(ibx^n)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},ibx^n\right)}{8n}-\frac{3^{-\frac{1+m}{n}}e^{3}\cdots}{\cdots}$$

Mathematica [A] time = 0.56, size = 221, normalized size = 0.97

$$-\frac{e^{-3ia}3^{-\frac{m+1}{n}}x^{m+1}\left(b^2x^{2n}\right)^{-\frac{m+1}{n}}\left(e^{2ia}3^{\frac{m+n+1}{n}}(-ibx^n)^{\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},ibx^n\right)+e^{4ia}3^{\frac{m+n+1}{n}}(ibx^n)^{\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-ibx^n\right)+e^{6ia}(ib\cdots\right.}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cos[a + b*x^n]^3,x]
```

```
[Out] -1/8*(x^(1 + m)*(3^((1 + m + n)/n)*E^((4*I)*a)*(I*b*x^n)^((1 + m)/n)*Gamma[
(1 + m)/n, (-I)*b*x^n] + 3^((1 + m + n)/n)*E^((2*I)*a)*((-I)*b*x^n)^((1 + m
)/n)*Gamma[(1 + m)/n, I*b*x^n] + E^((6*I)*a)*(I*b*x^n)^((1 + m)/n)*Gamma[(1
 + m)/n, (-3*I)*b*x^n] + ((-I)*b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, (3*I)*b*
x^n]))/(3^((1 + m)/n)*E^((3*I)*a)*n*(b^2*x^(2*n))^((1 + m)/n))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m\cos(bx^n+a)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*cos(b*x^n + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cos(bx^n + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*x^n)^3,x, algorithm="giac")
[Out] integrate(x^m*cos(b*x^n + a)^3, x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int x^m \left(\cos^3(a + b\,x^n)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a+b*x^n)^3,x)
[Out] int(x^m*cos(a+b*x^n)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cos(bx^n + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cos(a+b*x^n)^3,x, algorithm="maxima")
[Out] integrate(x^m*cos(b*x^n + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^m \cos(a + b\,x^n)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cos(a + b*x^n)^3,x)
[Out] int(x^m*cos(a + b*x^n)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cos^3(a + bx^n) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cos(a+b*x**n)**3,x)
[Out] Integral(x**m*cos(a + b*x**n)**3, x)
```

3.79 $\int x^{-1-n} \cos(a + bx^n) \, dx$

Optimal. Leaf size=47

$$-\frac{b \sin(a)\text{Ci}(bx^n)}{n} - \frac{b \cos(a)\text{Si}(bx^n)}{n} - \frac{x^{-n} \cos(a + bx^n)}{n}$$

```
[Out] -cos(a+b*x^n)/n/(x^n)-b*cos(a)*Si(b*x^n)/n-b*Ci(b*x^n)*sin(a)/n
```

Rubi [A] time = 0.09, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3380, 3297, 3303, 3299, 3302}

$$-\frac{b \sin(a)\text{CosIntegral}(bx^n)}{n} - \frac{b \cos(a)\text{Si}(bx^n)}{n} - \frac{x^{-n} \cos(a + bx^n)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Cos[a + b*x^n],x]
```

```
[Out] -(Cos[a + b*x^n]/(n*x^n)) - (b*CosIntegral[b*x^n]*Sin[a])/n - (b*Cos[a]*Sin
Integral[b*x^n])/n
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$
\begin{aligned}
\int x^{-1-n}\cos\left(a+bx^n\right)\,dx &= \frac{\text{Subst}\left(\int \frac{\cos(a+bx)}{x^2}\,dx,x,x^n\right)}{n}\\
&= -\frac{x^{-n}\cos\left(a+bx^n\right)}{n}-\frac{b\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x}\,dx,x,x^n\right)}{n}\\
&= -\frac{x^{-n}\cos\left(a+bx^n\right)}{n}-\frac{(b\cos(a))\,\text{Subst}\left(\int \frac{\sin(bx)}{x}\,dx,x,x^n\right)}{n}-\frac{(b\sin(a))\,\text{Subst}\left(\int \frac{\cos(}{x}\right.}{n}\\
&= -\frac{x^{-n}\cos\left(a+bx^n\right)}{n}-\frac{b\text{Ci}\left(bx^n\right)\sin(a)}{n}-\frac{b\cos(a)\text{Si}\left(bx^n\right)}{n}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 45, normalized size = 0.96

$$
-\frac{x^{-n}\left(b\sin(a)x^n\text{Ci}\left(bx^n\right)+b\cos(a)x^n\text{Si}\left(bx^n\right)+\cos\left(a+bx^n\right)\right)}{n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Cos[a + b*x^n],x]
```

```
[Out] -((Cos[a + b*x^n] + b*x^n*CosIntegral[b*x^n]*Sin[a] + b*x^n*Cos[a]*SinInteg
ral[b*x^n])/(n*x^n))
```

fricas [A] time = 0.84, size = 62, normalized size = 1.32

$$
-\frac{bx^n\,\text{Ci}\left(bx^n\right)\sin(a)+bx^n\,\text{Ci}\left(-bx^n\right)\sin(a)+2\,bx^n\cos(a)\,\text{Si}\left(bx^n\right)+2\,\cos\left(bx^n+a\right)}{2\,nx^n}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cos(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^n*cos_integral(b*x^n)*sin(a) + b*x^n*cos_integral(-b*x^n)*sin(a)
+ 2*b*x^n*cos(a)*sin_integral(b*x^n) + 2*cos(b*x^n + a))/(n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1}\cos\left(bx^n+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cos(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*cos(b*x^n + a), x)
```

maple [A] time = 0.04, size = 45, normalized size = 0.96

$$\frac{b\left(-\frac{\cos(a+b\,x^n)x^{-n}}{b}-\mathrm{Si}\left(b\,x^n\right)\cos(a)-\mathrm{Ci}\left(b\,x^n\right)\sin(a)\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*cos(a+b*x^n),x)
```

```
[Out] 1/n*b*(-cos(a+b*x^n)/(x^n)/b-Si(b*x^n)*cos(a)-Ci(b*x^n)*sin(a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1}\cos\left(bx^n+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cos(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(-n - 1)*cos(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(a+b\,x^n\right)}{x^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n)/x^(n + 1),x)
```

```
[Out] int(cos(a + b*x^n)/x^(n + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1}\cos\left(a+bx^n\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*cos(a+b*x**n),x)
```

```
[Out] Integral(x**(-n - 1)*cos(a + b*x**n), x)
```

3.80 $\int x^{-1-n}\cos^2(a+bx^n)\,dx$

Optimal. Leaf size=69

$$-\frac{b\sin(2a)\text{Ci}(2bx^n)}{n}-\frac{b\cos(2a)\text{Si}(2bx^n)}{n}-\frac{x^{-n}\cos(2(a+bx^n))}{2n}-\frac{x^{-n}}{2n}$$

```
[Out] -1/2/n/(x^n)-1/2*cos(2*a+2*b*x^n)/n/(x^n)-b*cos(2*a)*Si(2*b*x^n)/n-b*Ci(2*b
*x^n)*sin(2*a)/n
```

Rubi [A] time = 0.12, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3426, 3380, 3297, 3303, 3299, 3302}

$$-\frac{b\sin(2a)\text{CosIntegral}(2bx^n)}{n}-\frac{b\cos(2a)\text{Si}(2bx^n)}{n}-\frac{x^{-n}\cos(2(a+bx^n))}{2n}-\frac{x^{-n}}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Cos[a + b*x^n]^2,x]
```

```
[Out] -1/(2*n*x^n) - Cos[2*(a + b*x^n)]/(2*n*x^n) - (b*CosIntegral[2*b*x^n]*Sin[2
*a])/n - (b*Cos[2*a]*SinIntegral[2*b*x^n])/n
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3426

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-n}\cos^2(a+bx^n)\,dx &= \int\left(\frac{x^{-1-n}}{2}+\frac{1}{2}x^{-1-n}\cos(2a+2bx^n)\right)dx\\
&= -\frac{x^{-n}}{2n}+\frac{1}{2}\int x^{-1-n}\cos(2a+2bx^n)\,dx\\
&= -\frac{x^{-n}}{2n}+\frac{\text{Subst}\left(\int\frac{\cos(2a+2bx)}{x^2}\,dx,x,x^n\right)}{2n}\\
&= -\frac{x^{-n}}{2n}-\frac{x^{-n}\cos(2(a+bx^n))}{2n}-\frac{b\,\text{Subst}\left(\int\frac{\sin(2a+2bx)}{x}\,dx,x,x^n\right)}{n}\\
&= -\frac{x^{-n}}{2n}-\frac{x^{-n}\cos(2(a+bx^n))}{2n}-\frac{(b\cos(2a))\,\text{Subst}\left(\int\frac{\sin(2bx)}{x}\,dx,x,x^n\right)}{n}-\frac{(b\sin(2a)}{}\\
&= -\frac{x^{-n}}{2n}-\frac{x^{-n}\cos(2(a+bx^n))}{2n}-\frac{b\text{Ci}(2bx^n)\sin(2a)}{n}-\frac{b\cos(2a)\text{Si}(2bx^n)}{n}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 53, normalized size = 0.77

$$-\frac{x^{-n}\left(b\sin(2a)x^n\text{Ci}(2bx^n)+b\cos(2a)x^n\text{Si}(2bx^n)+\cos^2(a+bx^n)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Cos[a + b*x^n]^2,x]
```

```
[Out] -((Cos[a + b*x^n]^2 + b*x^n*CosIntegral[2*b*x^n]*Sin[2*a] + b*x^n*Cos[2*a]*
SinIntegral[2*b*x^n])/(n*x^n))
```

fricas [A] time = 0.96, size = 72, normalized size = 1.04

$$-\frac{bx^n \operatorname{Ci}(2\,bx^n)\sin(2\,a) + bx^n \operatorname{Ci}(-2\,bx^n)\sin(2\,a) + 2\,bx^n\cos(2\,a)\operatorname{Si}(2\,bx^n) + 2\,\cos(bx^n + a)^2}{2\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cos(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^n*cos_integral(2*b*x^n)*sin(2*a) + b*x^n*cos_integral(-2*b*x^n)*s
in(2*a) + 2*b*x^n*cos(2*a)*sin_integral(2*b*x^n) + 2*cos(b*x^n + a)^2)/(n*x
^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1}\cos(bx^n + a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cos(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*cos(b*x^n + a)^2, x)
```

maple [A] time = 0.06, size = 65, normalized size = 0.94

$$-\frac{x^{-n}}{2n} + \frac{b\left(-\frac{\cos(2a+2b\,x^n)x^{-n}}{2b} - \operatorname{Si}(2b\,x^n)\cos(2a) - \operatorname{Ci}(2b\,x^n)\sin(2a)\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*cos(a+b*x^n)^2,x)
```

```
[Out] -1/2/n/(x^n)+1/n*b*(-1/2*cos(2*a+2*b*x^n)/(x^n)/b-Si(2*b*x^n)*cos(2*a)-Ci(2
*b*x^n)*sin(2*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{nx^n\int\frac{\cos(2\,bx^n+2\,a)}{xx^n}\,dx - 1}{2\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cos(a+b*x^n)^2,x, algorithm="maxima")
[Out] 1/2*(n*x^n*integrate(cos(2*b*x^n + 2*a)/(x*x^n), x) - 1)/(n*x^n)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x^n\right)^2}{x^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n)^2/x^(n + 1),x)
[Out] int(cos(a + b*x^n)^2/x^(n + 1), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*cos(a+b*x**n)**2,x)
[Out] Timed out
```

3.81 $\int x^{-1-n}\cos^3\left(a+bx^n\right)\,dx$

Optimal. Leaf size=113

$$-\frac{3b\sin(a)\mathrm{Ci}\left(bx^n\right)}{4n}-\frac{3b\sin(3a)\mathrm{Ci}\left(3bx^n\right)}{4n}-\frac{3b\cos(a)\mathrm{Si}\left(bx^n\right)}{4n}-\frac{3b\cos(3a)\mathrm{Si}\left(3bx^n\right)}{4n}-\frac{3x^{-n}\cos\left(a+bx^n\right)}{4n}-\frac{x^{-n}\cos(3}{4}$$

```
[Out] -3/4*cos(a+b*x^n)/n/(x^n)-1/4*cos(3*a+3*b*x^n)/n/(x^n)-3/4*b*cos(a)*Si(b*x^
n)/n-3/4*b*cos(3*a)*Si(3*b*x^n)/n-3/4*b*Ci(b*x^n)*sin(a)/n-3/4*b*Ci(3*b*x^n
)*sin(3*a)/n
```

Rubi [A] time = 0.20, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3426, 3380, 3297, 3303, 3299, 3302}

$$-\frac{3b\sin(a)\mathrm{CosIntegral}\left(bx^n\right)}{4n}-\frac{3b\sin(3a)\mathrm{CosIntegral}\left(3bx^n\right)}{4n}-\frac{3b\cos(a)\mathrm{Si}\left(bx^n\right)}{4n}-\frac{3b\cos(3a)\mathrm{Si}\left(3bx^n\right)}{4n}-\frac{3x^{-n}\cos(}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)*Cos[a + b*x^n]^3,x]
```

```
[Out] (-3*Cos[a + b*x^n])/(4*n*x^n) - Cos[3*(a + b*x^n)]/(4*n*x^n) - (3*b*CosInte
gral[b*x^n]*Sin[a])/(4*n) - (3*b*CosIntegral[3*b*x^n]*Sin[3*a])/(4*n) - (3*
b*Cos[a]*SinIntegral[b*x^n])/(4*n) - (3*b*Cos[3*a]*SinIntegral[3*b*x^n])/(4
*n)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3426

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^{-1-n}\cos^3(a+bx^n)\,dx &= \int\left(\frac{3}{4}x^{-1-n}\cos(a+bx^n)+\frac{1}{4}x^{-1-n}\cos(3a+3bx^n)\right)dx\\
&= \frac{1}{4}\int x^{-1-n}\cos(3a+3bx^n)\,dx+\frac{3}{4}\int x^{-1-n}\cos(a+bx^n)\,dx\\
&= \frac{\text{Subst}\left(\int\frac{\cos(3a+3bx)}{x^2}dx,x,x^n\right)}{4n}+\frac{3\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x^2}dx,x,x^n\right)}{4n}\\
&= -\frac{3x^{-n}\cos(a+bx^n)}{4n}-\frac{x^{-n}\cos(3(a+bx^n))}{4n}-\frac{(3b)\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x}dx,x,x^n\right)}{4n}-\cdots\\
&= -\frac{3x^{-n}\cos(a+bx^n)}{4n}-\frac{x^{-n}\cos(3(a+bx^n))}{4n}-\frac{(3b\cos(a))\,\text{Subst}\left(\int\frac{\sin(bx)}{x}dx,x,x^n\right)}{4n}\cdots\\
&= -\frac{3x^{-n}\cos(a+bx^n)}{4n}-\frac{x^{-n}\cos(3(a+bx^n))}{4n}-\frac{3b\text{Ci}(bx^n)\sin(a)}{4n}-\frac{3b\text{Ci}(3bx^n)\sin\cdots}{4n}
\end{aligned}
$$

Mathematica [A] time = 0.24, size = 95, normalized size = 0.84

$$
-\frac{x^{-n}\left(3b\sin(a)x^n\text{Ci}(bx^n)+3b\sin(3a)x^n\text{Ci}(3bx^n)+3b\cos(a)x^n\text{Si}(bx^n)+3b\cos(3a)x^n\text{Si}(3bx^n)+3\cos(a+b\cdots\right.}{4n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)*Cos[a + b*x^n]^3,x]
```

```
[Out] -1/4*(3*Cos[a + b*x^n] + Cos[3*(a + b*x^n)] + 3*b*x^n*CosIntegral[b*x^n]*Si
n[a] + 3*b*x^n*CosIntegral[3*b*x^n]*Sin[3*a] + 3*b*x^n*Cos[a]*SinIntegral[b
*x^n] + 3*b*x^n*Cos[3*a]*SinIntegral[3*b*x^n])/(n*x^n)
```

fricas [A] time = 1.04, size = 117, normalized size = 1.04

$$-\frac{3\,bx^n\,\mathrm{Ci}\left(3\,bx^n\right)\sin\left(3\,a\right)+3\,bx^n\,\mathrm{Ci}\left(-3\,bx^n\right)\sin\left(3\,a\right)+3\,bx^n\,\mathrm{Ci}\left(bx^n\right)\sin(a)+3\,bx^n\,\mathrm{Ci}\left(-bx^n\right)\sin(a)+6\,bx^n\,\cos}{8\,nx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cos(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*b*x^n*cos_integral(3*b*x^n)*sin(3*a) + 3*b*x^n*cos_integral(-3*b*x^
n)*sin(3*a) + 3*b*x^n*cos_integral(b*x^n)*sin(a) + 3*b*x^n*cos_integral(-b*
x^n)*sin(a) + 6*b*x^n*cos(3*a)*sin_integral(3*b*x^n) + 6*b*x^n*cos(a)*sin_i
ntegral(b*x^n) + 8*cos(b*x^n + a)^3)/(n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1}\cos\left(bx^n+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cos(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)*cos(b*x^n + a)^3, x)
```

maple [A] time = 0.06, size = 101, normalized size = 0.89

$$\frac{3b\left(-\frac{\cos(a+b\,x^n)x^{-n}}{b}-\mathrm{Si}\left(b\,x^n\right)\cos(a)-\mathrm{Ci}\left(b\,x^n\right)\sin(a)\right)}{4n}+\frac{3b\left(-\frac{\cos(3a+3b\,x^n)x^{-n}}{3b}-\mathrm{Si}\left(3b\,x^n\right)\cos\left(3a\right)-\mathrm{Ci}\left(3b\,x^n\right)\sin(}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)*cos(a+b*x^n)^3,x)
```

```
[Out] 3/4/n*b*(-cos(a+b*x^n)/(x^n)/b-Si(b*x^n)*cos(a)-Ci(b*x^n)*sin(a))+3/4/n*b*(
-1/3*cos(3*a+3*b*x^n)/(x^n)/b-Si(3*b*x^n)*cos(3*a)-Ci(3*b*x^n)*sin(3*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-n-1}\cos\left(bx^n+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)*cos(a+b*x^n)^3,x, algorithm="maxima")
[Out] integrate(x^(-n - 1)*cos(b*x^n + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a + b\,x^n\right)^3}{x^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n)^3/x^(n + 1),x)
[Out] int(cos(a + b*x^n)^3/x^(n + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)*cos(a+b*x**n)**3,x)
[Out] Timed out
```

3.82 $\int x^{-1-2n} \cos(a+bx^n)\,dx$

Optimal. Leaf size=78

$$-\frac{b^2\cos(a)\mathrm{Ci}(bx^n)}{2n}+\frac{b^2\sin(a)\mathrm{Si}(bx^n)}{2n}+\frac{bx^{-n}\sin(a+bx^n)}{2n}-\frac{x^{-2n}\cos(a+bx^n)}{2n}$$

```
[Out] -1/2*b^2*Ci(b*x^n)*cos(a)/n-1/2*cos(a+b*x^n)/n/(x^(2*n))+1/2*b^2*Si(b*x^n)*
sin(a)/n+1/2*b*sin(a+b*x^n)/n/(x^n)
```

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3380, 3297, 3303, 3299, 3302}

$$-\frac{b^2\cos(a)\mathrm{CosIntegral}(bx^n)}{2n}+\frac{b^2\sin(a)\mathrm{Si}(bx^n)}{2n}+\frac{bx^{-n}\sin(a+bx^n)}{2n}-\frac{x^{-2n}\cos(a+bx^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)*Cos[a + b*x^n],x]
```

```
[Out] -Cos[a + b*x^n]/(2*n*x^(2*n)) - (b^2*Cos[a]*CosIntegral[b*x^n])/(2*n) + (b*
Sin[a + b*x^n])/(2*n*x^n) + (b^2*Sin[a]*SinIntegral[b*x^n])/(2*n)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}\int x^{-1-2n}\cos\left(a+bx^n\right)\,dx &= \frac{\text{Subst}\left(\int \frac{\cos(a+bx)}{x^3}\,dx, x, x^n\right)}{n}\\ &= -\frac{x^{-2n}\cos\left(a+bx^n\right)}{2n}-\frac{b\,\text{Subst}\left(\int \frac{\sin(a+bx)}{x^2}\,dx, x, x^n\right)}{2n}\\ &= -\frac{x^{-2n}\cos\left(a+bx^n\right)}{2n}+\frac{bx^{-n}\sin\left(a+bx^n\right)}{2n}-\frac{b^2\,\text{Subst}\left(\int \frac{\cos(a+bx)}{x}\,dx, x, x^n\right)}{2n}\\ &= -\frac{x^{-2n}\cos\left(a+bx^n\right)}{2n}+\frac{bx^{-n}\sin\left(a+bx^n\right)}{2n}-\frac{\left(b^2\cos(a)\right)\text{Subst}\left(\int \frac{\cos(bx)}{x}\,dx, x, x^n\right)}{2n}\\ &= -\frac{x^{-2n}\cos\left(a+bx^n\right)}{2n}-\frac{b^2\cos(a)\text{Ci}\left(bx^n\right)}{2n}+\frac{bx^{-n}\sin\left(a+bx^n\right)}{2n}+\frac{b^2\sin(a)\text{Si}\left(bx^n\right)}{2n}\end{aligned}$$

Mathematica [A] time = 0.14, size = 70, normalized size = 0.90

$$-\frac{x^{-2n}\left(b^2\cos(a)x^{2n}\text{Ci}\left(bx^n\right)-b^2\sin(a)x^{2n}\text{Si}\left(bx^n\right)-bx^n\sin\left(a+bx^n\right)+\cos\left(a+bx^n\right)\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*Cos[a + b*x^n],x]
```

```
[Out] -1/2*(Cos[a + b*x^n] + b^2*x^(2*n)*Cos[a]*CosIntegral[b*x^n] - b*x^n*Sin[a
+ b*x^n] - b^2*x^(2*n)*Sin[a]*SinIntegral[b*x^n])/(n*x^(2*n))
```

fricas [A] time = 0.92, size = 90, normalized size = 1.15

$$-\frac{b^2x^{2\,n}\cos(a)\,\text{Ci}\left(bx^n\right)+b^2x^{2\,n}\cos(a)\,\text{Ci}\left(-bx^n\right)-2\,b^2x^{2\,n}\sin(a)\,\text{Si}\left(bx^n\right)-2\,bx^n\sin\left(bx^n+a\right)+2\,\cos\left(bx^n+a\right)}{4\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*x^(2*n)*cos(a)*cos_integral(b*x^n) + b^2*x^(2*n)*cos(a)*cos_integ
ral(-b*x^n) - 2*b^2*x^(2*n)*sin(a)*sin_integral(b*x^n) - 2*b*x^n*sin(b*x^n
+ a) + 2*cos(b*x^n + a))/(n*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-2\,n-1} \cos\left(bx^{n} + a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)*cos(b*x^n + a), x)
```

maple [A] time = 0.04, size = 65, normalized size = 0.83

$$\frac{b^{2} \left(-\frac{\cos(a+b\,x^{n})x^{-2n}}{2b^{2}}+\frac{\sin(a+b\,x^{n})x^{-n}}{2b}+\frac{\mathrm{Si}(b\,x^{n})\sin(a)}{2}-\frac{\mathrm{Ci}(b\,x^{n})\cos(a)}{2}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*cos(a+b*x^n),x)
```

```
[Out] 1/n*b^2*(-1/2*cos(a+b*x^n)/(x^n)^2/b^2+1/2*sin(a+b*x^n)/(x^n)/b+1/2*Si(b*x^
n)*sin(a)-1/2*Ci(b*x^n)*cos(a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-2\,n-1} \cos\left(bx^{n} + a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(-2*n - 1)*cos(b*x^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x^{n}\right)}{x^{2\,n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n)/x^(2*n + 1),x)
```

```
[Out] int(cos(a + b*x^n)/x^(2*n + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-2n-1}\cos\left(a+bx^{n}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*cos(a+b*x**n),x)
```

```
[Out] Integral(x**(-2*n - 1)*cos(a + b*x**n), x)
```

3.83 $\int x^{-1-2n}\cos^2(a+bx^n)\,dx$

Optimal. Leaf size=95

$$-\frac{b^2\cos(2a)\text{Ci}(2bx^n)}{n}+\frac{b^2\sin(2a)\text{Si}(2bx^n)}{n}+\frac{bx^{-n}\sin(2(a+bx^n))}{2n}-\frac{x^{-2n}\cos(2(a+bx^n))}{4n}-\frac{x^{-2n}}{4n}$$

```
[Out] -1/4/n/(x^(2*n))-b^2*Ci(2*b*x^n)*cos(2*a)/n-1/4*cos(2*a+2*b*x^n)/n/(x^(2*n)
)+b^2*Si(2*b*x^n)*sin(2*a)/n+1/2*b*sin(2*a+2*b*x^n)/n/(x^n)
```

Rubi [A] time = 0.15, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3426, 3380, 3297, 3303, 3299, 3302}

$$-\frac{b^2\cos(2a)\text{CosIntegral}(2bx^n)}{n}+\frac{b^2\sin(2a)\text{Si}(2bx^n)}{n}+\frac{bx^{-n}\sin(2(a+bx^n))}{2n}-\frac{x^{-2n}\cos(2(a+bx^n))}{4n}-\frac{x^{-2n}}{4n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)*Cos[a + b*x^n]^2,x]
```

```
[Out] -1/(4*n*x^(2*n)) - Cos[2*(a + b*x^n)]/(4*n*x^(2*n)) - (b^2*Cos[2*a]*CosInte
gral[2*b*x^n])/n + (b*Sin[2*(a + b*x^n)])/(2*n*x^n) + (b^2*Sin[2*a]*SinInte
gral[2*b*x^n])/n
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3426

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^{-1-2n}\cos^2(a+bx^n)\,dx=\int\left(\frac{1}{2}x^{-1-2n}+\frac{1}{2}x^{-1-2n}\cos(2a+2bx^n)\right)dx$$

$$=-\frac{x^{-2n}}{4n}+\frac{1}{2}\int x^{-1-2n}\cos(2a+2bx^n)\,dx$$

$$=-\frac{x^{-2n}}{4n}+\frac{\mathrm{Subst}\left(\int\frac{\cos(2a+2bx)}{x^3}\,dx,x,x^n\right)}{2n}$$

$$=-\frac{x^{-2n}}{4n}-\frac{x^{-2n}\cos(2(a+bx^n))}{4n}-\frac{b\,\mathrm{Subst}\left(\int\frac{\sin(2a+2bx)}{x^2}\,dx,x,x^n\right)}{2n}$$

$$=-\frac{x^{-2n}}{4n}-\frac{x^{-2n}\cos(2(a+bx^n))}{4n}+\frac{bx^{-n}\sin(2(a+bx^n))}{2n}-\frac{b^2\,\mathrm{Subst}\left(\int\frac{\cos(2a+2bx)}{x}\right.}{n}$$

$$=-\frac{x^{-2n}}{4n}-\frac{x^{-2n}\cos(2(a+bx^n))}{4n}+\frac{bx^{-n}\sin(2(a+bx^n))}{2n}-\frac{\left(b^2\cos(2a)\right)\mathrm{Subst}\left(\int\right.}{n}$$

$$=-\frac{x^{-2n}}{4n}-\frac{x^{-2n}\cos(2(a+bx^n))}{4n}-\frac{b^2\cos(2a)\mathrm{Ci}(2bx^n)}{n}+\frac{bx^{-n}\sin(2(a+bx^n))}{2n}+$$

Mathematica [A] time = 0.22, size = 82, normalized size = 0.86

$$-\frac{x^{-2n}\left(4b^2\cos(2a)x^{2n}\mathrm{Ci}(2bx^n)-4b^2\sin(2a)x^{2n}\mathrm{Si}(2bx^n)-2bx^n\sin(2(a+bx^n))+\cos(2(a+bx^n))+1\right)}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*Cos[a + b*x^n]^2,x]
```

```
[Out] -1/4*(1 + Cos[2*(a + b*x^n)] + 4*b^2*x^(2*n)*Cos[2*a]*CosIntegral[2*b*x^n]
- 2*b*x^n*Sin[2*(a + b*x^n)] - 4*b^2*x^(2*n)*Sin[2*a]*SinIntegral[2*b*x^n])
/(n*x^(2*n))
```

fricas [A] time = 0.70, size = 106, normalized size = 1.12

$$-\frac{b^2x^{2\,n}\cos\left(2\,a\right)\operatorname{Ci}\left(2\,bx^n\right)+b^2x^{2\,n}\cos\left(2\,a\right)\operatorname{Ci}\left(-2\,bx^n\right)-2\,b^2x^{2\,n}\sin\left(2\,a\right)\operatorname{Si}\left(2\,bx^n\right)-2\,bx^n\cos\left(bx^n+a\right)\sin\left(bx^n\right.}{2\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*x^(2*n)*cos(2*a)*cos_integral(2*b*x^n) + b^2*x^(2*n)*cos(2*a)*cos
_integral(-2*b*x^n) - 2*b^2*x^(2*n)*sin(2*a)*sin_integral(2*b*x^n) - 2*b*x^
n*cos(b*x^n + a)*sin(b*x^n + a) + cos(b*x^n + a)^2)/(n*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-2\,n-1}\cos\left(bx^n+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)*cos(b*x^n + a)^2, x)
```

maple [A] time = 0.06, size = 89, normalized size = 0.94

$$-\frac{x^{-2n}}{4n}+\frac{2b^2\left(-\frac{x^{-2n}\cos(2a+2b\,x^n)}{8b^2}+\frac{\sin(2a+2b\,x^n)x^{-n}}{4b}+\frac{\operatorname{Si}(2b\,x^n)\sin(2a)}{2}-\frac{\operatorname{Ci}(2b\,x^n)\cos(2a)}{2}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*cos(a+b*x^n)^2,x)
```

```
[Out] -1/4/(x^n)^2/n+2/n*b^2*(-1/8/(x^n)^2/b^2*cos(2*a+2*b*x^n)+1/4*sin(2*a+2*b*x
^n)/(x^n)/b+1/2*Si(2*b*x^n)*sin(2*a)-1/2*Ci(2*b*x^n)*cos(2*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,nx^{2\,n}\int\frac{\cos\left(2\,bx^n+2\,a\right)}{xx^{2\,n}}\,dx-1}{4\,nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*n*x^(2*n)*integrate(cos(2*b*x^n + 2*a)/(x*x^(2*n)), x) - 1)/(n*x^(2*
n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x^n\right)^2}{x^{2\,n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n)^2/x^(2*n + 1),x)
```

```
[Out] int(cos(a + b*x^n)^2/x^(2*n + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*cos(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

3.84 $\int x^{-1-2n}\cos^3(a+bx^n)\,dx$

Optimal. Leaf size=165

$$-\frac{3b^2\cos(a)\text{Ci}(bx^n)}{8n}-\frac{9b^2\cos(3a)\text{Ci}(3bx^n)}{8n}+\frac{3b^2\sin(a)\text{Si}(bx^n)}{8n}+\frac{9b^2\sin(3a)\text{Si}(3bx^n)}{8n}+\frac{3bx^{-n}\sin(a+bx^n)}{8n}+\frac{3bx^{-}}{}$$

```
[Out] -3/8*b^2*Ci(b*x^n)*cos(a)/n-9/8*b^2*Ci(3*b*x^n)*cos(3*a)/n-3/8*cos(a+b*x^n)
/n/(x^(2*n))-1/8*cos(3*a+3*b*x^n)/n/(x^(2*n))+3/8*b^2*Si(b*x^n)*sin(a)/n+9/
8*b^2*Si(3*b*x^n)*sin(3*a)/n+3/8*b*sin(a+b*x^n)/n/(x^n)+3/8*b*sin(3*a+3*b*x
^n)/n/(x^n)
```

Rubi [A] time = 0.25, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3426, 3380, 3297, 3303, 3299, 3302}

$$-\frac{3b^2\cos(a)\text{CosIntegral}(bx^n)}{8n}-\frac{9b^2\cos(3a)\text{CosIntegral}(3bx^n)}{8n}+\frac{3b^2\sin(a)\text{Si}(bx^n)}{8n}+\frac{9b^2\sin(3a)\text{Si}(3bx^n)}{8n}+\frac{3bx^{-n}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)*Cos[a + b*x^n]^3,x]
```

```
[Out] (-3*Cos[a + b*x^n])/(8*n*x^(2*n)) - Cos[3*(a + b*x^n)]/(8*n*x^(2*n)) - (3*b
^2*Cos[a]*CosIntegral[b*x^n])/(8*n) - (9*b^2*Cos[3*a]*CosIntegral[3*b*x^n])
/(8*n) + (3*b*Sin[a + b*x^n])/(8*n*x^n) + (3*b*Sin[3*(a + b*x^n)])/(8*n*x^n
) + (3*b^2*Sin[a]*SinIntegral[b*x^n])/(8*n) + (9*b^2*Sin[3*a]*SinIntegral[3
*b*x^n])/(8*n)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3426

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.), x
_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cos[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^{-1-2n}\cos^3\left(a+bx^n\right)\,dx=\int\left(\frac{3}{4}x^{-1-2n}\cos\left(a+bx^n\right)+\frac{1}{4}x^{-1-2n}\cos\left(3a+3bx^n\right)\right)dx$$

$$=\frac{1}{4}\int x^{-1-2n}\cos\left(3a+3bx^n\right)\,dx+\frac{3}{4}\int x^{-1-2n}\cos\left(a+bx^n\right)\,dx$$

$$=\frac{\text{Subst}\left(\int\frac{\cos(3a+3bx)}{x^3}\,dx,x,x^n\right)}{4n}+\frac{3\,\text{Subst}\left(\int\frac{\cos(a+bx)}{x^3}\,dx,x,x^n\right)}{4n}$$

$$=-\frac{3x^{-2n}\cos\left(a+bx^n\right)}{8n}-\frac{x^{-2n}\cos\left(3\left(a+bx^n\right)\right)}{8n}-\frac{(3b)\,\text{Subst}\left(\int\frac{\sin(a+bx)}{x^2}\,dx,x,x^n\right)}{8n}-$$

$$=-\frac{3x^{-2n}\cos\left(a+bx^n\right)}{8n}-\frac{x^{-2n}\cos\left(3\left(a+bx^n\right)\right)}{8n}+\frac{3bx^{-n}\sin\left(a+bx^n\right)}{8n}+\frac{3bx^{-n}\sin(3}{8n}$$

$$=-\frac{3x^{-2n}\cos\left(a+bx^n\right)}{8n}-\frac{x^{-2n}\cos\left(3\left(a+bx^n\right)\right)}{8n}+\frac{3bx^{-n}\sin\left(a+bx^n\right)}{8n}+\frac{3bx^{-n}\sin(3}{8n}$$

$$=-\frac{3x^{-2n}\cos\left(a+bx^n\right)}{8n}-\frac{x^{-2n}\cos\left(3\left(a+bx^n\right)\right)}{8n}-\frac{3b^2\cos(a)\text{Ci}\left(bx^n\right)}{8n}-\frac{9b^2\cos(3a)\text{C}}{8n}$$

Mathematica [A] time = 0.36, size = 141, normalized size = 0.85

$$-\frac{x^{-2n}\left(3b^2\cos(a)x^{2n}\text{Ci}\left(bx^n\right)+9b^2\cos(3a)x^{2n}\text{Ci}\left(3bx^n\right)-3b^2\sin(a)x^{2n}\text{Si}\left(bx^n\right)-9b^2\sin(3a)x^{2n}\text{Si}\left(3bx^n\right)-3bx^n\right.}{8n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)*Cos[a + b*x^n]^3,x]
```

```
[Out] -1/8*(3*Cos[a + b*x^n] + Cos[3*(a + b*x^n)] + 3*b^2*x^(2*n)*Cos[a]*CosInteg
ral[b*x^n] + 9*b^2*x^(2*n)*Cos[3*a]*CosIntegral[3*b*x^n] - 3*b*x^n*Sin[a +
b*x^n] - 3*b*x^n*Sin[3*(a + b*x^n)] - 3*b^2*x^(2*n)*Sin[a]*SinIntegral[b*x^
n] - 9*b^2*x^(2*n)*Sin[3*a]*SinIntegral[3*b*x^n])/(n*x^(2*n))
```

fricas [A] time = 0.92, size = 167, normalized size = 1.01

$$-\frac{9\,b^2x^{2\,n}\cos\left(3\,a\right)\text{Ci}\left(3\,bx^n\right)+3\,b^2x^{2\,n}\cos(a)\,\text{Ci}\left(bx^n\right)+3\,b^2x^{2\,n}\cos(a)\,\text{Ci}\left(-bx^n\right)+9\,b^2x^{2\,n}\cos\left(3\,a\right)\text{Ci}\left(-3\,bx^n\right)-}{16\,nx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(9*b^2*x^(2*n)*cos(3*a)*cos_integral(3*b*x^n) + 3*b^2*x^(2*n)*cos(a)*
cos_integral(b*x^n) + 3*b^2*x^(2*n)*cos(a)*cos_integral(-b*x^n) + 9*b^2*x^(
```

```
2*n)*cos(3*a)*cos_integral(-3*b*x^n) - 24*b*x^n*cos(b*x^n + a)^2*sin(b*x^n
+ a) - 18*b^2*x^(2*n)*sin(3*a)*sin_integral(3*b*x^n) - 6*b^2*x^(2*n)*sin(a)
*sin_integral(b*x^n) + 8*cos(b*x^n + a)^3)/(n*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-2\,n-1} \cos\left(bx^{n} + a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)*cos(b*x^n + a)^3, x)
```

maple [A] time = 0.07, size = 144, normalized size = 0.87

$$\frac{3b^{2}\left(-\frac{\cos(a+b\,x^{n})x^{-2n}}{2b^{2}}+\frac{\sin(a+b\,x^{n})x^{-n}}{2b}+\frac{\mathrm{Si}(b\,x^{n})\sin(a)}{2}-\frac{\mathrm{Ci}(b\,x^{n})\cos(a)}{2}\right)}{4n}+\frac{9b^{2}\left(-\frac{\cos(3a+3b\,x^{n})x^{-2n}}{18b^{2}}+\frac{\sin(3a+3b\,x^{n})x^{-n}}{6b}+\frac{\mathrm{Si}(3b\,x^{n}}{}\right.}{4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)*cos(a+b*x^n)^3,x)
```

```
[Out] 3/4/n*b^2*(-1/2*cos(a+b*x^n)/(x^n)^2/b^2+1/2*sin(a+b*x^n)/(x^n)/b+1/2*Si(b*
x^n)*sin(a)-1/2*Ci(b*x^n)*cos(a))+9/4/n*b^2*(-1/18*cos(3*a+3*b*x^n)/(x^n)^2
/b^2+1/6*sin(3*a+3*b*x^n)/(x^n)/b+1/2*Si(3*b*x^n)*sin(3*a)-1/2*Ci(3*b*x^n)*
cos(3*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-2\,n-1} \cos\left(bx^{n} + a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)*cos(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^(-2*n - 1)*cos(b*x^n + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x^{n}\right)^{3}}{x^{2\,n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x^n)^3/x^(2*n + 1),x)
```

```
[Out] int(cos(a + b*x^n)^3/x^(2*n + 1), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)*cos(a+b*x**n)**3,x)
```

```
[Out] Timed out
```

3.85 $\int x^2 \cos\left((a+bx)^2\right) dx$

Optimal. Leaf size=99

$$\frac{\sqrt{\frac{\pi}{2}}\, a^2 C\left(\sqrt{\frac{2}{\pi}}\,(a+bx)\right)}{b^3}-\frac{\sqrt{\frac{\pi}{2}}\, S\left(\sqrt{\frac{2}{\pi}}\,(a+bx)\right)}{2b^3}-\frac{a\sin\left((a+bx)^2\right)}{b^3}+\frac{(a+bx)\sin\left((a+bx)^2\right)}{2b^3}$$

```
[Out] -a*sin((b*x+a)^2)/b^3+1/2*(b*x+a)*sin((b*x+a)^2)/b^3+1/2*a^2*FresnelC((b*x+
a)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b^3-1/4*FresnelS((b*x+a)*2^(1/2)/Pi^(
1/2))*2^(1/2)*Pi^(1/2)/b^3
```

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3434, 3352, 3380, 2637, 3386, 3351}

$$\frac{\sqrt{\frac{\pi}{2}}\, a^2 \text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,(a+bx)\right)}{b^3}-\frac{\sqrt{\frac{\pi}{2}}\, S\left(\sqrt{\frac{2}{\pi}}\,(a+bx)\right)}{2b^3}-\frac{a\sin\left((a+bx)^2\right)}{b^3}+\frac{(a+bx)\sin\left((a+bx)^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[(a + b*x)^2],x]
```

```
[Out] (a^2*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*(a + b*x)])/b^3 - (Sqrt[Pi/2]*FresnelS[
Sqrt[2/Pi]*(a + b*x)])/(2*b^3) - (a*Sin[(a + b*x)^2])/b^3 + ((a + b*x)*Sin[
(a + b*x)^2])/(2*b^3)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3386

```
Int[Cos[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_))^(m_.), x_Symbol] :> Simp[(e^(
n - 1)*(e*x)^(m - n + 1)*Sin[c + d*x^n])/(d*n), x] - Dist[(e^n*(m - n + 1))
/(d*n), Int[(e*x)^(m - n)*Sin[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] &&
IGtQ[n, 0] && LtQ[n, m + 1]
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int x^2\cos\left((a+bx)^2\right)dx = \frac{\text{Subst}\left(\int\left(a^2\cos\left(x^2\right)-2ax\cos\left(x^2\right)+x^2\cos\left(x^2\right)\right)dx,x,a+bx\right)}{b^3}$$
$$= \frac{\text{Subst}\left(\int x^2\cos\left(x^2\right)dx,x,a+bx\right)}{b^3} - \frac{(2a)\,\text{Subst}\left(\int x\cos\left(x^2\right)dx,x,a+bx\right)}{b^3} + \frac{a^2\,\text{Sub}}{}$$
$$= \frac{a^2\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{b^3} + \frac{(a+bx)\sin\left((a+bx)^2\right)}{2b^3} - \frac{\text{Subst}\left(\int\sin\left(x^2\right)dx,x,a+bx\right)}{2b^3} -$$
$$= \frac{a^2\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{b^3} - \frac{\sqrt{\frac{\pi}{2}}\,S\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)}{2b^3} - \frac{a\sin\left((a+bx)^2\right)}{b^3} + \frac{(a+bx)\sin\left((a-\right.}{2b^3}$$

Mathematica [A] time = 0.28, size = 76, normalized size = 0.77

$$-\frac{-2\sqrt{2\pi}\,a^2C\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)+\sqrt{2\pi}\,S\left(\sqrt{\frac{2}{\pi}}(a+bx)\right)+2(a-bx)\sin\left((a+bx)^2\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[(a + b*x)^2],x]
```

```
[Out] -1/4*(-2*a^2*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*(a + b*x)] + Sqrt[2*Pi]*Fresnel
S[Sqrt[2/Pi]*(a + b*x)] + 2*(a - b*x)*Sin[(a + b*x)^2])/b^3
```

fricas [A] time = 0.82, size = 112, normalized size = 1.13

$$\frac{2\sqrt{2}\,\pi a^2\sqrt{\frac{b^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\,(bx+a)\sqrt{\frac{b^2}{\pi}}}{b}\right)-\sqrt{2}\,\pi\sqrt{\frac{b^2}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}\,(bx+a)\sqrt{\frac{b^2}{\pi}}}{b}\right)+2\left(b^2x-ab\right)\sin\left(b^2x^2+2\,abx+a^2\right)}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos((b*x+a)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(2)*pi*a^2*sqrt(b^2/pi)*fresnel_cos(sqrt(2)*(b*x + a)*sqrt(b^2/p
i)/b) - sqrt(2)*pi*sqrt(b^2/pi)*fresnel_sin(sqrt(2)*(b*x + a)*sqrt(b^2/pi)/
b) + 2*(b^2*x - a*b)*sin(b^2*x^2 + 2*a*b*x + a^2))/b^4
```

giac [C] time = 0.46, size = 159, normalized size = 1.61

$$-\frac{\frac{(i+1)\,\sqrt{2}\,\sqrt{\pi}\left(2\,a^2+i\right)\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\left(x+\frac{a}{b}\right)|b|\right)}{|b|}+\frac{4\left(i\,b\left(x+\frac{a}{b}\right)-2i\,a\right)e^{\left(i\,b^2x^2+2i\,abx+i\,a^2\right)}}{b}}{16\,b^2}-\frac{-\frac{(i-1)\,\sqrt{2}\,\sqrt{\pi}\left(2\,a^2-i\right)\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\left(x+\frac{a}{b}\right)|b|\right)}{|b|}+\frac{4}{}}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/16*((I + 1)*sqrt(2)*sqrt(pi)*(2*a^2 + I)*erf((1/2*I - 1/2)*sqrt(2)*(x +
a/b)*abs(b))/abs(b) + 4*(I*b*(x + a/b) - 2*I*a)*e^(I*b^2*x^2 + 2*I*a*b*x +
I*a^2)/b)/b^2 - 1/16*(-(I - 1)*sqrt(2)*sqrt(pi)*(2*a^2 - I)*erf(-(1/2*I + 1
/2)*sqrt(2)*(x + a/b)*abs(b))/abs(b) + 4*(-I*b*(x + a/b) + 2*I*a)*e^(-I*b^2
*x^2 - 2*I*a*b*x - I*a^2)/b)/b^2
```

maple [A] time = 0.03, size = 131, normalized size = 1.32

$$\frac{x\sin\left(x^2b^2+2abx+a^2\right)}{2b^2}-\frac{a\left(\frac{\sin\left(x^2b^2+2abx+a^2\right)}{2b^2}-\frac{a\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\left(b^2x+ab\right)}{\sqrt{\pi}\,\sqrt{b^2}}\right)}{2b\sqrt{b^2}}\right)}{b}-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\left(b^2x+ab\right)}{\sqrt{\pi}\,\sqrt{b^2}}\right)}{4b^2\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos((b*x+a)^2),x)
```

```
[Out] 1/2/b^2*x*sin(b^2*x^2+2*a*b*x+a^2)-a/b*(1/2/b^2*sin(b^2*x^2+2*a*b*x+a^2)-1/
2*a/b*2^(1/2)*Pi^(1/2)/(b^2)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)/(b^2)^(1/2)*(b
```

```
^2*x+a*b)))-1/4/b^2*2^(1/2)*Pi^(1/2)/(b^2)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)/
(b^2)^(1/2)*(b^2*x+a*b))
```

maxima [C] time = 3.19, size = 256, normalized size = 2.59

$$\frac{abx\left(8i\,e^{\left(i\,b^2x^2+2i\,abx+i\,a^2\right)}-8i\,e^{\left(-i\,b^2x^2-2i\,abx-i\,a^2\right)}\right)+a^2\left(8i\,e^{\left(i\,b^2x^2+2i\,abx+i\,a^2\right)}-8i\,e^{\left(-i\,b^2x^2-2i\,abx-i\,a^2\right)}\right)+2\sqrt{b^2x^2+2\,abx}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos((b*x+a)^2),x, algorithm="maxima")
```

```
[Out] 1/16*(a*b*x*(8*I*e^(I*b^2*x^2 + 2*I*a*b*x + I*a^2) - 8*I*e^(-I*b^2*x^2 - 2*
I*a*b*x - I*a^2)) + a^2*(8*I*e^(I*b^2*x^2 + 2*I*a*b*x + I*a^2) - 8*I*e^(-I*
b^2*x^2 - 2*I*a*b*x - I*a^2)) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*((-(I - 1)*
sqrt(2)*sqrt(pi)*(erf(sqrt(I*b^2*x^2 + 2*I*a*b*x + I*a^2)) - 1) + (I + 1)*s
qrt(2)*sqrt(pi)*(erf(sqrt(-I*b^2*x^2 - 2*I*a*b*x - I*a^2)) - 1))*a^2 + (I +
 1)*sqrt(2)*gamma(3/2, I*b^2*x^2 + 2*I*a*b*x + I*a^2) - (I - 1)*sqrt(2)*gam
ma(3/2, -I*b^2*x^2 - 2*I*a*b*x - I*a^2)))/(b^4*x + a*b^3)
```

mupad [B] time = 0.15, size = 80, normalized size = 0.81

$$\frac{x\,\sin\left((a+b\,x)^2\right)}{2\,b^2}-\frac{a\,\sin\left((a+b\,x)^2\right)}{2\,b^3}-\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,(a+b\,x)}{\sqrt{\pi}}\right)}{4\,b^3}+\frac{\sqrt{2}\,a^2\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,(a+b\,x)}{\sqrt{\pi}}\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos((a + b*x)^2),x)
```

```
[Out] (x*sin((a + b*x)^2))/(2*b^2) - (a*sin((a + b*x)^2))/(2*b^3) - (2^(1/2)*pi^(
1/2)*fresnels((2^(1/2)*(a + b*x))/pi^(1/2)))/(4*b^3) + (2^(1/2)*a^2*pi^(1/2
)*fresnelc((2^(1/2)*(a + b*x))/pi^(1/2)))/(2*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cos\left(a^2+2abx+b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos((b*x+a)**2),x)
```

```
[Out] Integral(x**2*cos(a**2 + 2*a*b*x + b**2*x**2), x)
```

3.86 $\int x \cos\left((a+bx)^2\right) dx$

Optimal. Leaf size=47

$$\frac{\sin\left((a+bx)^2\right)}{2b^2} - \frac{\sqrt{\frac{\pi}{2}}\, a C\left(\sqrt{\frac{2}{\pi}}\,(a+bx)\right)}{b^2}$$

```
[Out] 1/2*sin((b*x+a)^2)/b^2-1/2*a*FresnelC((b*x+a)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^
(1/2)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3434, 3352, 3380, 2637}

$$\frac{\sin\left((a+bx)^2\right)}{2b^2} - \frac{\sqrt{\frac{\pi}{2}}\, a\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,(a+bx)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[(a + b*x)^2],x]
```

```
[Out] -((a*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*(a + b*x)])/b^2) + Sin[(a + b*x)^2]/(2*
b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3434

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Module[{k = If[FractionQ[n], Denominat
or[n], 1]}, Dist[k/f^(m + 1), Subst[Int[ExpandIntegrand[(a + b*Cos[c + d*x^
(k*n)])^p, x^(k - 1)*(f*g - e*h + h*x^k)^m, x], x], x, (e + f*x)^(1/k)], x]
] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int x\cos\left((a+bx)^2\right)\,dx &= \frac{\text{Subst}\left(\int\left(-a\cos\left(x^2\right)+x\cos\left(x^2\right)\right)\,dx,x,a+bx\right)}{b^2}\\ &= \frac{\text{Subst}\left(\int x\cos\left(x^2\right)\,dx,x,a+bx\right)}{b^2}-\frac{a\,\text{Subst}\left(\int\cos\left(x^2\right)\,dx,x,a+bx\right)}{b^2}\\ &= -\frac{a\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{\frac{2}{\pi}}\,(a+bx)\right)}{b^2}+\frac{\text{Subst}\left(\int\cos(x)\,dx,x,(a+bx)^2\right)}{2b^2}\\ &= -\frac{a\sqrt{\frac{\pi}{2}}\,C\left(\sqrt{\frac{2}{\pi}}\,(a+bx)\right)}{b^2}+\frac{\sin\left((a+bx)^2\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 0.89

$$\frac{\sin\left((a+bx)^2\right)-\sqrt{2\pi}\,aC\left(\sqrt{\frac{2}{\pi}}\,(a+bx)\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[(a + b*x)^2],x]
[Out] (-(a*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*(a + b*x)]) + Sin[(a + b*x)^2])/(2*b^2)
```

fricas [A] time = 0.99, size = 63, normalized size = 1.34

$$-\frac{\sqrt{2}\,\pi a\sqrt{\frac{b^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\,(bx+a)\sqrt{\frac{b^2}{\pi}}}{b}\right)-b\sin\left(b^2x^2+2\,abx+a^2\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((b*x+a)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*pi*a*sqrt(b^2/pi)*fresnel_cos(sqrt(2)*(b*x + a)*sqrt(b^2/pi)/
b) - b*sin(b^2*x^2 + 2*a*b*x + a^2))/b^3
```

giac [C] time = 0.55, size = 119, normalized size = 2.53

$$-\frac{-\frac{(i+1)\sqrt{2}\sqrt{\pi}a\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\left(x+\frac{a}{b}\right)|b|\right)}{|b|}+\frac{2ie^{\left(ib^2x^2+2iabx+ia^2\right)}}{b}}{8b}-\frac{\frac{(i-1)\sqrt{2}\sqrt{\pi}a\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\left(x+\frac{a}{b}\right)|b|\right)}{|b|}-\frac{2ie^{\left(-ib^2x^2-2iabx-ia^2\right)}}{b}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -1/8*(-(I + 1)*sqrt(2)*sqrt(pi)*a*erf((1/2*I - 1/2)*sqrt(2)*(x + a/b)*abs(b
))/abs(b) + 2*I*e^(I*b^2*x^2 + 2*I*a*b*x + I*a^2)/b)/b - 1/8*((I - 1)*sqrt(
2)*sqrt(pi)*a*erf(-(1/2*I + 1/2)*sqrt(2)*(x + a/b)*abs(b))/abs(b) - 2*I*e^(
-I*b^2*x^2 - 2*I*a*b*x - I*a^2)/b)/b
```

maple [A] time = 0.03, size = 63, normalized size = 1.34

$$\frac{\sin\left(x^2b^2+2abx+a^2\right)}{2b^2}-\frac{a\sqrt{2}\,\sqrt{\pi}\,\;\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\left(b^2x+ab\right)}{\sqrt{\pi}\,\sqrt{b^2}}\right)}{2b\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos((b*x+a)^2),x)
```

```
[Out] 1/2/b^2*sin(b^2*x^2+2*a*b*x+a^2)-1/2*a/b*2^(1/2)*Pi^(1/2)/(b^2)^(1/2)*Fresn
elC(2^(1/2)/Pi^(1/2)/(b^2)^(1/2)*(b^2*x+a*b))
```

maxima [C] time = 1.87, size = 197, normalized size = 4.19

$$-\frac{bx\left(4ie^{\left(ib^2x^2+2iabx+ia^2\right)}-4ie^{\left(-ib^2x^2-2iabx-ia^2\right)}\right)+2\sqrt{b^2x^2+2abx+a^2}\left(-(i-1)\sqrt{2}\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{ib^2x^2+2iabx+\dots}\right.\right.\right.}{16\left(b^3\dots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/16*(b*x*(4*I*e^(I*b^2*x^2 + 2*I*a*b*x + I*a^2) - 4*I*e^(-I*b^2*x^2 - 2*I
*a*b*x - I*a^2)) + 2*sqrt(b^2*x^2 + 2*a*b*x + a^2)*(-(I - 1)*sqrt(2)*sqrt(p
i)*(erf(sqrt(I*b^2*x^2 + 2*I*a*b*x + I*a^2)) - 1) + (I + 1)*sqrt(2)*sqrt(pi
)*(erf(sqrt(-I*b^2*x^2 - 2*I*a*b*x - I*a^2)) - 1))*a + a*(4*I*e^(I*b^2*x^2
+ 2*I*a*b*x + I*a^2) - 4*I*e^(-I*b^2*x^2 - 2*I*a*b*x - I*a^2)))/(b^3*x + a*
b^2)
```

mupad [B] time = 0.07, size = 39, normalized size = 0.83

$$\frac{\sin\left((a+b\,x)^2\right)}{2\,b^2}-\frac{\sqrt{2}\,a\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,(a+b\,x)}{\sqrt{\pi}}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos((a + b*x)^2),x)
```

```
[Out] sin((a + b*x)^2)/(2*b^2) - (2^(1/2)*a*pi^(1/2)*fresnelc((2^(1/2)*(a + b*x))
/pi^(1/2)))/(2*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(a^2+2abx+b^2x^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos((b*x+a)**2),x)
```

```
[Out] Integral(x*cos(a**2 + 2*a*b*x + b**2*x**2), x)
```

3.87 $\int \cos\left((a+bx)^2\right) dx$

Optimal. Leaf size=29

$$\frac{\sqrt{\frac{\pi}{2}}\, C\left(\sqrt{\frac{2}{\pi}}\,(a+bx)\right)}{b}$$

```
[Out] 1/2*FresnelC((b*x+a)*2^(1/2)/Pi^(1/2))*2^(1/2)*Pi^(1/2)/b
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3352}

$$\frac{\sqrt{\frac{\pi}{2}}\, \text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\,(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(a + b*x)^2],x]
```

```
[Out] (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*(a + b*x)])/b
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \cos\left((a+bx)^2\right) dx = \frac{\sqrt{\frac{\pi}{2}}\, C\left(\sqrt{\frac{2}{\pi}}\,(a+bx)\right)}{b}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{\sqrt{\frac{\pi}{2}}\, C\left(\sqrt{\frac{2}{\pi}}\,(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(a + b*x)^2],x]
```

```
[Out] (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*(a + b*x)])/b
```

fricas [A] time = 0.95, size = 40, normalized size = 1.38

$$\frac{\sqrt{2}\,\pi\sqrt{\frac{b^2}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}\,(bx+a)\sqrt{\frac{b^2}{\pi}}}{b}\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*pi*sqrt(b^2/pi)*fresnel_cos(sqrt(2)*(b*x + a)*sqrt(b^2/pi)/b)/b
^2
```

giac [C] time = 0.40, size = 55, normalized size = 1.90

$$-\frac{(i+1)\,\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{2}\left(x+\frac{a}{b}\right)|b|\right)}{8\,|b|}+\frac{(i-1)\,\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\left(x+\frac{a}{b}\right)|b|\right)}{8\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)^2),x, algorithm="giac")
```

```
[Out] -(1/8*I + 1/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*(x + a/b)*abs(b))
/abs(b) + (1/8*I - 1/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*(x + a/
b)*abs(b))/abs(b)
```

maple [A] time = 0.02, size = 36, normalized size = 1.24

$$\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\left(b^2x+ab\right)}{\sqrt{\pi}\,\sqrt{b^2}}\right)}{2\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((b*x+a)^2),x)
```

```
[Out] 1/2*2^(1/2)*Pi^(1/2)/(b^2)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)/(b^2)^(1/2)*(b^2
*x+a*b))
```

maxima [C] time = 0.82, size = 84, normalized size = 2.90

$$-\frac{\sqrt{\pi}\left((i-1)\,\sqrt{2}\,\operatorname{erf}\left(-(-1)^{\frac{3}{4}}\,(i\,bx+i\,a)\right)+(i-1)\,\sqrt{2}\,\operatorname{erf}\left(-\left(\frac{1}{4}i-\frac{1}{4}\right)\,\sqrt{2}\,(2i\,bx+2i\,a)\right)-(i+1)\,\sqrt{2}\,\operatorname{erf}\left(-\left(\frac{1}{4}i+\right.\right.\right.}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)^2),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(pi)*((I - 1)*sqrt(2)*erf(-(-1)^(3/4)*(I*b*x + I*a)) + (I - 1)*sq
rt(2)*erf(-(1/4*I - 1/4)*sqrt(2)*(2*I*b*x + 2*I*a)) - (I + 1)*sqrt(2)*erf(-
(1/4*I + 1/4)*sqrt(2)*(2*I*b*x + 2*I*a)) + (I + 1)*sqrt(2)*erf((I*b*x + I*a
)/sqrt(-I)))/b
```

mupad [B] time = 0.06, size = 32, normalized size = 1.10

$$\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,b\,\sqrt{\frac{1}{b^2}}\,(a+b\,x)}{\sqrt{\pi}}\right)\sqrt{\frac{1}{b^2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((a + b*x)^2),x)
```

```
[Out] (2^(1/2)*pi^(1/2)*fresnelc((2^(1/2)*b*(1/b^2)^(1/2)*(a + b*x))/pi^(1/2))*(1
/b^2)^(1/2))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left((a+bx)^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)**2),x)
```

```
[Out] Integral(cos((a + b*x)**2), x)
```

3.88 $\int \frac{\cos\left((a+bx)^2\right)}{x}\,dx$

Optimal. Leaf size=15

$$\operatorname{Int}\left(\frac{\cos\left((a+bx)^2\right)}{x},x\right)$$

```
[Out] Unintegrable(cos((b*x+a)^2)/x,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left((a+bx)^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[(a + b*x)^2]/x,x]
```

```
[Out] Defer[Int][Cos[(a + b*x)^2]/x, x]
```

Rubi steps

$$\int \frac{\cos\left((a+bx)^2\right)}{x}\,dx = \int \frac{\cos\left((a+bx)^2\right)}{x}\,dx$$

Mathematica [A] time = 2.65, size = 0, normalized size = 0.00

$$\int \frac{\cos\left((a+bx)^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[(a + b*x)^2]/x,x]
```

```
[Out] Integrate[Cos[(a + b*x)^2]/x, x]
```

fricas [A] time = 0.92, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\cos\left(b^2x^2+2\,abx+a^2\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)^2)/x,x, algorithm="fricas")
[Out] integral(cos(b^2*x^2 + 2*a*b*x + a^2)/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left((bx+a)^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)^2)/x,x, algorithm="giac")
[Out] integrate(cos((b*x + a)^2)/x, x)
```

maple [A] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\cos\left((bx+a)^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((b*x+a)^2)/x,x)
[Out] int(cos((b*x+a)^2)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left((bx+a)^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)^2)/x,x, algorithm="maxima")
[Out] integrate(cos((b*x + a)^2)/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cos\left((a+b\,x)^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((a + b*x)^2)/x,x)
[Out] int(cos((a + b*x)^2)/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a^2 + 2abx + b^2x^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)**2)/x,x)
```

```
[Out] Integral(cos(a**2 + 2*a*b*x + b**2*x**2)/x, x)
```

3.89 $\int \frac{\cos\left((a+bx)^2\right)}{x^2} \, dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\cos\left((a+bx)^2\right)}{x^2},x\right)$$

```
[Out] Unintegrable(cos((b*x+a)^2)/x^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos\left((a+bx)^2\right)}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[(a + b*x)^2]/x^2,x]
```

```
[Out] Defer[Int][Cos[(a + b*x)^2]/x^2, x]
```

Rubi steps

$$\int \frac{\cos\left((a+bx)^2\right)}{x^2} \, dx = \int \frac{\cos\left((a+bx)^2\right)}{x^2} \, dx$$

Mathematica [A] time = 5.22, size = 0, normalized size = 0.00

$$\int \frac{\cos\left((a+bx)^2\right)}{x^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[(a + b*x)^2]/x^2,x]
```

```
[Out] Integrate[Cos[(a + b*x)^2]/x^2, x]
```

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(b^2x^2+2\,abx+a^2\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)^2)/x^2,x, algorithm="fricas")
[Out] integral(cos(b^2*x^2 + 2*a*b*x + a^2)/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left((bx+a)^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)^2)/x^2,x, algorithm="giac")
[Out] integrate(cos((b*x + a)^2)/x^2, x)
```

maple [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\cos\left((bx+a)^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((b*x+a)^2)/x^2,x)
[Out] int(cos((b*x+a)^2)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left((bx+a)^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)^2)/x^2,x, algorithm="maxima")
[Out] integrate(cos((b*x + a)^2)/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\cos\left((a+b\,x)^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos((a + b*x)^2)/x^2,x)
[Out] int(cos((a + b*x)^2)/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a^2 + 2abx + b^2x^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos((b*x+a)**2)/x**2,x)
```

```
[Out] Integral(cos(a**2 + 2*a*b*x + b**2*x**2)/x**2, x)
```

3.90 $\int x^2 \cos\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal. Leaf size=346

$$\frac{240\cos\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{240\sqrt{c+dx}\,\sin\left(a+b\sqrt{c+dx}\right)}{b^5d^3}-\frac{120(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^3}+\frac{24c\cos\left(a+b\sqrt{c+\right.}}{b^4d^3}$$

```
[Out] 240*cos(a+b*(d*x+c)^(1/2))/b^6/d^3+24*c*cos(a+b*(d*x+c)^(1/2))/b^4/d^3+2*c^
2*cos(a+b*(d*x+c)^(1/2))/b^2/d^3-120*(d*x+c)*cos(a+b*(d*x+c)^(1/2))/b^4/d^3
-12*c*(d*x+c)*cos(a+b*(d*x+c)^(1/2))/b^2/d^3+10*(d*x+c)^2*cos(a+b*(d*x+c)^(
1/2))/b^2/d^3-40*(d*x+c)^(3/2)*sin(a+b*(d*x+c)^(1/2))/b^3/d^3-4*c*(d*x+c)^(
3/2)*sin(a+b*(d*x+c)^(1/2))/b/d^3+2*(d*x+c)^(5/2)*sin(a+b*(d*x+c)^(1/2))/b/
d^3+240*sin(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^5/d^3+24*c*sin(a+b*(d*x+c)^(
1/2))*(d*x+c)^(1/2)/b^3/d^3+2*c^2*sin(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b/d^
3
```

Rubi [A] time = 0.30, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3432, 3296, 2638}

$$\frac{2c^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{40(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^3}+\frac{24c\sqrt{c+dx}\,\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^3}+\frac{240\sqrt{c+dx}\,\sin\left(a\right.}{b^5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (240*Cos[a + b*Sqrt[c + d*x]])/(b^6*d^3) + (24*c*Cos[a + b*Sqrt[c + d*x]])/
(b^4*d^3) + (2*c^2*Cos[a + b*Sqrt[c + d*x]])/(b^2*d^3) - (120*(c + d*x)*Cos
[a + b*Sqrt[c + d*x]])/(b^4*d^3) - (12*c*(c + d*x)*Cos[a + b*Sqrt[c + d*x]]
)/(b^2*d^3) + (10*(c + d*x)^2*Cos[a + b*Sqrt[c + d*x]])/(b^2*d^3) + (240*Sq
rt[c + d*x]*Sin[a + b*Sqrt[c + d*x]])/(b^5*d^3) + (24*c*Sqrt[c + d*x]*Sin[a
 + b*Sqrt[c + d*x]])/(b^3*d^3) + (2*c^2*Sqrt[c + d*x]*Sin[a + b*Sqrt[c + d*
x]])/(b*d^3) - (40*(c + d*x)^(3/2)*Sin[a + b*Sqrt[c + d*x]])/(b^3*d^3) - (4
*c*(c + d*x)^(3/2)*Sin[a + b*Sqrt[c + d*x]])/(b*d^3) + (2*(c + d*x)^(5/2)*S
in[a + b*Sqrt[c + d*x]])/(b*d^3)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int x^2\cos\left(a+b\sqrt{c+dx}\right)dx = \frac{2\,\mathrm{Subst}\left(\int\left(\frac{c^2x\cos(a+bx)}{d^2}-\frac{2cx^3\cos(a+bx)}{d^2}+\frac{x^5\cos(a+bx)}{d^2}\right)dx,x,\sqrt{c+dx}\right)}{d}$$

$$= \frac{2\,\mathrm{Subst}\left(\int x^5\cos(a+bx)\,dx,x,\sqrt{c+dx}\right)}{d^3}-\frac{(4c)\,\mathrm{Subst}\left(\int x^3\cos(a+bx)\,dx,x,\right.}{d^3}$$

$$= \frac{2c^2\sqrt{c+dx}\,\sin\left(a+b\sqrt{c+dx}\right)}{bd^3}-\frac{4c(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^3}+\frac{2(c+dx)}{}$$

$$= \frac{2c^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{12c(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{10(c+dx)^2\cos\left(a\right.}{b^2d^3}$$

$$= \frac{2c^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{12c(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3}+\frac{10(c+dx)^2\cos\left(a\right.}{b^2d^3}$$

$$= \frac{24c\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^3}+\frac{2c^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{120(c+dx)\cos\left(a+b\sqrt{c\right.}}{b^4d^3}$$

$$= \frac{24c\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^3}+\frac{2c^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-\frac{120(c+dx)\cos\left(a+b\sqrt{c\right.}}{b^4d^3}$$

$$= \frac{240\cos\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{24c\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^3}+\frac{2c^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^3}-$$

Mathematica [C] time = 0.83, size = 224, normalized size = 0.65

$$\frac{e^{-i\left(a+b\sqrt{c+dx}\right)}\left(\left(-ib^5d^2x^2\sqrt{c+dx}+b^4dx(4c+5dx)+4ib^3\sqrt{c+dx}\,(2c+5dx)-12b^2(4c+5dx)-120ib\sqrt{c+dx}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (120 + (120*I)*b*Sqrt[c + d*x] + I*b^5*d^2*x^2*Sqrt[c + d*x] - (4*I)*b^3*Sq
rt[c + d*x]*(2*c + 5*d*x) - 12*b^2*(4*c + 5*d*x) + b^4*d*x*(4*c + 5*d*x) +
E^((2*I)*(a + b*Sqrt[c + d*x]))*(120 - (120*I)*b*Sqrt[c + d*x] - I*b^5*d^2*
x^2*Sqrt[c + d*x] + (4*I)*b^3*Sqrt[c + d*x]*(2*c + 5*d*x) - 12*b^2*(4*c + 5
*d*x) + b^4*d*x*(4*c + 5*d*x)))/(b^6*d^3*E^(I*(a + b*Sqrt[c + d*x])))
```

fricas [A] time = 1.75, size = 103, normalized size = 0.30

$$\frac{2\left(\left(b^5d^2x^2-20\,b^3dx-8\,b^3c+120\,b\right)\sqrt{dx+c}\,\sin\left(\sqrt{dx+c}\,b+a\right)+\left(5\,b^4d^2x^2-48\,b^2c+4\left(b^4c-15\,b^2\right)dx+120\right.}{b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*((b^5*d^2*x^2 - 20*b^3*d*x - 8*b^3*c + 120*b)*sqrt(d*x + c)*sin(sqrt(d*x
+ c)*b + a) + (5*b^4*d^2*x^2 - 48*b^2*c + 4*(b^4*c - 15*b^2)*d*x + 120)*cos
(sqrt(d*x + c)*b + a))/(b^6*d^3)
```

giac [A] time = 0.45, size = 480, normalized size = 1.39

$$2\left(\frac{\left(b^4c^2-6\left(\sqrt{dx+c}\,b+a\right)^2b^2c+12\left(\sqrt{dx+c}\,b+a\right)ab^2c-6\,a^2b^2c+5\left(\sqrt{dx+c}\,b+a\right)^4-20\left(\sqrt{dx+c}\,b+a\right)^3a+30\left(\sqrt{dx+c}\,b+a\right)^2a^2-20\left(\sqrt{dx+c}\,b+a\right)a^3+5\,a^4+12\,b\right.}{b^4d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*((b^4*c^2 - 6*(sqrt(d*x + c)*b + a)^2*b^2*c + 12*(sqrt(d*x + c)*b + a)*a*
b^2*c - 6*a^2*b^2*c + 5*(sqrt(d*x + c)*b + a)^4 - 20*(sqrt(d*x + c)*b + a)^
3*a + 30*(sqrt(d*x + c)*b + a)^2*a^2 - 20*(sqrt(d*x + c)*b + a)*a^3 + 5*a^4
 + 12*b^2*c - 60*(sqrt(d*x + c)*b + a)^2 + 120*(sqrt(d*x + c)*b + a)*a - 60
*a^2 + 120)*cos(sqrt(d*x + c)*b + a)/(b^4*d^2) + ((sqrt(d*x + c)*b + a)*b^4
*c^2 - a*b^4*c^2 - 2*(sqrt(d*x + c)*b + a)^3*b^2*c + 6*(sqrt(d*x + c)*b + a
)^2*a*b^2*c - 6*(sqrt(d*x + c)*b + a)*a^2*b^2*c + 2*a^3*b^2*c + (sqrt(d*x +
 c)*b + a)^5 - 5*(sqrt(d*x + c)*b + a)^4*a + 10*(sqrt(d*x + c)*b + a)^3*a^2
 - 10*(sqrt(d*x + c)*b + a)^2*a^3 + 5*(sqrt(d*x + c)*b + a)*a^4 - a^5 + 12*
(sqrt(d*x + c)*b + a)*b^2*c - 12*a*b^2*c - 20*(sqrt(d*x + c)*b + a)^3 + 60*
(sqrt(d*x + c)*b + a)^2*a - 60*(sqrt(d*x + c)*b + a)*a^2 + 20*a^3 + 120*sqr
t(d*x + c)*b)*sin(sqrt(d*x + c)*b + a)/(b^4*d^2))/(b^2*d)
```

maple [B] time = 0.05, size = 825, normalized size = 2.38

$$2c^2\left(\cos\left(a+b\sqrt{dx+c}\right)+\left(a+b\sqrt{dx+c}\right)\sin\left(a+b\sqrt{dx+c}\right)\right)-2a\,c^2\sin\left(a+b\sqrt{dx+c}\right)-\frac{4c\left(\left(a+b\sqrt{dx+c}\right)^3\sin\left(a+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/d^3/b^2*(c^2*(cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^
(1/2)))-a*c^2*sin(a+b*(d*x+c)^(1/2))-2/b^2*c*((a+b*(d*x+c)^(1/2))^3*sin(a+b
*(d*x+c)^(1/2))+3*(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x+c)^(1/2))-6*cos(a+b*(d
*x+c)^(1/2))-6*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))+6/b^2*a*c*((a+b*
(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))-2*sin(a+b*(d*x+c)^(1/2))+2*cos(a+b*
(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2)))-6/b^2*a^2*c*(cos(a+b*(d*x+c)^(1/2))+(a+
b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))+2/b^2*a^3*c*sin(a+b*(d*x+c)^(1/2))
+1/b^4*((a+b*(d*x+c)^(1/2))^5*sin(a+b*(d*x+c)^(1/2))+5*(a+b*(d*x+c)^(1/2))^
4*cos(a+b*(d*x+c)^(1/2))-20*(a+b*(d*x+c)^(1/2))^3*sin(a+b*(d*x+c)^(1/2))-60
*(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x+c)^(1/2))+120*cos(a+b*(d*x+c)^(1/2))+12
0*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))-5/b^4*a*((a+b*(d*x+c)^(1/2))^
4*sin(a+b*(d*x+c)^(1/2))+4*(a+b*(d*x+c)^(1/2))^3*cos(a+b*(d*x+c)^(1/2))-12*
(a+b*(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))+24*sin(a+b*(d*x+c)^(1/2))-24*c
os(a+b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2)))+10/b^4*a^2*((a+b*(d*x+c)^(1/2))^
3*sin(a+b*(d*x+c)^(1/2))+3*(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x+c)^(1/2))-6*c
os(a+b*(d*x+c)^(1/2))-6*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))-10/b^4*
a^3*((a+b*(d*x+c)^(1/2))^2*sin(a+b*(d*x+c)^(1/2))-2*sin(a+b*(d*x+c)^(1/2))+
2*cos(a+b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2)))+5/b^4*a^4*(cos(a+b*(d*x+c)^(1
/2))+(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))-1/b^4*a^5*sin(a+b*(d*x+c)^
(1/2)))
```

maxima [B] time = 0.65, size = 672, normalized size = 1.94

$$-\frac{2\left(ac^2\sin\left(\sqrt{dx+c}\,b+a\right)-\left(\left(\sqrt{dx+c}\,b+a\right)\sin\left(\sqrt{dx+c}\,b+a\right)+\cos\left(\sqrt{dx+c}\,b+a\right)\right)c^2-\frac{2\,a^3c\sin\left(\sqrt{dx+c}\,b+a\right)}{b^2}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*(a*c^2*sin(sqrt(d*x + c)*b + a) - ((sqrt(d*x + c)*b + a)*sin(sqrt(d*x +
c)*b + a) + cos(sqrt(d*x + c)*b + a))*c^2 - 2*a^3*c*sin(sqrt(d*x + c)*b + a
)/b^2 + 6*((sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a) + cos(sqrt(d*x +
c)*b + a))*a^2*c/b^2 + a^5*sin(sqrt(d*x + c)*b + a)/b^4 - 5*((sqrt(d*x + c)
*b + a)*sin(sqrt(d*x + c)*b + a) + cos(sqrt(d*x + c)*b + a))*a^4/b^4 - 6*(2
*(sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) + ((sqrt(d*x + c)*b + a)^2
- 2)*sin(sqrt(d*x + c)*b + a))*a*c/b^2 + 10*(2*(sqrt(d*x + c)*b + a)*cos(sq
rt(d*x + c)*b + a) + ((sqrt(d*x + c)*b + a)^2 - 2)*sin(sqrt(d*x + c)*b + a)
)*a^3/b^4 + 2*(3*((sqrt(d*x + c)*b + a)^2 - 2)*cos(sqrt(d*x + c)*b + a) + (
(sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*sin(sqrt(d*x + c)*b + a)
)*c/b^2 - 10*(3*((sqrt(d*x + c)*b + a)^2 - 2)*cos(sqrt(d*x + c)*b + a) + ((
```

```
sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*sin(sqrt(d*x + c)*b + a))
*a^2/b^4 + 5*(4*((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d*x + c)*b - 6*a)*cos(sqr
t(d*x + c)*b + a) + ((sqrt(d*x + c)*b + a)^4 - 12*(sqrt(d*x + c)*b + a)^2 +
 24)*sin(sqrt(d*x + c)*b + a))*a/b^4 - (5*((sqrt(d*x + c)*b + a)^4 - 12*(sq
rt(d*x + c)*b + a)^2 + 24)*cos(sqrt(d*x + c)*b + a) + ((sqrt(d*x + c)*b + a
)^5 - 20*(sqrt(d*x + c)*b + a)^3 + 120*sqrt(d*x + c)*b + 120*a)*sin(sqrt(d*
x + c)*b + a))/b^4)/(b^2*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \cos\left(a + b\sqrt{c + d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] int(x^2*cos(a + b*(c + d*x)^(1/2)), x)
```

sympy [A] time = 1.65, size = 269, normalized size = 0.78

$$\begin{cases} \frac{x^3\cos(a)}{3} \\ \frac{x^3\cos\left(a+b\sqrt{c}\right)}{3} \\ \frac{2x^2\sqrt{c+dx}\,\sin\left(a+b\sqrt{c+dx}\right)}{bd} + \frac{8cx\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{10x^2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d} - \frac{16c\sqrt{c+dx}\,\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^3} - \frac{40x\sqrt{c+dx}\,\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x**3*cos(a)/3, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**3*cos(a +
b*sqrt(c))/3, Eq(d, 0)), (2*x**2*sqrt(c + d*x)*sin(a + b*sqrt(c + d*x))/(b*
d) + 8*c*x*cos(a + b*sqrt(c + d*x))/(b**2*d**2) + 10*x**2*cos(a + b*sqrt(c
+ d*x))/(b**2*d) - 16*c*sqrt(c + d*x)*sin(a + b*sqrt(c + d*x))/(b**3*d**3)
- 40*x*sqrt(c + d*x)*sin(a + b*sqrt(c + d*x))/(b**3*d**2) - 96*c*cos(a + b*
sqrt(c + d*x))/(b**4*d**3) - 120*x*cos(a + b*sqrt(c + d*x))/(b**4*d**2) + 2
40*sqrt(c + d*x)*sin(a + b*sqrt(c + d*x))/(b**5*d**3) + 240*cos(a + b*sqrt(
c + d*x))/(b**6*d**3), True))
```

3.91 $\int x\cos\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal. Leaf size=167

$$-\frac{12\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^2}-\frac{12\sqrt{c+dx}\,\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^2}+\frac{6(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{2c\cos\left(a+b\sqrt{c+dx\right.}}{b^2d^2}$$

```
[Out] -12*cos(a+b*(d*x+c)^(1/2))/b^4/d^2-2*c*cos(a+b*(d*x+c)^(1/2))/b^2/d^2+6*(d*
x+c)*cos(a+b*(d*x+c)^(1/2))/b^2/d^2+2*(d*x+c)^(3/2)*sin(a+b*(d*x+c)^(1/2))/
b/d^2-12*sin(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b^3/d^2-2*c*sin(a+b*(d*x+c)^(
1/2))*(d*x+c)^(1/2)/b/d^2
```

Rubi [A] time = 0.14, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3432, 3296, 2638}

$$-\frac{12\sqrt{c+dx}\,\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^2}+\frac{6(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{2c\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2}-\frac{12\cos\left(a+b\sqrt{c+dx\right.}}{b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-12*Cos[a + b*Sqrt[c + d*x]])/(b^4*d^2) - (2*c*Cos[a + b*Sqrt[c + d*x]])/(
b^2*d^2) + (6*(c + d*x)*Cos[a + b*Sqrt[c + d*x]])/(b^2*d^2) - (12*Sqrt[c +
d*x]*Sin[a + b*Sqrt[c + d*x]])/(b^3*d^2) - (2*c*Sqrt[c + d*x]*Sin[a + b*Sqr
t[c + d*x]])/(b*d^2) + (2*(c + d*x)^(3/2)*Sin[a + b*Sqrt[c + d*x]])/(b*d^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
```

```
0] && IntegerQ[1/n]
```

Rubi steps

$$\int x\cos\left(a+b\sqrt{c+dx}\right)dx = \frac{2\,\text{Subst}\left(\int\left(-\frac{cx\cos(a+bx)}{d}+\frac{x^3\cos(a+bx)}{d}\right)dx,x,\sqrt{c+dx}\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int x^3\cos(a+bx)\,dx,x,\sqrt{c+dx}\right)}{d^2} - \frac{(2c)\,\text{Subst}\left(\int x\cos(a+bx)\,dx,x,\sqrt{c-}\right.}{d^2}$$
$$= -\frac{2c\sqrt{c+dx}\,\sin\left(a+b\sqrt{c+dx}\right)}{bd^2} + \frac{2(c+dx)^{3/2}\sin\left(a+b\sqrt{c+dx}\right)}{bd^2} - \frac{6\,\text{Subst}\left(\int x\right.}{}$$
$$= -\frac{2c\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{6(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2} - \frac{2c\sqrt{c+dx}\,\sin\left(a+b\right.}{bd^2}$$
$$= -\frac{2c\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{6(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2} - \frac{12\sqrt{c+dx}\,\sin\left(a+b\right.}{b^3d^2}$$
$$= -\frac{12\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^2} - \frac{2c\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{6(c+dx)\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2}$$

Mathematica [A] time = 0.28, size = 71, normalized size = 0.43

$$\frac{2\left(b\left(b^2dx-6\right)\sqrt{c+dx}\,\sin\left(a+b\sqrt{c+dx}\right)+\left(b^2(2c+3dx)-6\right)\cos\left(a+b\sqrt{c+dx}\right)\right)}{b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (2*((-6 + b^2*(2*c + 3*d*x))*Cos[a + b*Sqrt[c + d*x]] + b*Sqrt[c + d*x]*(-6
 + b^2*d*x)*Sin[a + b*Sqrt[c + d*x]]))/(b^4*d^2)
```

fricas [A] time = 0.80, size = 67, normalized size = 0.40

$$\frac{2\left(\left(b^3dx-6\,b\right)\sqrt{dx+c}\,\sin\left(\sqrt{dx+c}\,b+a\right)+\left(3\,b^2dx+2\,b^2c-6\right)\cos\left(\sqrt{dx+c}\,b+a\right)\right)}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*((b^3*d*x - 6*b)*sqrt(d*x + c)*sin(sqrt(d*x + c)*b + a) + (3*b^2*d*x + 2*
b^2*c - 6)*cos(sqrt(d*x + c)*b + a))/(b^4*d^2)
```

giac [A] time = 0.47, size = 166, normalized size = 0.99

$$-\frac{2\left(\frac{\left(b^{2}c-3\left(\sqrt{dx+c}\,b+a\right)^{2}+6\left(\sqrt{dx+c}\,b+a\right)a-3\,a^{2}+6\right)\cos\left(\sqrt{dx+c}\,b+a\right)}{b^{2}}+\frac{\left(\left(\sqrt{dx+c}\,b+a\right)b^{2}c-ab^{2}c-\left(\sqrt{dx+c}\,b+a\right)^{3}+3\left(\sqrt{dx+c}\,b+a\right)^{2}a-3\left(\sqrt{dx+c}\,b+\cdots\right.\right.}{b^{2}}\cdots\right.}{b^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*((b^2*c - 3*(sqrt(d*x + c)*b + a)^2 + 6*(sqrt(d*x + c)*b + a)*a - 3*a^2
+ 6)*cos(sqrt(d*x + c)*b + a)/b^2 + ((sqrt(d*x + c)*b + a)*b^2*c - a*b^2*c
- (sqrt(d*x + c)*b + a)^3 + 3*(sqrt(d*x + c)*b + a)^2*a - 3*(sqrt(d*x + c)*
b + a)*a^2 + a^3 + 6*sqrt(d*x + c)*b)*sin(sqrt(d*x + c)*b + a)/b^2)/(b^2*d^
2)
```

maple [A] time = 0.04, size = 299, normalized size = 1.79

$$\frac{-2c\left(\cos\left(a+b\sqrt{dx+c}\right)+\left(a+b\sqrt{dx+c}\right)\sin\left(a+b\sqrt{dx+c}\right)\right)+2ac\sin\left(a+b\sqrt{dx+c}\right)+\frac{2\left(\left(a+b\sqrt{dx+c}\right)^{3}\sin\left(a+\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/d^2/b^2*(-c*(cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(
1/2)))+a*c*sin(a+b*(d*x+c)^(1/2))+1/b^2*((a+b*(d*x+c)^(1/2))^3*sin(a+b*(d*x
+c)^(1/2))+3*(a+b*(d*x+c)^(1/2))^2*cos(a+b*(d*x+c)^(1/2))-6*cos(a+b*(d*x+c)
^(1/2))-6*(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2)))-3/b^2*a*((a+b*(d*x+c)
^(1/2))^2*sin(a+b*(d*x+c)^(1/2))-2*sin(a+b*(d*x+c)^(1/2))+2*cos(a+b*(d*x+c)
^(1/2))*(a+b*(d*x+c)^(1/2)))+3/b^2*a^2*(cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)
^(1/2))*sin(a+b*(d*x+c)^(1/2)))-1/b^2*a^3*sin(a+b*(d*x+c)^(1/2)))
```

maxima [A] time = 0.36, size = 263, normalized size = 1.57

$$\frac{2\left(ac\sin\left(\sqrt{dx+c}\,b+a\right)-\left(\left(\sqrt{dx+c}\,b+a\right)\sin\left(\sqrt{dx+c}\,b+a\right)+\cos\left(\sqrt{dx+c}\,b+a\right)\right)c-\frac{a^{3}\sin\left(\sqrt{dx+c}\,b+a\right)}{b^{2}}+\frac{3\left(\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*(a*c*sin(sqrt(d*x + c)*b + a) - ((sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*
b + a) + cos(sqrt(d*x + c)*b + a))*c - a^3*sin(sqrt(d*x + c)*b + a)/b^2 + 3
```

```
*((sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a) + cos(sqrt(d*x + c)*b + a)
)*a^2/b^2 - 3*(2*(sqrt(d*x + c)*b + a)*cos(sqrt(d*x + c)*b + a) + ((sqrt(d*
x + c)*b + a)^2 - 2)*sin(sqrt(d*x + c)*b + a))*a/b^2 + (3*((sqrt(d*x + c)*b
 + a)^2 - 2)*cos(sqrt(d*x + c)*b + a) + ((sqrt(d*x + c)*b + a)^3 - 6*sqrt(d
*x + c)*b - 6*a)*sin(sqrt(d*x + c)*b + a))/b^2)/(b^2*d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \cos\left(a + b\sqrt{c + d\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] int(x*cos(a + b*(c + d*x)^(1/2)), x)
```

sympy [A] time = 0.52, size = 151, normalized size = 0.90

$$\begin{cases} \frac{x^2\cos(a)}{2} & \text{for } b = 0 \wedge \\ \frac{x^2\cos\left(a+b\sqrt{c}\right)}{2} & \text{for } d = 0 \\ \frac{2x\sqrt{c+dx}\,\sin\left(a+b\sqrt{c+dx}\right)}{bd} + \frac{4c\cos\left(a+b\sqrt{c+dx}\right)}{b^2d^2} + \frac{6x\cos\left(a+b\sqrt{c+dx}\right)}{b^2d} - \frac{12\sqrt{c+dx}\,\sin\left(a+b\sqrt{c+dx}\right)}{b^3d^2} - \frac{12\cos\left(a+b\sqrt{c+dx}\right)}{b^4d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x**2*cos(a)/2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**2*cos(a +
b*sqrt(c))/2, Eq(d, 0)), (2*x*sqrt(c + d*x)*sin(a + b*sqrt(c + d*x))/(b*d)
+ 4*c*cos(a + b*sqrt(c + d*x))/(b**2*d**2) + 6*x*cos(a + b*sqrt(c + d*x))/(
b**2*d) - 12*sqrt(c + d*x)*sin(a + b*sqrt(c + d*x))/(b**3*d**2) - 12*cos(a
+ b*sqrt(c + d*x))/(b**4*d**2), True))
```

3.92 $\int \cos\left(a+b\sqrt{c+dx}\right)\,dx$

Optimal. Leaf size=54

$$\frac{2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d}+\frac{2\sqrt{c+dx}\,\sin\left(a+b\sqrt{c+dx}\right)}{bd}$$

```
[Out] 2*cos(a+b*(d*x+c)^(1/2))/b^2/d+2*sin(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/2)/b/d
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3362, 3296, 2638}

$$\frac{2\cos\left(a+b\sqrt{c+dx}\right)}{b^2d}+\frac{2\sqrt{c+dx}\,\sin\left(a+b\sqrt{c+dx}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (2*Cos[a + b*Sqrt[c + d*x]])/(b^2*d) + (2*Sqrt[c + d*x]*Sin[a + b*Sqrt[c +
d*x]])/(b*d)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3362

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\int \cos \left(a+b\sqrt{c+dx}\right) dx = \frac{2 \operatorname{Subst}\left(\int x \cos(a+bx) \, dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{2\sqrt{c+dx} \sin \left(a+b\sqrt{c+dx}\right)}{bd} - \frac{2 \operatorname{Subst}\left(\int \sin(a+bx) \, dx, x, \sqrt{c+dx}\right)}{bd}$$
$$= \frac{2 \cos \left(a+b\sqrt{c+dx}\right)}{b^2 d} + \frac{2\sqrt{c+dx} \sin \left(a+b\sqrt{c+dx}\right)}{bd}$$

Mathematica [A] time = 0.07, size = 48, normalized size = 0.89

$$\frac{2 \left(b\sqrt{c+dx} \sin \left(a+b\sqrt{c+dx}\right) + \cos \left(a+b\sqrt{c+dx}\right)\right)}{b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (2*(Cos[a + b*Sqrt[c + d*x]] + b*Sqrt[c + d*x]*Sin[a + b*Sqrt[c + d*x]]))/(
b^2*d)
```

fricas [A] time = 0.92, size = 42, normalized size = 0.78

$$\frac{2 \left(\sqrt{dx+c}\, b \sin \left(\sqrt{dx+c}\, b + a\right) + \cos \left(\sqrt{dx+c}\, b + a\right)\right)}{b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(d*x + c)*b*sin(sqrt(d*x + c)*b + a) + cos(sqrt(d*x + c)*b + a))/(b^
2*d)
```

giac [A] time = 0.33, size = 42, normalized size = 0.78

$$\frac{2 \left(\sqrt{dx+c}\, b \sin \left(\sqrt{dx+c}\, b + a\right) + \cos \left(\sqrt{dx+c}\, b + a\right)\right)}{b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*(sqrt(d*x + c)*b*sin(sqrt(d*x + c)*b + a) + cos(sqrt(d*x + c)*b + a))/(b^
2*d)
```

maple [A] time = 0.02, size = 61, normalized size = 1.13

$$\frac{2\cos\left(a+b\sqrt{dx+c}\right)+2\left(a+b\sqrt{dx+c}\right)\sin\left(a+b\sqrt{dx+c}\right)-2a\sin\left(a+b\sqrt{dx+c}\right)}{d\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*(d*x+c)^(1/2)),x)
```

```
[Out] 2/d/b^2*(cos(a+b*(d*x+c)^(1/2))+(a+b*(d*x+c)^(1/2))*sin(a+b*(d*x+c)^(1/2))-
a*sin(a+b*(d*x+c)^(1/2)))
```

maxima [A] time = 0.30, size = 60, normalized size = 1.11

$$\frac{2\left(\left(\sqrt{dx+c}\,b+a\right)\sin\left(\sqrt{dx+c}\,b+a\right)-a\sin\left(\sqrt{dx+c}\,b+a\right)+\cos\left(\sqrt{dx+c}\,b+a\right)\right)}{b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*((sqrt(d*x + c)*b + a)*sin(sqrt(d*x + c)*b + a) - a*sin(sqrt(d*x + c)*b +
 a) + cos(sqrt(d*x + c)*b + a))/(b^2*d)
```

mupad [B] time = 0.34, size = 42, normalized size = 0.78

$$\frac{2\,\left(\cos\left(a+b\,\sqrt{c+d\,x}\right)+b\,\sin\left(a+b\,\sqrt{c+d\,x}\right)\,\sqrt{c+d\,x}\right)}{b^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(cos(a + b*(c + d*x)^(1/2)) + b*sin(a + b*(c + d*x)^(1/2))*(c + d*x)^(1/
2)))/(b^2*d)
```

sympy [A] time = 0.40, size = 66, normalized size = 1.22

$$\begin{cases} x\cos\left(a\right) & \text{for}\: b=0\wedge d=0 \\ x\cos\left(a+b\sqrt{c}\right) & \text{for}\: d=0 \\ x\cos\left(a\right) & \text{for}\: b=0 \\ \frac{2\sqrt{c+dx}\,\sin\left(a+b\sqrt{c+dx}\right)}{bd}+\frac{2\cos\left(a+b\sqrt{c+dx}\right)}{b^{2}d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x*cos(a), Eq(b, 0) & Eq(d, 0)), (x*cos(a + b*sqrt(c)), Eq(d, 0))
, (x*cos(a), Eq(b, 0)), (2*sqrt(c + d*x)*sin(a + b*sqrt(c + d*x))/(b*d) + 2
*cos(a + b*sqrt(c + d*x))/(b**2*d), True))
```

3.93 $\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x}\,dx$

Optimal. Leaf size=126

$$\cos\left(a-b\sqrt{c}\right)\text{Ci}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)+\cos\left(a+b\sqrt{c}\right)\text{Ci}\left(b\sqrt{c}-b\sqrt{c+dx}\right)-\sin\left(a-b\sqrt{c}\right)\text{Si}\left(b\left(\sqrt{c}+\sqrt{c+d\right.\right.$$

```
[Out] Ci(b*(c^(1/2)+(d*x+c)^(1/2)))*cos(a-b*c^(1/2))+Ci(b*c^(1/2)-b*(d*x+c)^(1/2)
)*cos(a+b*c^(1/2))-Si(b*(c^(1/2)+(d*x+c)^(1/2)))*sin(a-b*c^(1/2))+Si(b*c^(1
/2)-b*(d*x+c)^(1/2))*sin(a+b*c^(1/2))
```

Rubi [A] time = 0.24, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3432, 3303, 3299, 3302}

$$\cos\left(a-b\sqrt{c}\right)\text{CosIntegral}\left(b\left(\sqrt{c+dx}+\sqrt{c}\right)\right)+\cos\left(a+b\sqrt{c}\right)\text{CosIntegral}\left(b\sqrt{c}-b\sqrt{c+dx}\right)-\sin\left(a-b\sqrt{c}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] Cos[a - b*Sqrt[c]]*CosIntegral[b*(Sqrt[c] + Sqrt[c + d*x])] + Cos[a + b*Sqr
t[c]]*CosIntegral[b*Sqrt[c] - b*Sqrt[c + d*x]] - Sin[a - b*Sqrt[c]]*SinInte
gral[b*(Sqrt[c] + Sqrt[c + d*x])] + Sin[a + b*Sqrt[c]]*SinIntegral[b*Sqrt[c
] - b*Sqrt[c + d*x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x}\,dx = \frac{2\,\text{Subst}\left(\int\left(-\frac{d\cos(a+bx)}{2\left(\sqrt{c}-x\right)}+\frac{d\cos(a+bx)}{2\left(\sqrt{c}+x\right)}\right)dx, x, \sqrt{c+dx}\right)}{d}$$

$$= -\text{Subst}\left(\int\frac{\cos(a+bx)}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)+\text{Subst}\left(\int\frac{\cos(a+bx)}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)$$

$$= \cos\left(a-b\sqrt{c}\right)\text{Subst}\left(\int\frac{\cos\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)-\cos\left(a+b\sqrt{c}\right)\text{Subst}\left(\int\right.$$

$$= \cos\left(a-b\sqrt{c}\right)\text{Ci}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)+\cos\left(a+b\sqrt{c}\right)\text{Ci}\left(b\sqrt{c}-b\sqrt{c+dx}\right)-\sin$$

Mathematica [C] time = 0.67, size = 145, normalized size = 1.15

$$\frac{1}{2}e^{-i\left(a+b\sqrt{c}\right)}\left(e^{2i\left(a+b\sqrt{c}\right)}\text{Ei}\left(ib\left(\sqrt{c+dx}-\sqrt{c}\right)\right)+e^{2ia}\text{Ei}\left(ib\left(\sqrt{c}+\sqrt{c+dx}\right)\right)+\text{Ei}\left(-ib\left(\sqrt{c+dx}-\sqrt{c}\right)\right)+e^{2ib\sqrt{c}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] (ExpIntegralEi[(-I)*b*(-Sqrt[c] + Sqrt[c + d*x])] + E^((2*I)*(a + b*Sqrt[c]
))*ExpIntegralEi[I*b*(-Sqrt[c] + Sqrt[c + d*x])] + E^((2*I)*b*Sqrt[c])*ExpI
ntegralEi[(-I)*b*(Sqrt[c] + Sqrt[c + d*x])] + E^((2*I)*a)*ExpIntegralEi[I*b
*(Sqrt[c] + Sqrt[c + d*x])])/(2*E^(I*(a + b*Sqrt[c])))
```

fricas [C] time = 0.92, size = 149, normalized size = 1.18

$$\frac{1}{2}\,\text{Ei}\left(i\sqrt{dx+c}\,b-\sqrt{-b^2c}\right)e^{\left(ia+\sqrt{-b^2c}\right)}+\frac{1}{2}\,\text{Ei}\left(i\sqrt{dx+c}\,b+\sqrt{-b^2c}\right)e^{\left(ia-\sqrt{-b^2c}\right)}+\frac{1}{2}\,\text{Ei}\left(-i\sqrt{dx+c}\,b-\sqrt{-b^2c}\right)e^{\left(-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] 1/2*Ei(I*sqrt(d*x + c)*b - sqrt(-b^2*c))*e^(I*a + sqrt(-b^2*c)) + 1/2*Ei(I*
sqrt(d*x + c)*b + sqrt(-b^2*c))*e^(I*a - sqrt(-b^2*c)) + 1/2*Ei(-I*sqrt(d*x
 + c)*b - sqrt(-b^2*c))*e^(-I*a + sqrt(-b^2*c)) + 1/2*Ei(-I*sqrt(d*x + c)*b
 + sqrt(-b^2*c))*e^(-I*a - sqrt(-b^2*c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\sqrt{dx+c}\,b+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(cos(sqrt(d*x + c)*b + a)/x, x)
```

maple [B] time = 0.06, size = 271, normalized size = 2.15

$$\frac{\frac{\left(a+b\sqrt{c}\right)b\left(\mathrm{Si}\left(b\sqrt{c}-b\sqrt{dx+c}\right)\sin\left(a+b\sqrt{c}\right)+\mathrm{Ci}\left(b\sqrt{dx+c}-b\sqrt{c}\right)\cos\left(a+b\sqrt{c}\right)\right)}{\sqrt{c}}-\frac{\left(a-b\sqrt{c}\right)b\left(-\mathrm{Si}\left(b\sqrt{dx+c}+b\sqrt{c}\right)\sin\left(a-b\sqrt{c}\right)+\mathrm{Ci}\left(b\sqrt{dx+c}+b\sqrt{c}\right)\mathrm{c}}{\sqrt{c}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*(d*x+c)^(1/2))/x,x)
```

```
[Out] 2/b^2*(1/2*(a+b*c^(1/2))*b/c^(1/2)*(Si(b*c^(1/2)-b*(d*x+c)^(1/2))*sin(a+b*c
^(1/2))+Ci(b*(d*x+c)^(1/2)-b*c^(1/2))*cos(a+b*c^(1/2)))-1/2*(a-b*c^(1/2))*b
/c^(1/2)*(-Si(b*(d*x+c)^(1/2)+b*c^(1/2))*sin(a-b*c^(1/2))+Ci(b*(d*x+c)^(1/2
)+b*c^(1/2))*cos(a-b*c^(1/2)))-b^2*a*(1/2/b/c^(1/2)*(Si(b*c^(1/2)-b*(d*x+c)
^(1/2))*sin(a+b*c^(1/2))+Ci(b*(d*x+c)^(1/2)-b*c^(1/2))*cos(a+b*c^(1/2)))-1/
2/b/c^(1/2)*(-Si(b*(d*x+c)^(1/2)+b*c^(1/2))*sin(a-b*c^(1/2))+Ci(b*(d*x+c)^(
1/2)+b*c^(1/2))*cos(a-b*c^(1/2)))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\sqrt{dx+c}\,b+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(cos(sqrt(d*x + c)*b + a)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,\sqrt{c+d\,x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*(c + d*x)^(1/2))/x,x)
```

```
[Out] int(cos(a + b*(c + d*x)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a + b\sqrt{c + dx}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)**(1/2))/x,x)
```

```
[Out] Integral(cos(a + b*sqrt(c + d*x))/x, x)
```

3.94 $\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$

Optimal. Leaf size=184

$$\frac{bd\sin\left(a-b\sqrt{c}\right)\text{Ci}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)}{2\sqrt{c}}-\frac{bd\sin\left(a+b\sqrt{c}\right)\text{Ci}\left(b\sqrt{c}-b\sqrt{c+dx}\right)}{2\sqrt{c}}+\frac{bd\cos\left(a-b\sqrt{c}\right)\text{Si}\left(b\left(\sqrt{c}\right.\right.}{2\sqrt{c}}$$

```
[Out] -cos(a+b*(d*x+c)^(1/2))/x+1/2*b*d*cos(a-b*c^(1/2))*Si(b*(c^(1/2)+(d*x+c)^(1
/2)))/c^(1/2)+1/2*b*d*cos(a+b*c^(1/2))*Si(b*c^(1/2)-b*(d*x+c)^(1/2))/c^(1/2
)+1/2*b*d*Ci(b*(c^(1/2)+(d*x+c)^(1/2)))*sin(a-b*c^(1/2))/c^(1/2)-1/2*b*d*Ci
(b*c^(1/2)-b*(d*x+c)^(1/2))*sin(a+b*c^(1/2))/c^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3432, 3342, 3333, 3303, 3299, 3302}

$$\frac{bd\sin\left(a-b\sqrt{c}\right)\text{CosIntegral}\left(b\left(\sqrt{c+dx}+\sqrt{c}\right)\right)}{2\sqrt{c}}-\frac{bd\sin\left(a+b\sqrt{c}\right)\text{CosIntegral}\left(b\sqrt{c}-b\sqrt{c+dx}\right)}{2\sqrt{c}}+\frac{bd\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] -(Cos[a + b*Sqrt[c + d*x]]/x) + (b*d*CosIntegral[b*(Sqrt[c] + Sqrt[c + d*x]
)]*Sin[a - b*Sqrt[c]])/(2*Sqrt[c]) - (b*d*CosIntegral[b*Sqrt[c] - b*Sqrt[c
+ d*x]]*Sin[a + b*Sqrt[c]])/(2*Sqrt[c]) + (b*d*Cos[a - b*Sqrt[c]]*SinIntegr
al[b*(Sqrt[c] + Sqrt[c + d*x])])/(2*Sqrt[c]) + (b*d*Cos[a + b*Sqrt[c]]*SinI
ntegral[b*Sqrt[c] - b*Sqrt[c + d*x]])/(2*Sqrt[c])
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3342

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_
), x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Cos[c + d*x])/(b*n*(p + 1)),
x] + Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sin[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3432

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x\cos(a+bx)}{\left(-\frac{c}{d}+\frac{x^2}{d}\right)^2}\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= -\frac{\cos\left(a+b\sqrt{c+dx}\right)}{x} - b\,\text{Subst}\left(\int \frac{\sin(a+bx)}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)$$
$$= -\frac{\cos\left(a+b\sqrt{c+dx}\right)}{x} - b\,\text{Subst}\left(\int\left(-\frac{d\sin(a+bx)}{2\sqrt{c}\left(\sqrt{c}-x\right)} - \frac{d\sin(a+bx)}{2\sqrt{c}\left(\sqrt{c}+x\right)}\right)dx, x, \sqrt{c}$$
$$= -\frac{\cos\left(a+b\sqrt{c+dx}\right)}{x} + \frac{(bd)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}} + \frac{(bd)\,\text{Subst}\left(\int \frac{\sin}{}\right.}{}$$
$$= -\frac{\cos\left(a+b\sqrt{c+dx}\right)}{x} + \frac{\left(bd\cos\left(a-b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{\sin\left(b\sqrt{c}+bx\right)}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)}{2\sqrt{c}} -$$
$$= -\frac{\cos\left(a+b\sqrt{c+dx}\right)}{x} + \frac{bd\text{Ci}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)\sin\left(a-b\sqrt{c}\right)}{2\sqrt{c}} - \frac{bd\text{Ci}\left(b\sqrt{c}-b\right.}{}$$

Mathematica [C] time = 1.20, size = 240, normalized size = 1.30

$$\frac{i\left(e^{-ia}\left(-bdxe^{-ib\sqrt{c}}\,\text{Ei}\left(-ib\left(\sqrt{c+dx}-\sqrt{c}\right)\right) + bdxe^{ib\sqrt{c}}\,\text{Ei}\left(-ib\left(\sqrt{c}+\sqrt{c+dx}\right)\right) + 2i\sqrt{c}\,e^{-ib\sqrt{c+dx}}\right) + e^{i\left(a-b\sqrt{c}\right)}\left(\right.\right.}{4\sqrt{c}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] ((I/4)*((((2*I)*Sqrt[c])/E^(I*b*Sqrt[c + d*x]) - (b*d*x*ExpIntegralEi[(-I)*
b*(-Sqrt[c] + Sqrt[c + d*x])])/E^(I*b*Sqrt[c]) + b*d*E^(I*b*Sqrt[c])*x*ExpI
ntegralEi[(-I)*b*(Sqrt[c] + Sqrt[c + d*x])])/E^(I*a) + E^(I*(a - b*Sqrt[c])
)*((2*I)*Sqrt[c]*E^(I*b*(Sqrt[c] + Sqrt[c + d*x])) + b*d*E^((2*I)*b*Sqrt[c]
)*x*ExpIntegralEi[I*b*(-Sqrt[c] + Sqrt[c + d*x])] - b*d*x*ExpIntegralEi[I*b
*(Sqrt[c] + Sqrt[c + d*x])])))/(Sqrt[c]*x)
```

fricas [C] time = 0.98, size = 210, normalized size = 1.14

$$\frac{\sqrt{-b^2c}\,dx\text{Ei}\left(i\sqrt{dx+c}\,b-\sqrt{-b^2c}\right)e^{\left(ia+\sqrt{-b^2c}\right)} - \sqrt{-b^2c}\,dx\text{Ei}\left(i\sqrt{dx+c}\,b+\sqrt{-b^2c}\right)e^{\left(ia-\sqrt{-b^2c}\right)} + \sqrt{-b^2c}\,dx\text{Ei}\left(\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(-b^2*c)*d*x*Ei(I*sqrt(d*x + c)*b - sqrt(-b^2*c))*e^(I*a + sqrt(-b
^2*c)) - sqrt(-b^2*c)*d*x*Ei(I*sqrt(d*x + c)*b + sqrt(-b^2*c))*e^(I*a - sqr
t(-b^2*c)) + sqrt(-b^2*c)*d*x*Ei(-I*sqrt(d*x + c)*b - sqrt(-b^2*c))*e^(-I*a
 + sqrt(-b^2*c)) - sqrt(-b^2*c)*d*x*Ei(-I*sqrt(d*x + c)*b + sqrt(-b^2*c))*e
^(-I*a - sqrt(-b^2*c)) - 4*c*cos(sqrt(d*x + c)*b + a))/(c*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\sqrt{dx+c}\,b+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos(sqrt(d*x + c)*b + a)/x^2, x)
```

maple [B] time = 0.08, size = 714, normalized size = 3.88

$$\frac{2d\left(\frac{\cos\left(a+b\sqrt{dx+c}\right)\left(-\frac{b^2a\left(a+b\sqrt{dx+c}\right)}{2c}+\frac{b^2\left(-c\,b^2+a^2\right)}{2c}\right)}{-c\,b^2+\left(a+b\sqrt{dx+c}\right)^2-2a\left(a+b\sqrt{dx+c}\right)+a^2}-\frac{ba\left(\mathrm{Si}\left(b\sqrt{c}-b\sqrt{dx+c}\right)\sin\left(a+b\sqrt{c}\right)+\mathrm{Ci}\left(b\sqrt{dx+c}-b\sqrt{c}\right)\cos\left(a+b\sqrt{c}\right)\right)}{4c^{\frac{3}{2}}}+\frac{ba\left(-\mathrm{Si}\left(b\sqrt{dx+c}+\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*(d*x+c)^(1/2))/x^2,x)
```

```
[Out] 2*d/b^2*(cos(a+b*(d*x+c)^(1/2))*(-1/2*b^2*a/c*(a+b*(d*x+c)^(1/2))+1/2*b^2*(
-b^2*c+a^2)/c)/(-c*b^2+(a+b*(d*x+c)^(1/2))^2-2*a*(a+b*(d*x+c)^(1/2))+a^2)-1
/4*b*a/c^(3/2)*(Si(b*c^(1/2)-b*(d*x+c)^(1/2))*sin(a+b*c^(1/2))+Ci(b*(d*x+c)
^(1/2)-b*c^(1/2))*cos(a+b*c^(1/2)))+1/4*b*a/c^(3/2)*(-Si(b*(d*x+c)^(1/2)+b*
c^(1/2))*sin(a-b*c^(1/2))+Ci(b*(d*x+c)^(1/2)+b*c^(1/2))*cos(a-b*c^(1/2)))-1
/4*b*(c*b^2+a*(a+b*c^(1/2))-a^2)/c^(3/2)*(-Si(b*c^(1/2)-b*(d*x+c)^(1/2))*co
s(a+b*c^(1/2))+Ci(b*(d*x+c)^(1/2)-b*c^(1/2))*sin(a+b*c^(1/2)))+1/4*b*(c*b^2
+a*(a-b*c^(1/2))-a^2)/c^(3/2)*(Si(b*(d*x+c)^(1/2)+b*c^(1/2))*cos(a-b*c^(1/2
))+Ci(b*(d*x+c)^(1/2)+b*c^(1/2))*sin(a-b*c^(1/2)))-a*b^4*(cos(a+b*(d*x+c)^(
1/2))*(-1/2/c/b^2*(a+b*(d*x+c)^(1/2))+1/2*a/c/b^2)/(-c*b^2+(a+b*(d*x+c)^(1/
2))^2-2*a*(a+b*(d*x+c)^(1/2))+a^2)-1/4/c^(3/2)/b^3*(Si(b*c^(1/2)-b*(d*x+c)^
(1/2))*sin(a+b*c^(1/2))+Ci(b*(d*x+c)^(1/2)-b*c^(1/2))*cos(a+b*c^(1/2)))+1/4
/c^(3/2)/b^3*(-Si(b*(d*x+c)^(1/2)+b*c^(1/2))*sin(a-b*c^(1/2))+Ci(b*(d*x+c)^
(1/2)+b*c^(1/2))*cos(a-b*c^(1/2)))-1/4/c/b^2*(-Si(b*c^(1/2)-b*(d*x+c)^(1/2)
```

```
)*cos(a+b*c^(1/2))+Ci(b*(d*x+c)^(1/2)-b*c^(1/2))*sin(a+b*c^(1/2)))-1/4/c/b^
2*(Si(b*(d*x+c)^(1/2)+b*c^(1/2))*cos(a-b*c^(1/2))+Ci(b*(d*x+c)^(1/2)+b*c^(1
/2))*sin(a-b*c^(1/2)))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(\sqrt{dx+c}\,b+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(sqrt(d*x + c)*b + a)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,\sqrt{c+d\,x}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*(c + d*x)^(1/2))/x^2,x)
```

```
[Out] int(cos(a + b*(c + d*x)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+b\sqrt{c+dx}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)**(1/2))/x**2,x)
```

```
[Out] Integral(cos(a + b*sqrt(c + d*x))/x**2, x)
```

3.95 $\int x^2 \cos\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=537

$$\frac{120960\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^9d^3}-\frac{120960\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3}-\frac{60480(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^7d^3}+\frac{20160(}{}$$

```
[Out] -720*c*cos(a+b*(d*x+c)^(1/3))/b^6/d^3-120960*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^
(1/3))/b^8/d^3+6*c^2*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^2/d^3+360*c*(d*
x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b^4/d^3+20160*(d*x+c)*cos(a+b*(d*x+c)^(1/
3))/b^6/d^3-30*c*(d*x+c)^(4/3)*cos(a+b*(d*x+c)^(1/3))/b^2/d^3-1008*(d*x+c)^
(5/3)*cos(a+b*(d*x+c)^(1/3))/b^4/d^3+24*(d*x+c)^(7/3)*cos(a+b*(d*x+c)^(1/3)
)/b^2/d^3+120960*sin(a+b*(d*x+c)^(1/3))/b^9/d^3-6*c^2*sin(a+b*(d*x+c)^(1/3)
)/b^3/d^3-720*c*(d*x+c)^(1/3)*sin(a+b*(d*x+c)^(1/3))/b^5/d^3-60480*(d*x+c)^
(2/3)*sin(a+b*(d*x+c)^(1/3))/b^7/d^3+3*c^2*(d*x+c)^(2/3)*sin(a+b*(d*x+c)^(1
/3))/b/d^3+120*c*(d*x+c)*sin(a+b*(d*x+c)^(1/3))/b^3/d^3+5040*(d*x+c)^(4/3)*
sin(a+b*(d*x+c)^(1/3))/b^5/d^3-6*c*(d*x+c)^(5/3)*sin(a+b*(d*x+c)^(1/3))/b/d
^3-168*(d*x+c)^2*sin(a+b*(d*x+c)^(1/3))/b^3/d^3+3*(d*x+c)^(8/3)*sin(a+b*(d*
x+c)^(1/3))/b/d^3
```

Rubi [A] time = 0.51, antiderivative size = 537, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3432, 3296, 2637, 2638}

$$-\frac{6c^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}+\frac{6c^2\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3}-\frac{168(c+dx)^2\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3}+\frac{5040(c+dx)^{4/3}\,\mathrm{si}}{b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (-720*c*Cos[a + b*(c + d*x)^(1/3)])/(b^6*d^3) - (120960*(c + d*x)^(1/3)*Cos
[a + b*(c + d*x)^(1/3)])/(b^8*d^3) + (6*c^2*(c + d*x)^(1/3)*Cos[a + b*(c +
d*x)^(1/3)])/(b^2*d^3) + (360*c*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/3)])
/(b^4*d^3) + (20160*(c + d*x)*Cos[a + b*(c + d*x)^(1/3)])/(b^6*d^3) - (30*c
*(c + d*x)^(4/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^2*d^3) - (1008*(c + d*x)^(5
/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^4*d^3) + (24*(c + d*x)^(7/3)*Cos[a + b*(
c + d*x)^(1/3)])/(b^2*d^3) + (120960*Sin[a + b*(c + d*x)^(1/3)])/(b^9*d^3)
- (6*c^2*Sin[a + b*(c + d*x)^(1/3)])/(b^3*d^3) - (720*c*(c + d*x)^(1/3)*Sin
[a + b*(c + d*x)^(1/3)])/(b^5*d^3) - (60480*(c + d*x)^(2/3)*Sin[a + b*(c +
d*x)^(1/3)])/(b^7*d^3) + (3*c^2*(c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(1/3)])
/(b*d^3) + (120*c*(c + d*x)*Sin[a + b*(c + d*x)^(1/3)])/(b^3*d^3) + (5040*(
c + d*x)^(4/3)*Sin[a + b*(c + d*x)^(1/3)])/(b^5*d^3) - (6*c*(c + d*x)^(5/3)
*Sin[a + b*(c + d*x)^(1/3)])/(b*d^3) - (168*(c + d*x)^2*Sin[a + b*(c + d*x)
^(1/3)])/(b^3*d^3) + (3*(c + d*x)^(8/3)*Sin[a + b*(c + d*x)^(1/3)])/(b*d^3)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int x^2 \cos\left(a+b\sqrt[3]{c+dx}\right) dx = \frac{3\,\text{Subst}\left(\int \left(\frac{c^2x^2\cos(a+bx)}{d^2} - \frac{2cx^5\cos(a+bx)}{d^2} + \frac{x^8\cos(a+bx)}{d^2}\right) dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= \frac{3\,\text{Subst}\left(\int x^8\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d^3} - \frac{(6c)\,\text{Subst}\left(\int x^5\cos(a+bx)\,dx, x, \sqrt[3]{\right.}}{d^3}$$

$$= \frac{3c^2(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{6c(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{3(c+dx}{}$$

$$= \frac{6c^2\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{30c(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{24(c+d}{}$$

$$= \frac{6c^2\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} - \frac{30c(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{24(c+d}{}$$

$$= \frac{6c^2\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{30c(c+}{}$$

$$= \frac{6c^2\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{30c(c+}{}$$

$$= -\frac{720c\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} + \frac{6c^2\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}}{}$$

$$= -\frac{720c\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} + \frac{6c^2\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}}{}$$

$$= -\frac{720c\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{120960\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3} + \frac{6c^2\sqrt[3]{c+dx}}{}$$

$$= -\frac{720c\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^3} - \frac{120960\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^8d^3} + \frac{6c^2\sqrt[3]{c+dx}}{}$$

Mathematica [C] time = 1.10, size = 382, normalized size = 0.71

$$\frac{3e^{-i\left(a+b\sqrt[3]{c+dx}\right)}\left(-ib^8d^2x^2(c+dx)^{2/3}\left(-1+e^{2i\left(a+b\sqrt[3]{c+dx}\right)}\right)+2b^7dx\sqrt[3]{c+dx}\,(3c+4dx)\left(1+e^{2i\left(a+b\sqrt[3]{c+dx}\right)}\right)+2ib^6\left(9c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (3*((-40320*I)*(-1 + E^((2*I)*(a + b*(c + d*x)^(1/3)))) - 40320*b*(1 + E^((
2*I)*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(1/3) + (20160*I)*b^2*(-1 + E^((2*
I)*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(2/3) - I*b^8*d^2*(-1 + E^((2*I)*(a
+ b*(c + d*x)^(1/3))))*x^2*(c + d*x)^(2/3) + 2*b^7*d*(1 + E^((2*I)*(a + b*(
```

```
c + d*x)^(1/3))))*x*(c + d*x)^(1/3)*(3*c + 4*d*x) - (240*I)*b^4*(-1 + E^((2
*I)*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(1/3)*(6*c + 7*d*x) - 24*b^5*(1 + E
^((2*I)*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(2/3)*(9*c + 14*d*x) + 240*b^3*
(1 + E^((2*I)*(a + b*(c + d*x)^(1/3))))*(27*c + 28*d*x) + (2*I)*b^6*(-1 + E
^((2*I)*(a + b*(c + d*x)^(1/3))))*(9*c^2 + 36*c*d*x + 28*d^2*x^2)))/(2*b^9*
d^3*E^(I*(a + b*(c + d*x)^(1/3))))
```

fricas [A] time = 0.95, size = 182, normalized size = 0.34

$$\frac{3\left(2\left(3360\,b^{3}dx+3240\,b^{3}c-12\left(14\,b^{5}dx+9\,b^{5}c\right)(dx+c)^{\frac{2}{3}}+\left(4\,b^{7}d^{2}x^{2}+3\,b^{7}cdx-20160\,b\right)(dx+c)^{\frac{1}{3}}\right)\cos\left((dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*(2*(3360*b^3*d*x + 3240*b^3*c - 12*(14*b^5*d*x + 9*b^5*c)*(d*x + c)^(2/3)
 + (4*b^7*d^2*x^2 + 3*b^7*c*d*x - 20160*b)*(d*x + c)^(1/3))*cos((d*x + c)^(
1/3)*b + a) - (56*b^6*d^2*x^2 + 72*b^6*c*d*x + 18*b^6*c^2 - (b^8*d^2*x^2 -
20160*b^2)*(d*x + c)^(2/3) - 240*(7*b^4*d*x + 6*b^4*c)*(d*x + c)^(1/3) - 40
320)*sin((d*x + c)^(1/3)*b + a))/(b^9*d^3)
```

giac [B] time = 0.50, size = 1104, normalized size = 2.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3*(2*(((d*x + c)^(1/3)*b + a)*b^6*c^2 - a*b^6*c^2 - 5*((d*x + c)^(1/3)*b +
a)^4*b^3*c + 20*((d*x + c)^(1/3)*b + a)^3*a*b^3*c - 30*((d*x + c)^(1/3)*b +
 a)^2*a^2*b^3*c + 20*((d*x + c)^(1/3)*b + a)*a^3*b^3*c - 5*a^4*b^3*c + 4*((
d*x + c)^(1/3)*b + a)^7 - 28*((d*x + c)^(1/3)*b + a)^6*a + 84*((d*x + c)^(1
/3)*b + a)^5*a^2 - 140*((d*x + c)^(1/3)*b + a)^4*a^3 + 140*((d*x + c)^(1/3)
*b + a)^3*a^4 - 84*((d*x + c)^(1/3)*b + a)^2*a^5 + 28*((d*x + c)^(1/3)*b +
a)*a^6 - 4*a^7 + 60*((d*x + c)^(1/3)*b + a)^2*b^3*c - 120*((d*x + c)^(1/3)*
b + a)*a*b^3*c + 60*a^2*b^3*c - 168*((d*x + c)^(1/3)*b + a)^5 + 840*((d*x +
 c)^(1/3)*b + a)^4*a - 1680*((d*x + c)^(1/3)*b + a)^3*a^2 + 1680*((d*x + c)
^(1/3)*b + a)^2*a^3 - 840*((d*x + c)^(1/3)*b + a)*a^4 + 168*a^5 - 120*b^3*c
 + 3360*((d*x + c)^(1/3)*b + a)^3 - 10080*((d*x + c)^(1/3)*b + a)^2*a + 100
80*((d*x + c)^(1/3)*b + a)*a^2 - 3360*a^3 - 20160*(d*x + c)^(1/3)*b)*cos((d
*x + c)^(1/3)*b + a)/(b^8*d^2) + (((d*x + c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d
*x + c)^(1/3)*b + a)*a*b^6*c^2 + a^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)^5*
b^3*c + 10*((d*x + c)^(1/3)*b + a)^4*a*b^3*c - 20*((d*x + c)^(1/3)*b + a)^3
*a^2*b^3*c + 20*((d*x + c)^(1/3)*b + a)^2*a^3*b^3*c - 10*((d*x + c)^(1/3)*b
```

```
 + a)*a^4*b^3*c + 2*a^5*b^3*c + ((d*x + c)^(1/3)*b + a)^8 - 8*((d*x + c)^(1
/3)*b + a)^7*a + 28*((d*x + c)^(1/3)*b + a)^6*a^2 - 56*((d*x + c)^(1/3)*b +
 a)^5*a^3 + 70*((d*x + c)^(1/3)*b + a)^4*a^4 - 56*((d*x + c)^(1/3)*b + a)^3
*a^5 + 28*((d*x + c)^(1/3)*b + a)^2*a^6 - 8*((d*x + c)^(1/3)*b + a)*a^7 + a
^8 - 2*b^6*c^2 + 40*((d*x + c)^(1/3)*b + a)^3*b^3*c - 120*((d*x + c)^(1/3)*
b + a)^2*a*b^3*c + 120*((d*x + c)^(1/3)*b + a)*a^2*b^3*c - 40*a^3*b^3*c - 5
6*((d*x + c)^(1/3)*b + a)^6 + 336*((d*x + c)^(1/3)*b + a)^5*a - 840*((d*x +
 c)^(1/3)*b + a)^4*a^2 + 1120*((d*x + c)^(1/3)*b + a)^3*a^3 - 840*((d*x + c
)^(1/3)*b + a)^2*a^4 + 336*((d*x + c)^(1/3)*b + a)*a^5 - 56*a^6 - 240*((d*x
 + c)^(1/3)*b + a)*b^3*c + 240*a*b^3*c + 1680*((d*x + c)^(1/3)*b + a)^4 - 6
720*((d*x + c)^(1/3)*b + a)^3*a + 10080*((d*x + c)^(1/3)*b + a)^2*a^2 - 672
0*((d*x + c)^(1/3)*b + a)*a^3 + 1680*a^4 - 20160*((d*x + c)^(1/3)*b + a)^2
+ 40320*((d*x + c)^(1/3)*b + a)*a - 20160*a^2 + 40320)*sin((d*x + c)^(1/3)*
b + a)/(b^8*d^2))/(b*d)
```

maple [B] time = 0.05, size = 1809, normalized size = 3.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] 3/d^3/b^3*(c^2*((a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-2*sin(a+b*(d*x
+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3)))-2*a*c^2*(cos(a+b*(
d*x+c)^(1/3))+(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+a^2*c^2*sin(a+b*(
d*x+c)^(1/3))-2/b^3*c*((a+b*(d*x+c)^(1/3))^5*sin(a+b*(d*x+c)^(1/3))+5*(a+b*
(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))-20*(a+b*(d*x+c)^(1/3))^3*sin(a+b*(d
*x+c)^(1/3))-60*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+120*cos(a+b*(d
*x+c)^(1/3))+120*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+10/b^3*a*c*((a
+b*(d*x+c)^(1/3))^4*sin(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3))^3*cos(a+b*
(d*x+c)^(1/3))-12*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+24*sin(a+b*(
d*x+c)^(1/3))-24*cos(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3)))-20/b^3*a^2*c*(
(a+b*(d*x+c)^(1/3))^3*sin(a+b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^2*cos(a+
b*(d*x+c)^(1/3))-6*cos(a+b*(d*x+c)^(1/3))-6*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*
x+c)^(1/3)))+20/b^3*a^3*c*((a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-2*s
in(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3)))-10/b^3*
a^4*c*(cos(a+b*(d*x+c)^(1/3))+(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+2
/b^3*a^5*c*sin(a+b*(d*x+c)^(1/3))+1/b^6*((a+b*(d*x+c)^(1/3))^8*sin(a+b*(d*x
+c)^(1/3))+8*(a+b*(d*x+c)^(1/3))^7*cos(a+b*(d*x+c)^(1/3))-56*(a+b*(d*x+c)^(
1/3))^6*sin(a+b*(d*x+c)^(1/3))-336*(a+b*(d*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1
/3))+1680*(a+b*(d*x+c)^(1/3))^4*sin(a+b*(d*x+c)^(1/3))+6720*(a+b*(d*x+c)^(1
/3))^3*cos(a+b*(d*x+c)^(1/3))-20160*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(
1/3))+40320*sin(a+b*(d*x+c)^(1/3))-40320*cos(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c
)^(1/3)))-8/b^6*a*((a+b*(d*x+c)^(1/3))^7*sin(a+b*(d*x+c)^(1/3))+7*(a+b*(d*x
+c)^(1/3))^6*cos(a+b*(d*x+c)^(1/3))-42*(a+b*(d*x+c)^(1/3))^5*sin(a+b*(d*x+c
```

```
)^(1/3))-210*(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))+840*(a+b*(d*x+c)^
(1/3))^3*sin(a+b*(d*x+c)^(1/3))+2520*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^
(1/3))-5040*cos(a+b*(d*x+c)^(1/3))-5040*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)
^(1/3)))+28/b^6*a^2*((a+b*(d*x+c)^(1/3))^6*sin(a+b*(d*x+c)^(1/3))+6*(a+b*(d
*x+c)^(1/3))^5*cos(a+b*(d*x+c)^(1/3))-30*(a+b*(d*x+c)^(1/3))^4*sin(a+b*(d*x
+c)^(1/3))-120*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))+360*(a+b*(d*x+c
)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-720*sin(a+b*(d*x+c)^(1/3))+720*cos(a+b*(d
*x+c)^(1/3))*(a+b*(d*x+c)^(1/3)))-56/b^6*a^3*((a+b*(d*x+c)^(1/3))^5*sin(a+b
*(d*x+c)^(1/3))+5*(a+b*(d*x+c)^(1/3))^4*cos(a+b*(d*x+c)^(1/3))-20*(a+b*(d*x
+c)^(1/3))^3*sin(a+b*(d*x+c)^(1/3))-60*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c
)^(1/3))+120*cos(a+b*(d*x+c)^(1/3))+120*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)
^(1/3)))+70/b^6*a^4*((a+b*(d*x+c)^(1/3))^4*sin(a+b*(d*x+c)^(1/3))+4*(a+b*(d
*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1/3))-12*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x
+c)^(1/3))+24*sin(a+b*(d*x+c)^(1/3))-24*cos(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)
^(1/3)))-56/b^6*a^5*((a+b*(d*x+c)^(1/3))^3*sin(a+b*(d*x+c)^(1/3))+3*(a+b*(d
*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))-6*cos(a+b*(d*x+c)^(1/3))-6*(a+b*(d*x+
c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+28/b^6*a^6*((a+b*(d*x+c)^(1/3))^2*sin(a+b
*(d*x+c)^(1/3))-2*sin(a+b*(d*x+c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))*(a+b*(d*x
+c)^(1/3)))-8/b^6*a^7*(cos(a+b*(d*x+c)^(1/3))+(a+b*(d*x+c)^(1/3))*sin(a+b*(
d*x+c)^(1/3)))+1/b^6*a^8*sin(a+b*(d*x+c)^(1/3)))
```

maxima [B] time = 0.75, size = 1349, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 3*(a^2*c^2*sin((d*x + c)^(1/3)*b + a) - 2*(((d*x + c)^(1/3)*b + a)*sin((d*x
 + c)^(1/3)*b + a) + cos((d*x + c)^(1/3)*b + a))*a*c^2 + 2*a^5*c*sin((d*x +
 c)^(1/3)*b + a)/b^3 - 10*(((d*x + c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b +
a) + cos((d*x + c)^(1/3)*b + a))*a^4*c/b^3 + (2*((d*x + c)^(1/3)*b + a)*cos
((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^2 - 2)*sin((d*x + c)^(1/
3)*b + a))*c^2 + a^8*sin((d*x + c)^(1/3)*b + a)/b^6 - 8*(((d*x + c)^(1/3)*b
 + a)*sin((d*x + c)^(1/3)*b + a) + cos((d*x + c)^(1/3)*b + a))*a^7/b^6 + 20
*(2*((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*
b + a)^2 - 2)*sin((d*x + c)^(1/3)*b + a))*a^3*c/b^3 + 28*(2*((d*x + c)^(1/3
)*b + a)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^2 - 2)*sin((
d*x + c)^(1/3)*b + a))*a^6/b^6 - 20*(3*(((d*x + c)^(1/3)*b + a)^2 - 2)*cos(
(d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b -
 6*a)*sin((d*x + c)^(1/3)*b + a))*a^2*c/b^3 - 56*(3*(((d*x + c)^(1/3)*b + a
)^2 - 2)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^3 - 6*(d*x +
 c)^(1/3)*b - 6*a)*sin((d*x + c)^(1/3)*b + a))*a^5/b^6 + 10*(4*(((d*x + c)^
(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a) + ((
(d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*sin((d*x + c)
```

```
^(1/3)*b + a))*a*c/b^3 + 70*(4*(((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/
3)*b - 6*a)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^4 - 12*((
d*x + c)^(1/3)*b + a)^2 + 24)*sin((d*x + c)^(1/3)*b + a))*a^4/b^6 - 2*(5*((
(d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)
^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)*b + a)^3 +
 120*(d*x + c)^(1/3)*b + 120*a)*sin((d*x + c)^(1/3)*b + a))*c/b^3 - 56*(5*(
((d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c
)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)*b + a)^3
+ 120*(d*x + c)^(1/3)*b + 120*a)*sin((d*x + c)^(1/3)*b + a))*a^3/b^6 + 28*(
6*(((d*x + c)^(1/3)*b + a)^5 - 20*((d*x + c)^(1/3)*b + a)^3 + 120*(d*x + c)
^(1/3)*b + 120*a)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^6 -
 30*((d*x + c)^(1/3)*b + a)^4 + 360*((d*x + c)^(1/3)*b + a)^2 - 720)*sin((d
*x + c)^(1/3)*b + a))*a^2/b^6 - 8*(7*(((d*x + c)^(1/3)*b + a)^6 - 30*((d*x
+ c)^(1/3)*b + a)^4 + 360*((d*x + c)^(1/3)*b + a)^2 - 720)*cos((d*x + c)^(1
/3)*b + a) + (((d*x + c)^(1/3)*b + a)^7 - 42*((d*x + c)^(1/3)*b + a)^5 + 84
0*((d*x + c)^(1/3)*b + a)^3 - 5040*(d*x + c)^(1/3)*b - 5040*a)*sin((d*x + c
)^(1/3)*b + a))*a/b^6 + (8*(((d*x + c)^(1/3)*b + a)^7 - 42*((d*x + c)^(1/3)
*b + a)^5 + 840*((d*x + c)^(1/3)*b + a)^3 - 5040*(d*x + c)^(1/3)*b - 5040*a
)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^8 - 56*((d*x + c)^(
1/3)*b + a)^6 + 1680*((d*x + c)^(1/3)*b + a)^4 - 20160*((d*x + c)^(1/3)*b +
 a)^2 + 40320)*sin((d*x + c)^(1/3)*b + a))/b^6)/(b^3*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \cos\left(a + b\,(c + d\,x)^{1/3}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(a + b*(c + d*x)^(1/3)),x)
```

```
[Out] int(x^2*cos(a + b*(c + d*x)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cos\left(a + b\sqrt[3]{c + dx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral(x**2*cos(a + b*(c + d*x)**(1/3)), x)
```

3.96 $\int x \cos\left(a + b\sqrt[3]{c + dx}\right) dx$

Optimal. Leaf size=261

$$\frac{360\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2}+\frac{360\sqrt[3]{c+dx}\,\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2}-\frac{180(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2}-\frac{60(c+dx)\sin}{b^3}$$

```
[Out] 360*cos(a+b*(d*x+c)^(1/3))/b^6/d^2-6*c*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(1/3))
/b^2/d^2-180*(d*x+c)^(2/3)*cos(a+b*(d*x+c)^(1/3))/b^4/d^2+15*(d*x+c)^(4/3)*
cos(a+b*(d*x+c)^(1/3))/b^2/d^2+6*c*sin(a+b*(d*x+c)^(1/3))/b^3/d^2+360*(d*x+
c)^(1/3)*sin(a+b*(d*x+c)^(1/3))/b^5/d^2-3*c*(d*x+c)^(2/3)*sin(a+b*(d*x+c)^(
1/3))/b/d^2-60*(d*x+c)*sin(a+b*(d*x+c)^(1/3))/b^3/d^2+3*(d*x+c)^(5/3)*sin(a
+b*(d*x+c)^(1/3))/b/d^2
```

Rubi [A] time = 0.23, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3432, 3296, 2637, 2638}

$$-\frac{60(c+dx)\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}+\frac{360\sqrt[3]{c+dx}\,\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2}+\frac{6c\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2}+\frac{15(c+dx)^{4/3}\cos\left(\right.}{b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (360*Cos[a + b*(c + d*x)^(1/3)])/(b^6*d^2) - (6*c*(c + d*x)^(1/3)*Cos[a + b
*(c + d*x)^(1/3)])/(b^2*d^2) - (180*(c + d*x)^(2/3)*Cos[a + b*(c + d*x)^(1/
3)])/(b^4*d^2) + (15*(c + d*x)^(4/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^2*d^2)
+ (6*c*Sin[a + b*(c + d*x)^(1/3)])/(b^3*d^2) + (360*(c + d*x)^(1/3)*Sin[a +
 b*(c + d*x)^(1/3)])/(b^5*d^2) - (3*c*(c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(
1/3)])/(b*d^2) - (60*(c + d*x)*Sin[a + b*(c + d*x)^(1/3)])/(b^3*d^2) + (3*(
c + d*x)^(5/3)*Sin[a + b*(c + d*x)^(1/3)])/(b*d^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3432

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int x\cos\left(a+b\sqrt[3]{c+dx}\right)dx=\frac{3\,\text{Subst}\left(\int\left(-\frac{cx^2\cos(a+bx)}{d}+\frac{x^5\cos(a+bx)}{d}\right)dx,x,\sqrt[3]{c+dx}\right)}{d}$$

$$=\frac{3\,\text{Subst}\left(\int x^5\cos(a+bx)\,dx,x,\sqrt[3]{c+dx}\right)}{d^2}-\frac{(3c)\,\text{Subst}\left(\int x^2\cos(a+bx)\,dx,x,\sqrt[3]{c\right.}}{d^2}$$

$$=-\frac{3c(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}+\frac{3(c+dx)^{5/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}-\frac{15\,\text{Subst}\left(\right.}{}$$

$$=-\frac{6c\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}+\frac{15(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{3c(c+dx)}{}$$

$$=-\frac{6c\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}+\frac{15(c+dx)^{4/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}+\frac{6c\sin\left(a\right.}{b}$$

$$=-\frac{6c\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{180(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2}+\frac{15(c+d}{}$$

$$=-\frac{6c\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{180(c+dx)^{2/3}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2}+\frac{15(c+d}{}$$

$$=\frac{360\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2}-\frac{6c\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2}-\frac{180(c+dx)^{2/3}\cos\left(a\right.}{b^4d^2}$$

Mathematica [A] time = 0.42, size = 117, normalized size = 0.45

$$\frac{3\left(b\left(b^4dx(c+dx)^{2/3}-2b^2(9c+10dx)+120\sqrt[3]{c+dx}\right)\sin\left(a+b\sqrt[3]{c+dx}\right)+\left(b^4\sqrt[3]{c+dx}(3c+5dx)-60b^2(c+dx\right.\right.}{b^6d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (3*((120 - 60*b^2*(c + d*x)^(2/3) + b^4*(c + d*x)^(1/3)*(3*c + 5*d*x))*Cos[
a + b*(c + d*x)^(1/3)] + b*(120*(c + d*x)^(1/3) + b^4*d*x*(c + d*x)^(2/3) -
 2*b^2*(9*c + 10*d*x))*Sin[a + b*(c + d*x)^(1/3)]))/(b^6*d^2)
```

fricas [A] time = 0.77, size = 110, normalized size = 0.42

$$-\frac{3\left(\left(60\,(dx+c)^{\frac{2}{3}}b^2-\left(5\,b^4dx+3\,b^4c\right)(dx+c)^{\frac{1}{3}}-120\right)\cos\left((dx+c)^{\frac{1}{3}}b+a\right)-\left((dx+c)^{\frac{2}{3}}b^5dx-20\,b^3dx-18\,b^3\right.}{b^6d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] -3*((60*(d*x + c)^(2/3)*b^2 - (5*b^4*d*x + 3*b^4*c)*(d*x + c)^(1/3) - 120)*
cos((d*x + c)^(1/3)*b + a) - ((d*x + c)^(2/3)*b^5*d*x - 20*b^3*d*x - 18*b^3
*c + 120*(d*x + c)^(1/3)*b)*sin((d*x + c)^(1/3)*b + a))/(b^6*d^2)
```

giac [A] time = 1.26, size = 370, normalized size = 1.42

$$-\frac{3\left(\frac{\left(2\left((dx+c)^{\frac{1}{3}}b+a\right)b^3c-2\,ab^3c-5\left((dx+c)^{\frac{1}{3}}b+a\right)^4+20\left((dx+c)^{\frac{1}{3}}b+a\right)^3a-30\left((dx+c)^{\frac{1}{3}}b+a\right)^2a^2+20\left((dx+c)^{\frac{1}{3}}b+a\right)a^3-5\,a^4+60\left((dx+c)^{\frac{1}{3}}b+a\right)^2-120\left(\right.\right.}{b^5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] -3*((2*((d*x + c)^(1/3)*b + a)*b^3*c - 2*a*b^3*c - 5*((d*x + c)^(1/3)*b + a
)^4 + 20*((d*x + c)^(1/3)*b + a)^3*a - 30*((d*x + c)^(1/3)*b + a)^2*a^2 + 2
0*((d*x + c)^(1/3)*b + a)*a^3 - 5*a^4 + 60*((d*x + c)^(1/3)*b + a)^2 - 120*
((d*x + c)^(1/3)*b + a)*a + 60*a^2 - 120)*cos((d*x + c)^(1/3)*b + a)/b^5 +
(((d*x + c)^(1/3)*b + a)^2*b^3*c - 2*((d*x + c)^(1/3)*b + a)*a*b^3*c + a^2*
b^3*c - ((d*x + c)^(1/3)*b + a)^5 + 5*((d*x + c)^(1/3)*b + a)^4*a - 10*((d*
x + c)^(1/3)*b + a)^3*a^2 + 10*((d*x + c)^(1/3)*b + a)^2*a^3 - 5*((d*x + c)
^(1/3)*b + a)*a^4 + a^5 - 2*b^3*c + 20*((d*x + c)^(1/3)*b + a)^3 - 60*((d*x
 + c)^(1/3)*b + a)^2*a + 60*((d*x + c)^(1/3)*b + a)*a^2 - 20*a^3 - 120*(d*x
 + c)^(1/3)*b)*sin((d*x + c)^(1/3)*b + a)/b^5)/(b*d^2)
```

maple [B] time = 0.04, size = 655, normalized size = 2.51

$$\frac{-3c\left(\left(a+b\,(dx+c)^{\frac{1}{3}}\right)^2\sin\left(a+b\,(dx+c)^{\frac{1}{3}}\right)-2\sin\left(a+b\,(dx+c)^{\frac{1}{3}}\right)+2\cos\left(a+b\,(dx+c)^{\frac{1}{3}}\right)\left(a+b\,(dx+c)^{\frac{1}{3}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] 3/d^2/b^3*(-c*((a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-2*sin(a+b*(d*x+
c)^(1/3))+2*cos(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3)))+2*a*c*(cos(a+b*(d*x
+c)^(1/3))+(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-a^2*c*sin(a+b*(d*x+c
)^(1/3))+1/b^3*((a+b*(d*x+c)^(1/3))^5*sin(a+b*(d*x+c)^(1/3))+5*(a+b*(d*x+c)
^(1/3))^4*cos(a+b*(d*x+c)^(1/3))-20*(a+b*(d*x+c)^(1/3))^3*sin(a+b*(d*x+c)^(
1/3))-60*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1/3))+120*cos(a+b*(d*x+c)^(
1/3))+120*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-5/b^3*a*((a+b*(d*x+c)
^(1/3))^4*sin(a+b*(d*x+c)^(1/3))+4*(a+b*(d*x+c)^(1/3))^3*cos(a+b*(d*x+c)^(1
/3))-12*(a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))+24*sin(a+b*(d*x+c)^(1/
3))-24*cos(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3)))+10/b^3*a^2*((a+b*(d*x+c)
^(1/3))^3*sin(a+b*(d*x+c)^(1/3))+3*(a+b*(d*x+c)^(1/3))^2*cos(a+b*(d*x+c)^(1
/3))-6*cos(a+b*(d*x+c)^(1/3))-6*(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))
-10/b^3*a^3*((a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-2*sin(a+b*(d*x+c)
^(1/3))+2*cos(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3)))+5/b^3*a^4*(cos(a+b*(d
*x+c)^(1/3))+(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))-1/b^3*a^5*sin(a+b*
(d*x+c)^(1/3)))
```

maxima [B] time = 0.70, size = 523, normalized size = 2.00

$$-\frac{3\left(a^2c\sin\left((dx+c)^{\frac{1}{3}}b+a\right)-2\left(\left((dx+c)^{\frac{1}{3}}b+a\right)\sin\left((dx+c)^{\frac{1}{3}}b+a\right)+\cos\left((dx+c)^{\frac{1}{3}}b+a\right)\right)ac+\frac{a^5\sin\left((dx+c)^{\frac{1}{3}}b\right.}{b^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -3*(a^2*c*sin((d*x + c)^(1/3)*b + a) - 2*(((d*x + c)^(1/3)*b + a)*sin((d*x
+ c)^(1/3)*b + a) + cos((d*x + c)^(1/3)*b + a))*a*c + a^5*sin((d*x + c)^(1/
3)*b + a)/b^3 - 5*(((d*x + c)^(1/3)*b + a)*sin((d*x + c)^(1/3)*b + a) + cos
((d*x + c)^(1/3)*b + a))*a^4/b^3 + (2*((d*x + c)^(1/3)*b + a)*cos((d*x + c)
^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^2 - 2)*sin((d*x + c)^(1/3)*b + a))
*c + 10*(2*((d*x + c)^(1/3)*b + a)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)
^(1/3)*b + a)^2 - 2)*sin((d*x + c)^(1/3)*b + a))*a^3/b^3 - 10*(3*(((d*x + c
)^(1/3)*b + a)^2 - 2)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)
^3 - 6*(d*x + c)^(1/3)*b - 6*a)*sin((d*x + c)^(1/3)*b + a))*a^2/b^3 + 5*(4*
(((d*x + c)^(1/3)*b + a)^3 - 6*(d*x + c)^(1/3)*b - 6*a)*cos((d*x + c)^(1/3)
*b + a) + (((d*x + c)^(1/3)*b + a)^4 - 12*((d*x + c)^(1/3)*b + a)^2 + 24)*s
in((d*x + c)^(1/3)*b + a))*a/b^3 - (5*(((d*x + c)^(1/3)*b + a)^4 - 12*((d*x
 + c)^(1/3)*b + a)^2 + 24)*cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b
```

```
 + a)^5 - 20*((d*x + c)^(1/3)*b + a)^3 + 120*(d*x + c)^(1/3)*b + 120*a)*sin
((d*x + c)^(1/3)*b + a))/b^3)/(b^3*d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\cos\left(a + b\left(c + d\,x\right)^{1/3}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*(c + d*x)^(1/3)),x)
```

```
[Out] int(x*cos(a + b*(c + d*x)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(a + b\sqrt[3]{c + dx}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Integral(x*cos(a + b*(c + d*x)**(1/3)), x)
```

3.97 $\int \cos\left(a+b\sqrt[3]{c+dx}\right)\,dx$

Optimal. Leaf size=85

$$-\frac{6\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{6\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{3(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

```
[Out] 6*(d*x+c)^(1/3)*cos(a+b*(d*x+c)^(1/3))/b^2/d-6*sin(a+b*(d*x+c)^(1/3))/b^3/d
+3*(d*x+c)^(2/3)*sin(a+b*(d*x+c)^(1/3))/b/d
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3362, 3296, 2637}

$$-\frac{6\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}+\frac{6\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d}+\frac{3(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (6*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)])/(b^2*d) - (6*Sin[a + b*(c +
d*x)^(1/3)])/(b^3*d) + (3*(c + d*x)^(2/3)*Sin[a + b*(c + d*x)^(1/3)])/(b*d)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3362

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rubi steps

$$\int \cos\left(a+b\sqrt[3]{c+dx}\right)dx = \frac{3\,\text{Subst}\left(\int x^2\cos(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$
$$= \frac{3(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{6\,\text{Subst}\left(\int x\sin(a+bx)\,dx, x, \sqrt[3]{c+dx}\right)}{bd}$$
$$= \frac{6\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} + \frac{3(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd} - \frac{6\,\text{Subst}\left(\int \text{cos}\right.}{}$$
$$= \frac{6\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} - \frac{6\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} + \frac{3(c+dx)^{2/3}\sin\left(a+b\sqrt[3]{c\right.}}{bd}$$

Mathematica [A] time = 0.11, size = 65, normalized size = 0.76

$$\frac{3\left(b^2(c+dx)^{2/3}-2\right)\sin\left(a+b\sqrt[3]{c+dx}\right)+6b\sqrt[3]{c+dx}\,\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*(c + d*x)^(1/3)],x]
```

```
[Out] (6*b*(c + d*x)^(1/3)*Cos[a + b*(c + d*x)^(1/3)] + 3*(-2 + b^2*(c + d*x)^(2/
3))*Sin[a + b*(c + d*x)^(1/3)])/(b^3*d)
```

fricas [A] time = 1.09, size = 57, normalized size = 0.67

$$\frac{3\left(2\,(dx+c)^{\frac{1}{3}}b\cos\left((dx+c)^{\frac{1}{3}}b+a\right)+\left((dx+c)^{\frac{2}{3}}b^2-2\right)\sin\left((dx+c)^{\frac{1}{3}}b+a\right)\right)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*(2*(d*x + c)^(1/3)*b*cos((d*x + c)^(1/3)*b + a) + ((d*x + c)^(2/3)*b^2 -
2)*sin((d*x + c)^(1/3)*b + a))/(b^3*d)
```

giac [A] time = 0.43, size = 81, normalized size = 0.95

$$\frac{3\left(\frac{2\,(dx+c)^{\frac{1}{3}}\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{b}+\frac{\left(\left((dx+c)^{\frac{1}{3}}b+a\right)^2-2\left((dx+c)^{\frac{1}{3}}b+a\right)a+a^2-2\right)\sin\left((dx+c)^{\frac{1}{3}}b+a\right)}{b^2}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

```
[Out] 3*(2*(d*x + c)^(1/3)*cos((d*x + c)^(1/3)*b + a)/b + (((d*x + c)^(1/3)*b + a
)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2)*sin((d*x + c)^(1/3)*b + a)/b^2
)/(b*d)
```

maple [A] time = 0.03, size = 131, normalized size = 1.54

$$\frac{3\left(a+b\left(dx+c\right)^{\frac{1}{3}}\right)^{2}\sin\left(a+b\left(dx+c\right)^{\frac{1}{3}}\right)-6\sin\left(a+b\left(dx+c\right)^{\frac{1}{3}}\right)+6\cos\left(a+b\left(dx+c\right)^{\frac{1}{3}}\right)\left(a+b\left(dx+c\right)^{\frac{1}{3}}\right)-6}{d\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*(d*x+c)^(1/3)),x)
```

```
[Out] 3/d/b^3*((a+b*(d*x+c)^(1/3))^2*sin(a+b*(d*x+c)^(1/3))-2*sin(a+b*(d*x+c)^(1/
3))+2*cos(a+b*(d*x+c)^(1/3))*(a+b*(d*x+c)^(1/3))-2*a*(cos(a+b*(d*x+c)^(1/3)
)+(a+b*(d*x+c)^(1/3))*sin(a+b*(d*x+c)^(1/3)))+a^2*sin(a+b*(d*x+c)^(1/3)))
```

maxima [A] time = 0.44, size = 118, normalized size = 1.39

$$\frac{3\left(a^{2}\sin\left(\left(dx+c\right)^{\frac{1}{3}}b+a\right)-2\left(\left(\left(dx+c\right)^{\frac{1}{3}}b+a\right)\sin\left(\left(dx+c\right)^{\frac{1}{3}}b+a\right)+\cos\left(\left(dx+c\right)^{\frac{1}{3}}b+a\right)\right)a+2\left(\left(dx+c\right)^{\frac{1}{3}}b+\right.}{b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] 3*(a^2*sin((d*x + c)^(1/3)*b + a) - 2*(((d*x + c)^(1/3)*b + a)*sin((d*x + c
)^(1/3)*b + a) + cos((d*x + c)^(1/3)*b + a))*a + 2*((d*x + c)^(1/3)*b + a)*
cos((d*x + c)^(1/3)*b + a) + (((d*x + c)^(1/3)*b + a)^2 - 2)*sin((d*x + c)^
(1/3)*b + a))/(b^3*d)
```

mupad [B] time = 0.38, size = 68, normalized size = 0.80

$$\frac{6\,b\,\cos\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c+d\,x\right)}^{1/3}-6\,\sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)+3\,b^2\,\sin\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)\,{\left(c+d\,x\right)}^{2/3}}{b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*(c + d*x)^(1/3)),x)
```

```
[Out] (6*b*cos(a + b*(c + d*x)^(1/3))*(c + d*x)^(1/3) - 6*sin(a + b*(c + d*x)^(1/
3)) + 3*b^2*sin(a + b*(c + d*x)^(1/3))*(c + d*x)^(2/3))/(b^3*d)
```

sympy [A] time = 1.16, size = 94, normalized size = 1.11

$$\begin{cases} x\cos(a) & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ x\cos\left(a + b\sqrt[3]{c}\right) & \text{for } d = 0 \\ \frac{3(c+dx)^{\frac{2}{3}}\sin\left(a+b\sqrt[3]{c+dx}\right)}{bd} + \frac{6\sqrt[3]{c+dx}\cos\left(a+b\sqrt[3]{c+dx}\right)}{b^2d} - \frac{6\sin\left(a+b\sqrt[3]{c+dx}\right)}{b^3d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Piecewise((x*cos(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*cos(a + b*c**(1/
3)), Eq(d, 0)), (3*(c + d*x)**(2/3)*sin(a + b*(c + d*x)**(1/3))/(b*d) + 6*(
c + d*x)**(1/3)*cos(a + b*(c + d*x)**(1/3))/(b**2*d) - 6*sin(a + b*(c + d*x
)**(1/3))/(b**3*d), True))
```

3.98 $\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx$

Optimal. Leaf size=234

$$\cos\left(a+b\sqrt[3]{c}\right)\text{Ci}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)+\cos\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Ci}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)+\cos\left(a-\sqrt[3]{-1}\,b\sqrt[3]{c}\right)\text{Ci}$$

```
[Out] Ci(b*c^(1/3)-b*(d*x+c)^(1/3))*cos(a+b*c^(1/3))+Ci((-1)^(1/3)*b*c^(1/3)+b*(d
*x+c)^(1/3))*cos(a-(-1)^(1/3)*b*c^(1/3))+Ci((-1)^(2/3)*b*c^(1/3)-b*(d*x+c)^
(1/3))*cos(a+(-1)^(2/3)*b*c^(1/3))+Si(b*c^(1/3)-b*(d*x+c)^(1/3))*sin(a+b*c^
(1/3))-Si((-1)^(1/3)*b*c^(1/3)+b*(d*x+c)^(1/3))*sin(a-(-1)^(1/3)*b*c^(1/3))
+Si((-1)^(2/3)*b*c^(1/3)-b*(d*x+c)^(1/3))*sin(a+(-1)^(2/3)*b*c^(1/3))
```

Rubi [A] time = 0.47, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3432, 3303, 3299, 3302}

$$\cos\left(a+b\sqrt[3]{c}\right)\text{CosIntegral}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)+\cos\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{CosIntegral}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)+\text{co}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*(c + d*x)^(1/3)]/x,x]
```

```
[Out] Cos[a + b*c^(1/3)]*CosIntegral[b*c^(1/3) - b*(c + d*x)^(1/3)] + Cos[a + (-1
)^(2/3)*b*c^(1/3)]*CosIntegral[(-1)^(2/3)*b*c^(1/3) - b*(c + d*x)^(1/3)] +
Cos[a - (-1)^(1/3)*b*c^(1/3)]*CosIntegral[(-1)^(1/3)*b*c^(1/3) + b*(c + d*x
)^(1/3)] + Sin[a + b*c^(1/3)]*SinIntegral[b*c^(1/3) - b*(c + d*x)^(1/3)] +
Sin[a + (-1)^(2/3)*b*c^(1/3)]*SinIntegral[(-1)^(2/3)*b*c^(1/3) - b*(c + d*x
)^(1/3)] - Sin[a - (-1)^(1/3)*b*c^(1/3)]*SinIntegral[(-1)^(1/3)*b*c^(1/3) +
 b*(c + d*x)^(1/3)]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx = \frac{3\,\text{Subst}\left(\int\left(-\frac{d\cos(a+bx)}{3\left(\sqrt[3]{c}-x\right)}-\frac{d\cos(a+bx)}{3\left(-\sqrt[3]{-1}\sqrt[3]{c}-x\right)}-\frac{d\cos(a+bx)}{3\left((-1)^{2/3}\sqrt[3]{c}-x\right)}\right)dx,x,\sqrt[3]{c+dx}\right)}{d}$$

$$= -\text{Subst}\left(\int\frac{\cos(a+bx)}{\sqrt[3]{c}-x}\,dx,x,\sqrt[3]{c+dx}\right)-\text{Subst}\left(\int\frac{\cos(a+bx)}{-\sqrt[3]{-1}\sqrt[3]{c}-x}\,dx,x,\sqrt[3]{c+dx}\right.$$

$$= -\left(\cos\left(a+b\sqrt[3]{c}\right)\text{Subst}\left(\int\frac{\cos\left(b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}-x}\,dx,x,\sqrt[3]{c+dx}\right)\right)-\cos\left(a-\sqrt[3]{-1}\,b\sqrt[3]{c}\right.$$

$$= \cos\left(a+b\sqrt[3]{c}\right)\text{Ci}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)+\cos\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Ci}\left((-1)^{2/3}b\sqrt[3]{c}-b\sqrt[3]{\right.$$

Mathematica [C] time = 0.43, size = 243, normalized size = 1.04

$$\frac{1}{2}\left(\text{RootSum}\left[c-\#1^3\&,-i\sin(\#1b+a)\text{Ci}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)+\cos(\#1b+a)\text{Ci}\left(b\left(\sqrt[3]{c+dx}-\#1\right)\right)-\sin(\#1b\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*(c + d*x)^(1/3)]/x,x]
```

```
[Out] (RootSum[c - #1^3 & , Cos[a + b*#1]*CosIntegral[b*((c + d*x)^(1/3) - #1)] -
 I*CosIntegral[b*((c + d*x)^(1/3) - #1)]*Sin[a + b*#1] - I*Cos[a + b*#1]*Si
nIntegral[b*((c + d*x)^(1/3) - #1)] - Sin[a + b*#1]*SinIntegral[b*((c + d*x
)^(1/3) - #1)] & ] + RootSum[c - #1^3 & , Cos[a + b*#1]*CosIntegral[b*((c +
 d*x)^(1/3) - #1)] + I*CosIntegral[b*((c + d*x)^(1/3) - #1)]*Sin[a + b*#1]
+ I*Cos[a + b*#1]*SinIntegral[b*((c + d*x)^(1/3) - #1)] - Sin[a + b*#1]*Sin
Integral[b*((c + d*x)^(1/3) - #1)] & ])/2
```

fricas [C] time = 1.09, size = 287, normalized size = 1.23

$$\frac{1}{2}\,\mathrm{Ei}\left(i\,(dx+c)^{\frac{1}{3}}b+\frac{1}{2}\,\left(i\,b^3c\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}\,-1\right)\right)e^{\left(\frac{1}{2}\,\left(i\,b^3c\right)^{\frac{1}{3}}\left(i\,\sqrt{3}+1\right)+i\,a\right)}+\frac{1}{2}\,\mathrm{Ei}\left(-i\,(dx+c)^{\frac{1}{3}}b+\frac{1}{2}\,\left(-i\,b^3c\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}\,-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/3))/x,x, algorithm="fricas")
```

```
[Out] 1/2*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*b^3*c)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(
I*b^3*c)^(1/3)*(I*sqrt(3) + 1) + I*a) + 1/2*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(
-I*b^3*c)^(1/3)*(-I*sqrt(3) - 1))*e^(1/2*(-I*b^3*c)^(1/3)*(I*sqrt(3) + 1) -
 I*a) + 1/2*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*b^3*c)^(1/3)*(I*sqrt(3) - 1))*e
^(1/2*(I*b^3*c)^(1/3)*(-I*sqrt(3) + 1) + I*a) + 1/2*Ei(-I*(d*x + c)^(1/3)*b
 + 1/2*(-I*b^3*c)^(1/3)*(I*sqrt(3) - 1))*e^(1/2*(-I*b^3*c)^(1/3)*(-I*sqrt(3
) + 1) - I*a) + 1/2*Ei(I*(d*x + c)^(1/3)*b + (I*b^3*c)^(1/3))*e^(I*a - (I*b
^3*c)^(1/3)) + 1/2*Ei(-I*(d*x + c)^(1/3)*b + (-I*b^3*c)^(1/3))*e^(-I*a - (-
I*b^3*c)^(1/3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/3))/x,x, algorithm="giac")
```

```
[Out] integrate(cos((d*x + c)^(1/3)*b + a)/x, x)
```

maple [C] time = 0.07, size = 279, normalized size = 1.19

$$\frac{b^3\left(\sum_{\_R1=\mathrm{RootOf}\left(-c\,b^3+\_Z^3-3a\,\_Z^2+3a^2\,\_Z-a^3\right)}\frac{\_R1^2\left(\mathrm{Si}\left(-b(dx+c)^{\frac{1}{3}}+\_R1-a\right)\sin(\_R1)+\mathrm{Ci}\left(b(dx+c)^{\frac{1}{3}}-\_R1+a\right)\cos(\_R1)\right)}{\_R1^2-2\,\_R1a+a^2}\right)-2b^3a\left(\sum_{\_R1=\mathrm{RootC}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*(d*x+c)^(1/3))/x,x)
```

```
[Out] 3/b^3*(1/3*b^3*sum(_R1^2/(_R1^2-2*_R1*a+a^2)*(Si(-b*(d*x+c)^(1/3)+_R1-a)*si
n(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*cos(_R1)),_R1=RootOf(-b^3*c+_Z^3-3*_Z^2*a+
3*_Z*a^2-a^3))-2/3*b^3*a*sum(_R1/(_R1^2-2*_R1*a+a^2)*(Si(-b*(d*x+c)^(1/3)+_
R1-a)*sin(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*cos(_R1)),_R1=RootOf(-b^3*c+_Z^3-3
```

```
*_Z^2*a+3*_Z*a^2-a^3))+1/3*a^2*b^3*sum(1/(_R1^2-2*_R1*a+a^2)*(Si(-b*(d*x+c)
^(1/3)+_R1-a)*sin(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*cos(_R1)),_R1=RootOf(-b^3*
c+_Z^3-3*_Z^2*a+3*_Z*a^2-a^3)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/3))/x,x, algorithm="maxima")
```

```
[Out] integrate(cos((d*x + c)^(1/3)*b + a)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,(c+d\,x)^{1/3}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*(c + d*x)^(1/3))/x,x)
```

```
[Out] int(cos(a + b*(c + d*x)^(1/3))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)**(1/3))/x,x)
```

```
[Out] Integral(cos(a + b*(c + d*x)**(1/3))/x, x)
```

3.99 $\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx$

Optimal. Leaf size=332

$$-\frac{bd\sin\left(a+b\sqrt[3]{c}\right)\text{Ci}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}}+\frac{\sqrt[3]{-1}\,bd\sin\left(a-\sqrt[3]{-1}\,b\sqrt[3]{c}\right)\text{Ci}\left(\sqrt[3]{-1}\,\sqrt[3]{c}\,b+\sqrt[3]{c+dx}\,b\right)}{3c^{2/3}}-\frac{(-1)^{2/3}bd\sin}{}$$

```
[Out] -cos(a+b*(d*x+c)^(1/3))/x+1/3*b*d*cos(a+b*c^(1/3))*Si(b*c^(1/3)-b*(d*x+c)^(
1/3))/c^(2/3)+1/3*(-1)^(2/3)*b*d*cos(a+(-1)^(2/3)*b*c^(1/3))*Si((-1)^(2/3)*
b*c^(1/3)-b*(d*x+c)^(1/3))/c^(2/3)+1/3*(-1)^(1/3)*b*d*cos(a-(-1)^(1/3)*b*c^
(1/3))*Si((-1)^(1/3)*b*c^(1/3)+b*(d*x+c)^(1/3))/c^(2/3)-1/3*b*d*Ci(b*c^(1/3
)-b*(d*x+c)^(1/3))*sin(a+b*c^(1/3))/c^(2/3)+1/3*(-1)^(1/3)*b*d*Ci((-1)^(1/3
)*b*c^(1/3)+b*(d*x+c)^(1/3))*sin(a-(-1)^(1/3)*b*c^(1/3))/c^(2/3)-1/3*(-1)^(
2/3)*b*d*Ci((-1)^(2/3)*b*c^(1/3)-b*(d*x+c)^(1/3))*sin(a+(-1)^(2/3)*b*c^(1/3
))/c^(2/3)
```

Rubi [A] time = 0.74, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3432, 3342, 3333, 3303, 3299, 3302}

$$-\frac{bd\sin\left(a+b\sqrt[3]{c}\right)\text{CosIntegral}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)}{3c^{2/3}}+\frac{\sqrt[3]{-1}\,bd\sin\left(a-\sqrt[3]{-1}\,b\sqrt[3]{c}\right)\text{CosIntegral}\left(b\sqrt[3]{c+dx}+\sqrt[3]{-1}\right.}{3c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*(c + d*x)^(1/3)]/x^2,x]
```

```
[Out] -(Cos[a + b*(c + d*x)^(1/3)]/x) - (b*d*CosIntegral[b*c^(1/3) - b*(c + d*x)^
(1/3)]*Sin[a + b*c^(1/3)])/(3*c^(2/3)) + ((-1)^(1/3)*b*d*CosIntegral[(-1)^(
1/3)*b*c^(1/3) + b*(c + d*x)^(1/3)]*Sin[a - (-1)^(1/3)*b*c^(1/3)])/(3*c^(2/
3)) - ((-1)^(2/3)*b*d*CosIntegral[(-1)^(2/3)*b*c^(1/3) - b*(c + d*x)^(1/3)]
*Sin[a + (-1)^(2/3)*b*c^(1/3)])/(3*c^(2/3)) + (b*d*Cos[a + b*c^(1/3)]*SinIn
tegral[b*c^(1/3) - b*(c + d*x)^(1/3)])/(3*c^(2/3)) + ((-1)^(2/3)*b*d*Cos[a
+ (-1)^(2/3)*b*c^(1/3)]*SinIntegral[(-1)^(2/3)*b*c^(1/3) - b*(c + d*x)^(1/3
)])/(3*c^(2/3)) + ((-1)^(1/3)*b*d*Cos[a - (-1)^(1/3)*b*c^(1/3)]*SinIntegral
[(-1)^(1/3)*b*c^(1/3) + b*(c + d*x)^(1/3)])/(3*c^(2/3))
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3333

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3342

```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_
), x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Cos[c + d*x])/(b*n*(p + 1)),
x] + Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sin[c + d*x], x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, -1] && EqQ[m, n - 1] && (I
ntegerQ[n] || GtQ[e, 0])
```

Rule 3432

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rubi steps

$$\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx = \frac{3\,\text{Subst}\left(\int \frac{x^2\cos(a+bx)}{\left(-\frac{c}{d}+\frac{x^3}{d}\right)^2}\,dx, x, \sqrt[3]{c+dx}\right)}{d}$$

$$= -\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x} - b\,\text{Subst}\left(\int \frac{\sin(a+bx)}{-\frac{c}{d}+\frac{x^3}{d}}\,dx, x, \sqrt[3]{c+dx}\right)$$

$$= -\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x} - b\,\text{Subst}\left(\int\left(-\frac{d\sin(a+bx)}{3c^{2/3}\left(\sqrt[3]{c}-x\right)} - \frac{d\sin(a+bx)}{3c^{2/3}\left(\sqrt[3]{c}+\sqrt[3]{-1}\,x\right)} - \frac{d}{3c^{2/3}}\right.\right.$$

$$= -\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x} + \frac{(bd)\,\text{Subst}\left(\int \frac{\sin(a+bx)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} + \frac{(bd)\,\text{Subst}\left(\int \frac{\sin(}{\sqrt[3]{c}+}\right.}{3}$$

$$= -\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x} - \frac{\left(bd\cos\left(a+b\sqrt[3]{c}\right)\right)\text{Subst}\left(\int \frac{\sin\left(b\sqrt[3]{c}-bx\right)}{\sqrt[3]{c}-x}\,dx, x, \sqrt[3]{c+dx}\right)}{3c^{2/3}} +$$

$$= -\frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x} - \frac{bd\text{Ci}\left(b\sqrt[3]{c}-b\sqrt[3]{c+dx}\right)\sin\left(a+b\sqrt[3]{c}\right)}{3c^{2/3}} + \frac{\sqrt[3]{-1}\,bd\text{Ci}\left(\sqrt[3]{-1}\,b\right.}{}$$

Mathematica [C] time = 0.60, size = 138, normalized size = 0.42

$$-\frac{1}{6}ibd\text{RootSum}\left[c-\#1^3\&, \frac{e^{-i\#1b-ia}\text{Ei}\left(-ib\left(\sqrt[3]{c+dx}-\#1\right)\right)}{\#1^2}\&\right] + \frac{1}{6}ibd\text{RootSum}\left[c-\#1^3\&, \frac{e^{i\#1b+ia}\text{Ei}\left(ib\left(\sqrt[3]{c+dx}\right.\right.}{\#1^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*(c + d*x)^(1/3)]/x^2,x]
```

```
[Out] -(Cos[a + b*(c + d*x)^(1/3)]/x) - (I/6)*b*d*RootSum[c - #1^3 & , (E^((-I)*a
 - I*b*#1)*ExpIntegralEi[(-I)*b*((c + d*x)^(1/3) - #1)])/#1^2 & ] + (I/6)*b
*d*RootSum[c - #1^3 & , (E^(I*a + I*b*#1)*ExpIntegralEi[I*b*((c + d*x)^(1/3
) - #1)])/#1^2 & ]
```

fricas [C] time = 0.79, size = 406, normalized size = 1.22

$$-\frac{2\left(ib^3c\right)^{\frac{1}{3}}dx\text{Ei}\left(i(dx+c)^{\frac{1}{3}}b+\left(ib^3c\right)^{\frac{1}{3}}\right)e^{\left(ia-\left(ib^3c\right)^{\frac{1}{3}}\right)} + 2\left(-ib^3c\right)^{\frac{1}{3}}dx\text{Ei}\left(-i(dx+c)^{\frac{1}{3}}b+\left(-ib^3c\right)^{\frac{1}{3}}\right)e^{\left(-ia-\left(-ib^3c\right)^{\frac{1}{3}}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="fricas")
```

```
[Out] -1/12*(2*(I*b^3*c)^(1/3)*d*x*Ei(I*(d*x + c)^(1/3)*b + (I*b^3*c)^(1/3))*e^(I
*a - (I*b^3*c)^(1/3)) + 2*(-I*b^3*c)^(1/3)*d*x*Ei(-I*(d*x + c)^(1/3)*b + (-
I*b^3*c)^(1/3))*e^(-I*a - (-I*b^3*c)^(1/3)) - (I*b^3*c)^(1/3)*(I*sqrt(3)*d*
x + d*x)*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*b^3*c)^(1/3)*(-I*sqrt(3) - 1))*e^(
1/2*(I*b^3*c)^(1/3)*(I*sqrt(3) + 1) + I*a) - (-I*b^3*c)^(1/3)*(I*sqrt(3)*d*
x + d*x)*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*b^3*c)^(1/3)*(-I*sqrt(3) - 1))*e
^(1/2*(-I*b^3*c)^(1/3)*(I*sqrt(3) + 1) - I*a) - (I*b^3*c)^(1/3)*(-I*sqrt(3)
*d*x + d*x)*Ei(I*(d*x + c)^(1/3)*b + 1/2*(I*b^3*c)^(1/3)*(I*sqrt(3) - 1))*e
^(1/2*(I*b^3*c)^(1/3)*(-I*sqrt(3) + 1) + I*a) - (-I*b^3*c)^(1/3)*(-I*sqrt(3
)*d*x + d*x)*Ei(-I*(d*x + c)^(1/3)*b + 1/2*(-I*b^3*c)^(1/3)*(I*sqrt(3) - 1)
)*e^(1/2*(-I*b^3*c)^(1/3)*(-I*sqrt(3) + 1) - I*a) + 12*c*cos((d*x + c)^(1/3
)*b + a))/(c*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="giac")
```

```
[Out] integrate(cos((d*x + c)^(1/3)*b + a)/x^2, x)
```

maple [C] time = 0.10, size = 933, normalized size = 2.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+b*(d*x+c)^(1/3))/x^2,x)
```

```
[Out] 3*d/b^3*(cos(a+b*(d*x+c)^(1/3))*(-2/3*a*b^3/c*(a+b*(d*x+c)^(1/3))^2+a^2*b^3
/c*(a+b*(d*x+c)^(1/3))-1/3*b^3*(b^3*c+a^3)/c)/(-c*b^3+(a+b*(d*x+c)^(1/3))^3
-3*a*(a+b*(d*x+c)^(1/3))^2+3*(a+b*(d*x+c)^(1/3))*a^2-a^3)-2/9*a*b^3/c*sum(_
R1/(_R1^2-2*_R1*a+a^2)*(Si(-b*(d*x+c)^(1/3)+_R1-a)*sin(_R1)+Ci(b*(d*x+c)^(1
/3)-_R1+a)*cos(_R1)),_R1=RootOf(-b^3*c+_Z^3-3*_Z^2*a+3*_Z*a^2-a^3))-1/9*b^3
/c*sum((b^3*c+2*_RR1^2*a-3*_RR1*a^2+a^3)/(_RR1^2-2*_RR1*a+a^2)*(-Si(-b*(d*x
+c)^(1/3)+_RR1-a)*cos(_RR1)+Ci(b*(d*x+c)^(1/3)-_RR1+a)*sin(_RR1)),_RR1=Root
Of(-b^3*c+_Z^3-3*_Z^2*a+3*_Z*a^2-a^3))+cos(a+b*(d*x+c)^(1/3))*(2/3*a*b^3/c*
(a+b*(d*x+c)^(1/3))^2-2/3*a^2*b^3/c*(a+b*(d*x+c)^(1/3)))/(-c*b^3+(a+b*(d*x+
c)^(1/3))^3-3*a*(a+b*(d*x+c)^(1/3))^2+3*(a+b*(d*x+c)^(1/3))*a^2-a^3)+2/9*a*
```

```
b^3/c*sum((_R1+a)/(_R1^2-2*_R1*a+a^2)*(Si(-b*(d*x+c)^(1/3)+_R1-a)*sin(_R1)+
Ci(b*(d*x+c)^(1/3)-_R1+a)*cos(_R1)),_R1=RootOf(-b^3*c+_Z^3-3*_Z^2*a+3*_Z*a^
2-a^3))+2/9*a*b^3/c*sum(_RR1/(_RR1-a)*(-Si(-b*(d*x+c)^(1/3)+_RR1-a)*cos(_RR
1)+Ci(b*(d*x+c)^(1/3)-_RR1+a)*sin(_RR1)),_RR1=RootOf(-b^3*c+_Z^3-3*_Z^2*a+3
*_Z*a^2-a^3))+a^2*b^6*(cos(a+b*(d*x+c)^(1/3))*(-1/3/c/b^3*(a+b*(d*x+c)^(1/3
))+1/3*a/c/b^3)/(-c*b^3+(a+b*(d*x+c)^(1/3))^3-3*a*(a+b*(d*x+c)^(1/3))^2+3*(
a+b*(d*x+c)^(1/3))*a^2-a^3)-2/9/c/b^3*sum(1/(_R1^2-2*_R1*a+a^2)*(Si(-b*(d*x
+c)^(1/3)+_R1-a)*sin(_R1)+Ci(b*(d*x+c)^(1/3)-_R1+a)*cos(_R1)),_R1=RootOf(-b
^3*c+_Z^3-3*_Z^2*a+3*_Z*a^2-a^3))-1/9/c/b^3*sum(1/(_RR1-a)*(-Si(-b*(d*x+c)^
(1/3)+_RR1-a)*cos(_RR1)+Ci(b*(d*x+c)^(1/3)-_RR1+a)*sin(_RR1)),_RR1=RootOf(-
b^3*c+_Z^3-3*_Z^2*a+3*_Z*a^2-a^3))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left((dx+c)^{\frac{1}{3}}b+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(cos((d*x + c)^(1/3)*b + a)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,{\left(c+d\,x\right)}^{1/3}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*(c + d*x)^(1/3))/x^2,x)
```

```
[Out] int(cos(a + b*(c + d*x)^(1/3))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(a+b\sqrt[3]{c+dx}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+b*(d*x+c)**(1/3))/x**2,x)
```

```
[Out] Integral(cos(a + b*(c + d*x)**(1/3))/x**2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
   ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```